# OPTIMIZING SLS-2 NONLINEARITIES USING A MULTI-OBJECTIVE GENETIC OPTIMIZER

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# Abstract

An upgrade to the SLS is currently under development. This upgrade will likely utilize the same hall and same machine circumference, 288 m, of the SLS. Achieving a sufficiently low emittance with such a small circumference requires tight focusing and low dispersion. These conditions make chromaticity correction difficult and minimization of 1st and 2nd order non-linear driving terms does not yield sufficient dynamic aperture and Touschek lifetime. In this proceeding, we discuss the multi-objective genetic optimization method implemented at SLS to aid the design of a chromaticity correction scheme for SLS-2.

# **INTRODUCTION**

An upgrade to the SLS, which we will refer to as SLS-2 in this paper, is under development [1]. The upgrade is envisioned as a complete replacement of the SLS storage ring while keeping the same hall, shielding, and booster. The beamline locations will be kept, but the beamlines themselves may be upgraded. The SLS-2 will utilize small vacuum chambers, as pioneered by MAX-IV, to obtain higher field strengths, and also longitudinal gradient bends and anti-bends [2] to reduce the horizontal emittance from 5.6 nm to approximately 130 pm. The new ring will maintain the 12 cell topology of the existing SLS. 3-fold periodic lattices, like the existing SLS, and also 12-fold periodic lattices have been considered.

The new lattice uses strong quadrupole fields which induce a large chromaticity. This, in combination with small dispersion, necessitates a scheme of strong chromatic sextupoles to correct the chromaticity. These strong sextupoles generate strong nonlinearities which must in turn be compensated using harmonic sextupoles and octupoles.

Layout constraints prevent zeroing resonant driving terms (RDTs) beyond 1st order (in sextupole strength). Therefore, the optimal map has some combination driving terms and tune shift terms of 2nd order and higher. In fact, the optimal map may even have non-zero 1st order terms. Weighting 1st, 2nd, and higher order RDTs to obtain the best map, and locating the global minimum, is not straightforward. This is especially so given that RDTs are only a heuristic for the actual objectives, which are acceptance and lifetime.

Iffetime. An application of perturbation theory to 1st and 2nd order terms as described in [3] yields only marginally acceptable dynamic aperture (DA) and beam lifetime. We find that the direct genetic optimizer described in this proceeding consistently does better. For example, the direct optimizer finds high order corrections to the tune footprint far away from the closed orbit.

Elements of the traditional approach are maintained in our genetic optimization scheme. Chromatic and harmonic sextupoles and octupole locations are set by hand taking into consideration  $\beta$ -functions and phase advances. The working point is positioned considering the locations of low order resonances such that the tune footprint does not cross any dangerous resonances.

Features of the optimization scheme described here are: 1) It does not use RDTs. 2) It requires only moderate computing resources, typically about 12 hours on 64 PC cores. 3) A robust constraint system. 4) It seems to reliably converge to the global minimum. 5) Does not require seeding.

### **SYSTEM**

The accelerator physics simulation is developed using the Bmad [4] subroutine library. The multi-objective genetic optimization scheme is the aPISA extension [5] to the PISA framework [6]. The sorting algorithm is aspea2, a version of the spea2 [7] sorting algorithm modified to support dominance constraints. The parallelization is implemented using Fortran COARRAYs, which are implemented as a high level layer on top of MPI. The computing resource is a Linux cluster running SGE consisting of distributed Intel Xeon compute nodes. The scheme is naturally loadbalancing and works fine in heterogeneous environments.

The results from the optimizer are portable to other codes, in particular OPA [8], for further analysis. Effort in this project includes understanding the usage and modeling differences of different calculation codes so as to keep the development process consistent and flexible.

# **OPTIMIZATION PROBLEM**

The goal of the optimization problem is to maximize injection acceptance and maximize the Touschek lifetime. Injection acceptance is maximized by maximizing the onenergy DA. The Touschek lifetime is maximized by maximizing the momentum aperture. However, the elementby-element momentum aperture is expensive to calculate. Therefore, instead of calculating the momentum aperture we calculate the off-momentum DA and constrain the chromatic tune footprint. As will be shown later, we find this is an effective and efficient proxy for maximizing the Touschek lifetime.

# **Objectives**

Three objectives are used by the optimizer. The objective function, depicted for  $N_{angle} = 7$  in Fig. 1, is calculated as

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$$\min f(x) = \frac{1}{N_{ang}} \sum_{N_{ang}} \begin{cases} \left(\frac{L_{LA} - L_{DA}}{L_{LA}}\right)^2, & \text{if } L_{DA} < L_{LA} \\ 0, & \text{otherwise} \end{cases}$$
(1)

The objectives consist of Eq. 1 evaluated on-energy, at  $\Delta E = -3\%$ , and at  $\Delta E = +3\%$ .



Figure 1: The DA is found by tracking, and found using a binary search along a ray. The linear aperture is obtained by projecting the beam chamber profile to the injection point using linearized optics. The linear aperture is generally  $\Delta E$ -dependent.

#### *Constraints*

The following constraints are applied:

1. Constrained variable space

(a) Horizontal and vertical corrected chromaticity

- 2. Dominance constraints
  - (a) Magnet strength
  - (b) Global nonlinear dispersion
  - (c) Chromatic tune footprint
- 3. Modified objective function
  - (a) Amplitude-dependent tune shift (ADTS)
  - (b) Linear aperture size

Chromaticity is constrained by restricting the optimizer to the  $(N_{sext} - 2)$ -dimensional subspace of sextupole strengths on which the horizontal and vertical chromaticity is at the desired value. Given a chromaticity-sextupole response matrix **A**, obtain the Monroe-Penrose pseudoinverse **A**<sub>p</sub> and assemble **B** = **I** - **A**<sub>p</sub>**A**, where **I** is the identity matrix. Obtain **Q**<sub>1</sub> from the thin-QR decomposition of **B**. **Q**<sub>1</sub>  $\in \mathbb{R}^{N_{sext} \times (N_{sext} - 2)}$ . From any  $\omega \in \mathbb{R}^{N_{sext} - 2}$ , **K**<sub>2</sub> calculated from

$$\mathbf{K}_2 = \mathbf{A}_p \, \boldsymbol{\chi} + \mathbf{Q}_1 \boldsymbol{\omega},\tag{2}$$

yields sextupole strengths with the chromaticity  $\chi = (\chi_x, \chi_y)^T$ . In other words,  $\mathbf{A}_p \chi$  is the least-squares solution, and  $\mathbf{Q}_1 \omega$  spans the space of sextupole strengths that leave the chromaticities unchanged.

The next three constraints are implemented as dominance constraints [5]. Sextupole strength is limited to 500 m<sup>-3</sup>. Global nonlinear dispersion is constrained as a limit on the difference between the off-momentum closed orbit and linear dispersive orbit. The chromatic tune footprint is found from the linearization of the optics about the off-momentum closed orbit. The footprint is constrained to the  $\frac{1}{2}$ -integer box.

The last two constraints are implemented by modifying the objective function. The ADTS is calculated by summing the element-by-element phase advance in normalized coordinates during DA tracking. If the particle per-turn averaged phase advance exceeds the ADTS constraint, then it is treated as lost. The linear aperture size constraint is a minimum bound on the size of the linear aperture. This is effectively a constraint on non-linear focusing at the injection point. If the minimum linear aperture size constraint is not met, then a perfectly bad objective value is returned. The minimum linear aperture size is typical set to 2 mm.

#### Variables

Variables are sextupole and octupole strengths. The optimizer operates directly on the harmonic sextupole and octupole strengths. The optimizer does not operate directly on the chromatic sextupoles, but rather  $\omega$  as in Eq. 2.

The magnets are set as families that are symmetric about the cells, and periodic with the Twiss optics.

### **APPLICATION TO SLS-2**

The genetic optimizer is applied to a periodicity 12 SLS-2 prototype. Figure 2 demonstrates the effectiveness of the chromatic and ADTS constraints. The resonance lines in Fig. 2a are of the format (p, q, r, n) where  $pQ_x + qQ_y + rQ_s = n$ .  $Q_x$  and  $Q_y$  are full, integer + fractional tunes. Resonance lines which are excluded due to periodicity are not shown.

Figure 3 compares the on- and off-momentum DA obtained from an application of perturbation theory and the genetic optimizer.

The perturbation theory result is obtained by applying a local numerical optimizer to minimize the *J*-weighted 1st and 2nd order (in sextupole strength) terms of the Hamiltonian. Found local minima are evaluated by tracking the DA. The process may be repeated by adjusting the weights and tunes.

The resulting momentum aperture is restored to approximately  $\pm 5\%$ , which is the RF-bucket height. The Touschek lifetime from the momentum aperture calculated using only linear optics is 3.69 h. For the lattice optimized using perturbation theory, the lifetime is 3.48 h. And for the lattice optimized with the genetic algorithm, the lifetime is 3.64 h.

### CONCLUSION

The multi-objective genetic optimizer described here finds sextupole and octupole strengths that constrain the

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Figure 2: Effectiveness of chromatic and amplitudedependent tune shift constraints. The tunes are constrained to the half-integer box. Point labels in (a) are  $\Delta E$  (%).

tune footprint and yield on- and off-momentum DA improvements over an application of traditional perturbation theory. Yet, the scheme observes many traditional design principles such as placement, symmetry, periodicity, and resonance avoidance. The optimizer requires only modest computing resources to converge in about 12 hours.

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Figure 3: DA results. Left are from the genetic optimizer. Right are from applying perturbation theory. Blue curves are linear aperture. (a) and (b) on-energy. (c) and (d) -3%. (e) and (f) +3%. (g) and (h) Horizontal DA vs.  $\Delta E$ .

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