INTRA-BEAM AND TOUSCHEK SCATTERING COMPUTATIONS FOR BEAM WITH NON-GAUSSIAN LONGITUDINAL DISTRIBUTIONS*

A. Xiao[†], M. Borland, ANL, Argonne, IL 60439, USA

Abstract

Both intra-beam scattering (IBS) and the Touschek effect become prominent for multi-bend-achromat- (MBA-) based ultra-low-emittance storage rings. To mitigate the transverse emittance degradation and obtain a reasonably long beam lifetime, a higher harmonic rf cavity (HHC) is often proposed to lengthen the bunch. The use of such a cavity results in a non-gaussian longitudinal distribution. However, common methods for computing IBS and Touschek scattering assume Gaussian distributions. Modifications have been made to several simulation codes that are part of the `elegant` [1] toolkit to allow these computations for arbitrary longitudinal distributions. After describing these modifications, we review the results of detailed simulations for the proposed hybrid seven-bend-achromat (H7BA) upgrade lattice [2] for the Advanced Photon Source.

INTRODUCTION

The natural emittance of next-generation storage ring light sources will be reduced by more than an order of magnitude compared to present rings. As a consequence, the Coulomb scattering rate among particles, which is inversely proportional to the bunch volume, increases rapidly. The small angle multiple scattering process (IBS effect) significantly increases beam emittance at the operating current, and limits benefits obtained from a ultra-low emittance machine. The large angle single scattering process (Touschek effect) puts more particles outside the rf acceptance, resulting in a much shorter beam lifetime.

To mitigate these problems, an HHC is often proposed to lengthen the bunch. Due to the distortion of the rf potential well, the particles are no longer Gaussian-distributed longitudinally. To accurately simulate beam scattering effects, all of our original beam scattering simulation tools—developed based on the assumption of Gaussian distributions—have been updated to allow arbitrary distributions. This paper first describes the technique used to deal with non-Gaussian distributed beam, then gives calculation results from the original and new methods for the same Gaussian distributed bunch to verify the code. The results of detailed simulations for the H7BA lattice with HHC are presented at the end.

TOUSCHEK SCATTERING

The Touschek scattering rate R is given by Eq. 28 in [3] and is an integral of the local scattering rate over all beam

coordinates:

$$R = 2\beta c \int P_1 P_2 \sigma \chi dV, \qquad (1)$$

where βc is the particle's velocity, σ is the Møller cross-section transformed into the laboratory system, χ is the half angle between the scattered particles' momenta vectors $\vec{p_1}$ and $\vec{p_2}$, $P_{1,2}$ is bunch density function, and dV is given by

$$dV = d\Delta s_1 dx_{\beta 1} dy_{\beta 1} d\Delta p_{s1} d\Delta p_{s2} dx'_{\beta 1} dx'_{\beta 2} dy'_{\beta 1} dy'_{\beta 2}. \quad (2)$$

The density function P is well known for a Gaussian distributed beam, giving an analytical expression for R [3]. For a non-Gaussian beam, the integral is less straightforward. One option is the Monte-Carlo integration method [4], which is already coded in `elegant`. However, this method is computationally-demanding, and we elected to also enhance the faster-running `touschekLifetime` program. The quantity $\sigma \chi$ is dependent on the momenta and thus on the transverse coordinates owing to $x - p_x$ and $y - p_y$ correlations. In general, there is less correlation between $s - p_s$ in a storage ring, thus the integral over ds can be done separately, i.e., the beam can be sliced longitudinally.

For each longitudinal slice, particles are Gaussian distributed in all dimensions except in s, which is approximately uniformly distributed. One can show that the scattering rate R_U for a uniform longitudinal distribution of length L is related to the rate R_G for a Gaussian distrubtion with $\sigma_s = L$ by

$$R_U = 2\sqrt{\pi} R_G. \qquad (3)$$

Since R_G is already computed by `touschekLifetime`, relation 3 is used to calculate the scattering rate from each slice $R_{U,m}$. The total scattering rate will be the sum of $R_{U,m}$ over all slices m, giving a lifetime

$$\frac{1}{T_l} = \left\langle \frac{R}{N_0} \right\rangle = \left\langle \frac{\sum_m R_{U,m}}{\sum_m N_m} \right\rangle, \qquad (4)$$

where N_m is the population of slice m.

To verify the code, the total local scattering rate of a Gaussian distributed bunch was calculated with slicing (21 slices) and without slicing. Results shown in Fig.1 agree very well with one another.

INTRA-BEAM SCATTERING

A similar slicing technique can be used for IBS calculations. Since IBS is a multiple scattering process and the beam emittance is diluted over time, the equilibrium beam emittance results from the interplay of many factors, such as synchrotron radiation damping, quantum excitation, and beam optics. The standalone `ibsEmittance` tool is
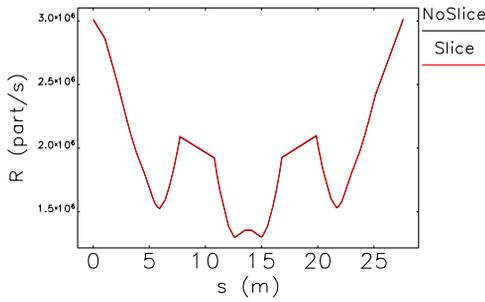
Figure 1: Calculated total local Touschek scattering rate for a Gaussian distributed bunch with and without slicing.

thus much harder to modify to accommodate non-Gaussian bunches. Using the IBSCATTER element in elegant, which uses particle tracking, is the most expedient way to proceed.

Using Eq. 30 and 31 in [5], the change per unit time of the mean values of ε_k (transverse emittance) and H (longitudinal emittance), averaged over all particles, is given by,

$$\frac{d}{dt}\langle\varepsilon_k\rangle = \left\langle \frac{2}{N\gamma^2}\int \bar{v}P_1 P_2 \int_0^{2\pi}\int_{\bar{\psi}_{min}}^{\pi} \delta\varepsilon_k d\bar{\sigma}dV \right\rangle \quad (5)$$

$$\frac{d}{dt}\langle H\rangle = \left\langle \frac{2}{N\gamma^2}\int \bar{v}P_1 P_2 \int_0^{2\pi}\int_{\bar{\psi}_{min}}^{\pi} \delta H d\bar{\sigma}dV \right\rangle \quad (6)$$

where, γ is the relativistic factor, $\bar{v}$ is the particle velocity, $d\bar{\sigma}$ is the Møller cross-section in the center of mass system, and $\bar{\psi}_{min}$ is the minimum scattering angle determined by the maximum impact parameter. P_1, P_2 and dV have the same definitions as Eq. 1. As above, the integration over s can be done separately, so the same relationship holds for the IBS emittance growth rates between Gaussian (τ_G) and Uniformly (τ_U) distributed bunches,

$$\frac{1}{\tau_U} = 2\sqrt{\pi}\frac{1}{\tau_G} \quad (7)$$

In elegant, non-Gaussian distributed bunch particles have been tracked through a beam line for multiple turns. Each time the particles reach an IBSCATTER element, it is sliced longitudinally and each slice is treated as Gaussian-distributed in all dimensions except s, which is approximated as uniformly distributed. The accumulated emittance dilutions between two successive IBSCATTER elements are calculated using the BM formula [6] (for a Gaussian bunch) and the relationship list in Eq. 7 is applied. Particle coordinates for that slice are adjusted to reflect these dilutions, then the next slice is processed. Equilibrium emittances should be obtained if the tracking also includes synchrotron radiation (SR) effects and enough passes are tracked.

To check the code validity, we have computed IBS scattering effects from ibsEmittance (using the BM formula without tracking) and elegant tracking for the same Gaussian distributed bunch under the following conditions, assuming the present APS operating optics and bunch charge of 400, 2000, 3600 nC (the charge is deliberately made very large to amplify the effects):

- With SR effects, but without slicing. See Fig.2.

- With SR effects, but using 11 longituidinal slices in tracking. See Fig.3.

All results show very good agreement, thus verifying the implementation of the slicing-based method in elegant. Note that the new method is also available in the parallel version [7], Pelegant.
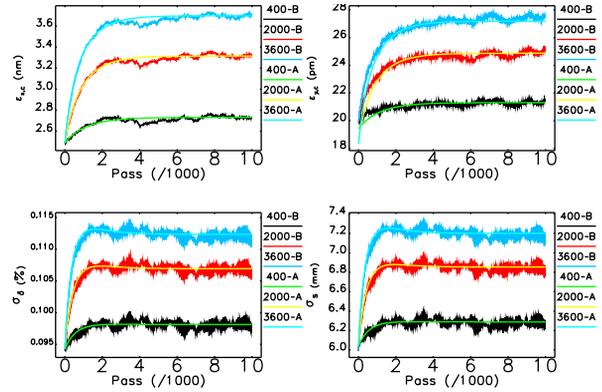

Figure 2: IBS-induced bunch parameter evolution for different bunch charge (400/2000/3600 nC): A - from ibsEmittance; B - from elegant tracking. Condition: With SR effects, no slicing.
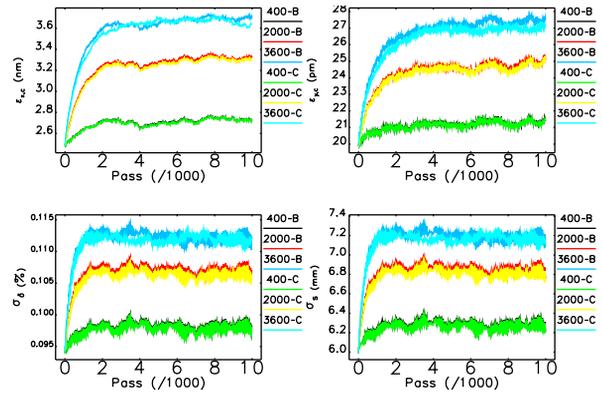

Figure 3: IBS caused bunch parameter evolution at different bunch charge (400/2000/3600 nC): B - no longitudinal slicing; C - with longitudinal slicing (11 slices). Condition: With SR effects.

APPLICATION TO APSU H7BA DESIGN

The APS upgrade hybrid seven-bend achromat lattice [2] produces a natural emittance of $\epsilon_0 = 67pm$, leading to the potential for short lifetimes and significant IBS. An HHC is planned to lengthen the bunch and will lengthen it considerably.

The default fill patterns for the upgrade lattice consist of 48 or 324 uniformly-spaced bunches with a total current of 200 mA. Examples of simulated bunch profiles can be seen

in [8]. Since the longitudinal impedance [9] was included in the tracking, the energy spread is inflated by the microwave instability, but still has a Gaussian character. Thus, our assumptions for the slice-based Touschek lifetime calculation are valid. The Touschek lifetime was computed using such profiles and the upgraded version of `touschekLifetime`.

As part of the HHC tracking simulations, the beam phase space for a representative bunch was sampled at 50-turn intervals over 2000 turns. Slice analysis was performed for each of the 40 samples using 20 slices, giving the 40 sets of slice population and energy spread. Since IBS tracking as described above has yet to be undertaken for this lattice, the emittances of each slice were assumed to be identical and given by the IBS-inflated emittance computed with `ibsEmittance`, starting from coupled emittances determined from lattice simulations, along with the rms bunch duration determined by tracking.

The lattice simulations make use of 100 error ensembles [10], giving 100 sets of values for the coupled emittances and local momentum acceptance. Hence, for each HHC tracking run, 4000 lifetime values are computed (40 beam samples times 100 error ensembles). Averaging over the beam samples gives 100 predicted lifetime values.

The bunch duration and profile depend not only on the charge per bunch, but also on the detuning Δf_h of the HHC. As the passive cavity is tuned closer to resonance ($\Delta f_h \to 0$), the induced voltage increases and the effect of the HHC on the distribution increases. Hence, the HHC tracking was performed with various values of Δf_h, to determine trends in the lifetime. Figure 4 shows the cumulative distribution functions for the lifetime values from the 100 error ensembles as a function of Δf_h, for $\kappa = \epsilon_y/\epsilon_x \approx 1$. From Fig. 5, which shows a reduced set of data, it is clear that the optimum detuning for lifetime is about 12.5 kHz, which correspond to a slightly bifurcated bunch. Figure 6 shows similar data for a 48-bunch fill. The improvement is not as great and the total lifetime is still quite short, albeit workable.
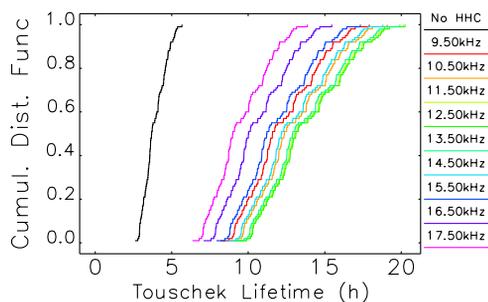

Figure 4: Cumulative distribution of lifetime values over 100 error ensembles, as the harmonic cavity detuning is varied, for 200 mA in 324 bunches.

CONCLUSIONS

We have described newly-implemented methods for computing Touschek lifetime and intrabeam scattering using
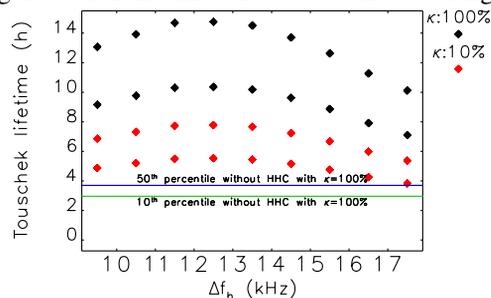

Figure 5: 10th- and 50th-percentile Touschek lifetimes for 324 bunches with different emittance ratios. The upper set of points within any color shows the 50th percentile, while the lower set of points shows the 10th percentile.
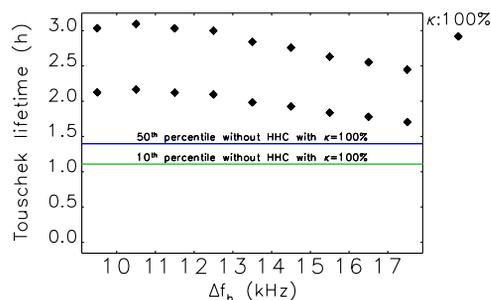

Figure 6: 10th- and 50th-percentile Touschek lifetimes for 48 bunches. The upper set of shows the 50th percentile, while the lower set of points shows the 10th percentile.

slice-based techniques that allow higher-fidelity computations for cases with non-Gaussian bunches. The Touschek lifetime calculation was added as a feature of the existing program `touschekLifetime`. The IBS calculation was added as a feature of tracking-based IBS simulation provided by `elegant` and `Pelegant`. The slice-based Touschek lifetime calculation was applied to the APS MBA lattice to find the lifetime for many error ensembles as a function of the detuning of the passive higher-harmonic cavity.

Computations used the Blues cluster at Argonne's Laboratory Computing Resources Center.

REFERENCES

[1] M. Borland. ANL/APS LS-287, Advanced Photon Source (2000).

[2] M. Borland. *Proc. IPAC 2015*. TUPJE063, to be published.

[3] A. Piwinski. 98-179, DESY (1998).

[4] A. Xiao et al. *Phys Rev ST Accel Beams*, 13:074201 (2010).

[5] A. Piwinski. DESY 90-113, DESY (1990).

[6] J. D. Bjorken et al. *Part Accel*, 13:115 (1983).

[7] Y. Wang et al. *Proc. of PAC 2007*, 3444–3446 (2007).

[8] M. Borland et al. *MOPMA007, IPAC 2015, these proc.*

[9] R. R. Lindberg et al. *TUPJE078, IPAC 2015, these proc.*

[10] M. Borland et al. *TUPJE063, IPAC 2015, these proc.*