

LONGITUDINAL STABILITY IN MULTI-HARMONIC ACCELERATING CAVITIES

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Abstract

Accelerating cavities that excite multiple modes at integer harmonics of the fundamental frequency can potentially be used to limit the effects of rf breakdown and pulsed surface heating at high accelerating gradients. Understanding the longitudinal stability and the acceptance of such a cavity is important to their development and use. The general Hamiltonian for longitudinal stability in multi harmonic cavities is derived and the particle dynamics are explored.

INTRODUCTION

A multi-harmonic cavity that operates at high gradients could act as an alternative cavity design for CLIC. Cavities of this type have unconventional surface electric and magnetic field profiles that can potentially lower the surface field emission and/or pulsed surface heating without compromising the gradient [1]. Two particular phenomena found in multi-harmonic cavities provide the main motivation for their use: (a) the anode-cathode effect, which can be found in an asymmetric multi-harmonic cavity that relies on fields pointing into one wall (cathode-like) to be significantly smaller than fields pointing away (anode-like) from the same wall. This effect will raise the work function barrier to suppress field and secondary emission, and (b) a reduction in the surface heating by lowering the average H_{\parallel}^2 along the surface.

For cavities of this type to be used in an accelerator, the effect of the additional mode on a bunch of traversing particles needs to be explored. To achieve this, a Hamiltonian is derived that describes the behaviour of particles with small deviations from the synchronous particle in energy and/or phase. Typical formalisms of this kind only account for a single TM_{010} mode which follow a $\cos(kz)$ longitudinal distribution [2, 3] (where k is the wave number and z is the longitudinal coordinate) and not for a combination of modes with different longitudinal field profiles.

Harmonic rf systems are often used to linearise the energy gain [4] and reduce the energy spread of the bunch [5, 6]. These typically require an additional cavity operating in a TM_{010} mode with a frequency that is an integer harmonic of the main cavities. Here however, we excite two modes simultaneously within one cavity.

The first section presents tracking results for a particle traversing a linac and this is compared with the Hamiltonian found in literature. In subsequent sections, a general Hamiltonian for multi-harmonic cavities is derived and applied to two separate modal configurations.

SINGLE MODE PARTICLE TRACKING

The electric field of a TM_{010} standing wave in a cavity with amplitude E_0 is

$$E_T = E_0 \cos(kz) \cos(\omega t + \phi). \quad (1)$$

where ϕ is the phase of the field when the particle is at $z = 0$, ω is the angular frequency and $k = 2\pi/\beta_s \lambda$. The energy gain of a charge q as it crosses a single cavity gap g is given by

$$W = qE_0 \int_{-g/2}^{g/2} \cos(kz) \cos(\omega t + \phi) dz \quad (2)$$

The energy relative to the synchronous particle is $w = W - W_s$. The relevant differential equations in terms of the Hamiltonian H are given by [2]

$$\frac{dw}{ds} = qE_1 T(\beta) (\cos(\phi) - \cos(\phi_s)) = -\frac{\partial H}{\partial \phi} \quad (3)$$

and

$$\frac{d\phi}{ds} = -2\pi \frac{w}{\gamma_s^3 \beta_s^3 mc^2 \lambda} = \frac{\partial H}{\partial w}, \quad (4)$$

where $V_0 = E_0 \int_{-g/2}^{g/2} \cos(kz) dz$, and T is the transit time factor given by

$$T(\beta) = \frac{\int_{-g/2}^{g/2} \cos(kz) \cos(\frac{\omega z}{\beta c}) dz}{V_0}, \quad (5)$$

with $E_1 = V_0/g$. For the synchronous particle, T remains constant as the cavity gap increases with β_s . For negligibly small acceleration rates, the differential equations can be integrated, and the Hamiltonian is

$$H = \frac{\pi}{\beta_s^3 \gamma_s^3 mc^2 \lambda} w^2 + qE_1 T [\sin \phi - \phi \cos(\phi_s)], \quad (6)$$

where β_s and γ_s refer to the velocity and gamma factor of the synchronous particle.

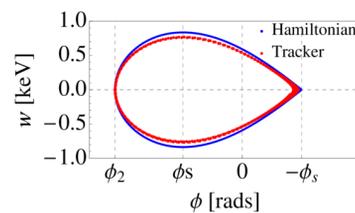


Figure 1: Comparison of Hamiltonian with particle tracked according to Eq. 2.

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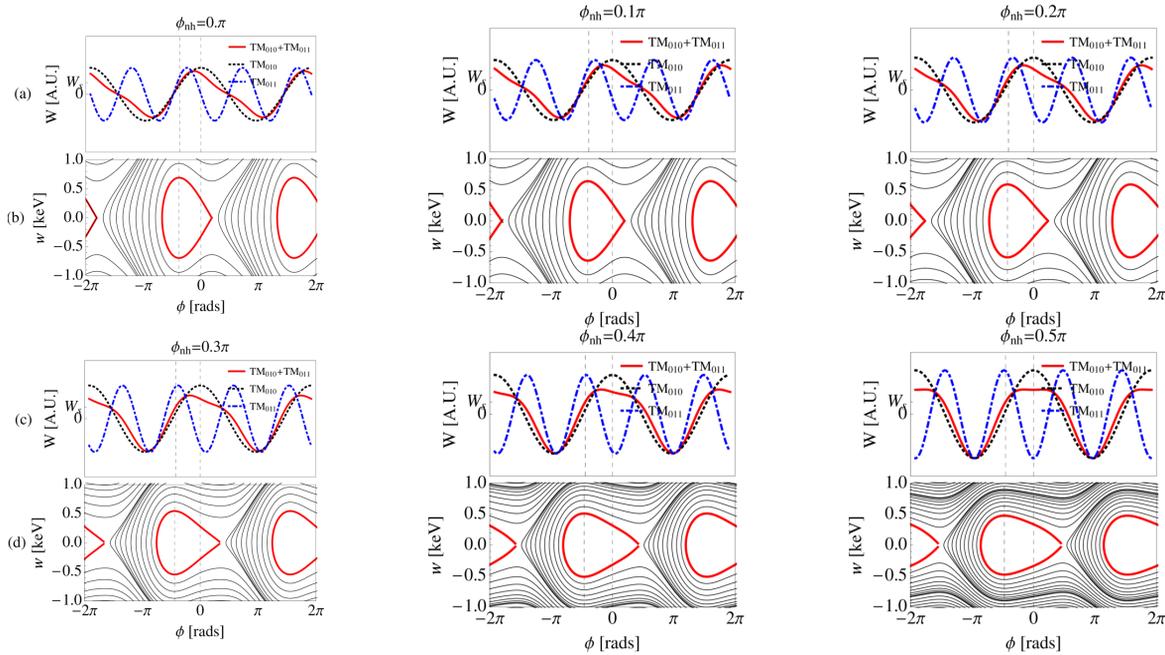


Figure 2: Phase space plots for a cavity exciting both the TM_{010} and TM_{011} modes. A phase shift is gradually applied to the TM_{011} mode and the gradient is kept constant for each step. For all steps, $\alpha = 0.222$. Rows (a) and (c) show the energy gain of a traversing particle as a function of its initial phase offset. Rows (b) and (d) show the phase space contours, with the separatrix marked in red.

A comparison between tracking the particle directly under the influence of the field with the derived Hamiltonian can be found in Fig. 1. The parameters used were consistent with an X-band cavity, with $E_0 = 1$ mV/m and $\phi_s = -\pi/3$. To assess the accuracy of the Hamiltonian, we have subjected the particles to an extremely small acceleration (1 mV/m). We injected particles with an initial energy of 1 MeV and this makes the effect of acceleration negligible within a few cavities (as it is less than a second order effect). Nonetheless, there is an 8% discrepancy discernible in Fig. 1 between the two methods and this we attribute to the change in velocity which accumulates after transit through several thousand cavities.

GENERAL HAMILTONIAN FOR MULTI-HARMONIC CAVITIES

The electric field in a multi-harmonic cavity is given by

$$E_T = E_0[(1 - \alpha) \cos(kz) \cos(\omega t + \phi) + \alpha \cos(hkz + \phi_{zh}) \cos(h(\omega t + \phi) + \phi_{nh})] \quad (7)$$

where α is the fractional contribution from the additional harmonic, h is the harmonic number, ϕ_{zh} is a phase shift relating to the longitudinal distribution and ϕ_{nh} is a phase shift in time for the harmonic mode that is additional to the shift from the synchronous phase. The energy gain of a particle traversing this field is given by

$$W = q[E_1(1 - \alpha) \cos(\phi) + \alpha E_h(\cos(\phi_{nh}) \sin(h\phi) + \sin(\phi_{nh}) \cos(h\phi))], \quad (8)$$

where

$$E_1 = \frac{E_0}{g} \int_{-g/2}^{g/2} \cos(kz) \cos(\omega \frac{z}{\beta c}) dz \quad (9)$$

and

$$E_h = -\frac{E_0}{g} \int_{-g/2}^{g/2} \cos(hkz + \phi_{zh}) \cos(h\omega \frac{z}{\beta c}) dz. \quad (10)$$

Using Eq. 8, a differential equation for energy can be obtained, and a general Hamiltonian can be derived by following the same methodology as that found in the previous section.

$$H = \frac{\pi}{\beta_s^3 \gamma_s^3 m c^2 \lambda} w^2 + qW_0[E_1(1 - \alpha)(\sin \phi - \phi \cos \phi_s) + E_h \alpha [\cos(\phi_{nh}) (\frac{\cos(h\phi)}{h} - \phi \sin(h\phi_s)) + \sin(\phi_{nh}) (\frac{\sin(h\phi)}{h} + \phi \cos(h\phi_s))]], \quad (11)$$

where W_0 is a scaling factor that is used to ensure the energy gain of the synchronous particle is the same for each step within each study. We now apply this Hamiltonian to glean some insight into the beam dynamics in second harmonic and third harmonic cavities.

SECOND HARMONIC CAVITY

In the case of a cavity that excites a fundamental TM_{010} mode with a second harmonic TM_{011} mode, $h = 2$, $\phi_{zh} =$

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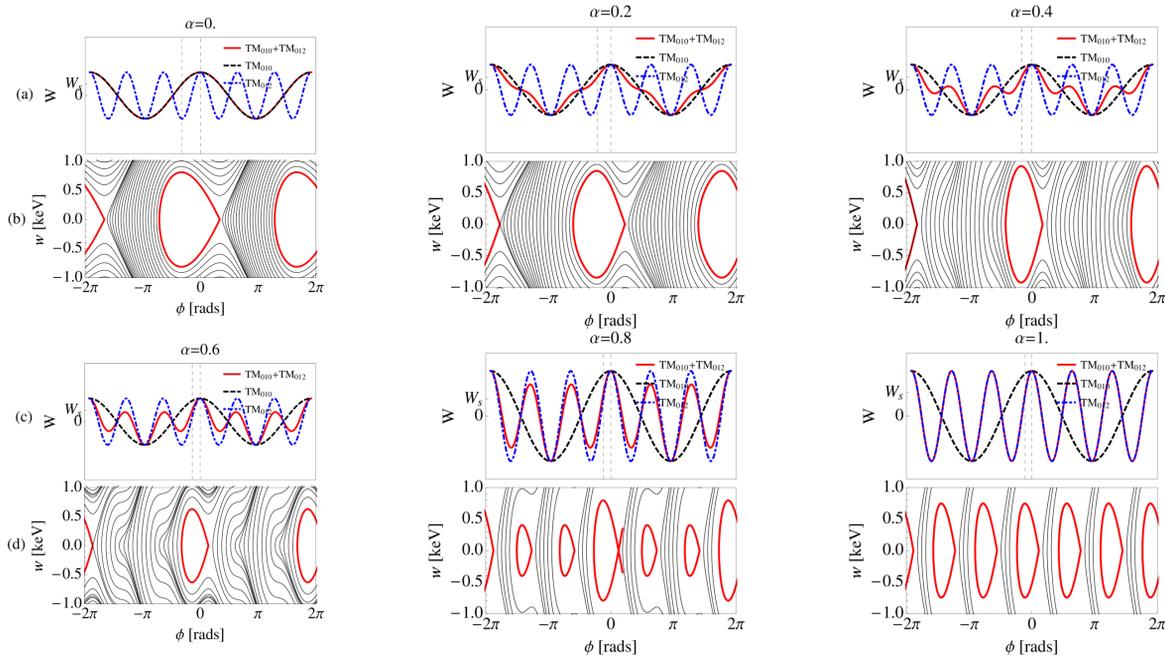


Figure 4: Phase space plots for a cavity exciting both the TM_{010} and TM_{012} modes. α is varied to determine at what harmonic field strength secondary buckets begin to emerge. Rows (a) and (c) show the energy gain of a traversing particle as a function of its initial phase offset. Rows (b) and (d) show the phase space contours, with the separatrix marked in red.

$-\pi/2$. We prescribe the synchronous phase of the second harmonic by assigning it a value that reduces the accelerating gradient to half the peak. This is in order to be consistent with the single mode case. This synchronous phase varies for each step. Here, ϕ_{nh} is varied, in order to determine the variation of the acceptance.

The energy gain and phase space contours for a second harmonic cavity are displayed in Fig. 2. It can be seen that as ϕ_{nh} increases, a plateau begins to form in the energy gain of particles crossing the cavity. This allows a reduction in the energy spread and a large increase in the phase width of the bucket. The acceptance for each step can be found in Fig. 3

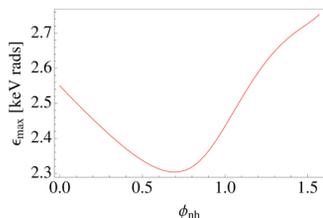


Figure 3: Acceptance of the accelerating bucket in a second harmonic cavity with $\alpha = 0.222$ as a function of ϕ_{nh} .

THIRD HARMONIC CAVITY

Here, we consider simultaneous excitation of a TM_{010} mode with a third harmonic TM_{012} mode, which corresponds to $h = 3$ and $\phi_{zh,nh} = 0$. The contribution from the harmonic mode α will be varied in order to determine when

additional stable regions form in between the main accelerating buckets.

The beam dynamics in this case is investigated and the results are shown in Fig. 4. The contours begin to distort as α increases, however it is not until $\alpha = 0.6$ that stable regions begin to form, shown in Fig. 5. The additional buckets emerge when the peak of the field in between the main buckets exceeds the energy gain of the synchronous particle.

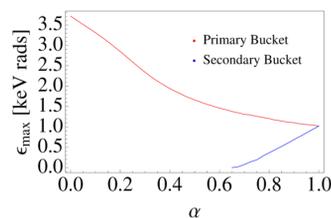


Figure 5: Acceptance of the accelerating bucket as a function of α . The red line is the primary bucket and the blue line is for the secondary buckets.

CONCLUSION

A Hamiltonian has been derived that allows an arbitrary combination of harmonic modes to be modelled. This was then applied to two different mode configurations in order to gain an insight into the phase space behaviour of the particles. Future studies will be focused on beam dynamics with accelerating gradients appropriate to linear colliders and light sources.

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