

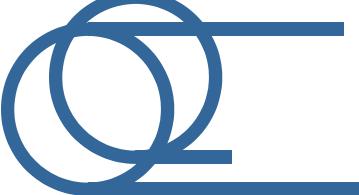
# **Ultralow Emittance Beam Production Based on Doppler Laser Cooling and Coupling Resonance**

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**and**

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H. Tongu, K. Jimbo, M. Grieser, Z. He, A. Smirnov**



# Contents

## 1. History of Approach to Low Emittance Beam

**Beam Cooling** → **Reduce Beam Temperature**  $k_B T_{\parallel} = m_0 c^2 \beta^2 (\delta p / p)^2,$

**Temperature**  $k_B T_{\perp} = 1/2 m_0 c^2 \beta^2 \gamma^2 (y'^2 + z'^2)$

- Motivation of Beam Cooling → Creation of anti- proton beam  
→ Creation of Ultra-cold Beam!! (Our Goal)
- Electron Beam Cooling → 1D Ordering
- PALLAS 3D Crystal by Laser Cooling

## 2. Beam Temperature so far attained by SBRC at S-LSR

- Doppler Laser Cooling: 1D → 3D

40 keV  $^{24}\text{Mg}^+$  Ion Beam Intensity  $10^7 \Rightarrow$  Scraper  $\Rightarrow 10^4$

## 3. Future Prospect with MD Simulation based on the Experiment

1D Longitudinal String (Bunched), 3D Ordered State (Coasting)



# Motive Force of Beam Cooling

## - Creation of Good Quality Secondary Beam - (Anti-proton Beam)

- Electron Cooling  
(Proposed by G.I. Budker)

Electron Cooling is rather oriented for relatively cooler beam compared with Stochastic Cooling!!

by Late Dr. Dieter Möhl (Proc.of ECOOL84, 293 (1984))

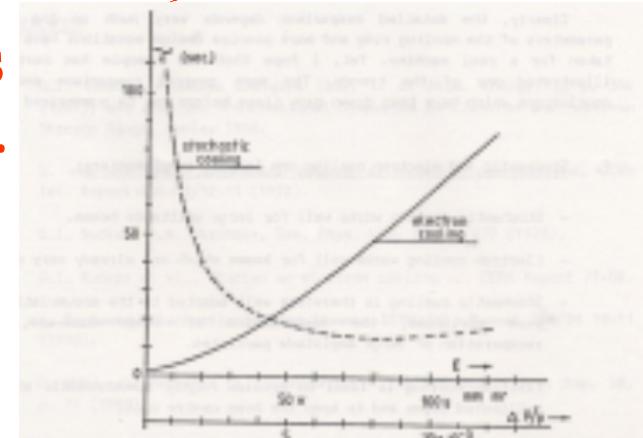


Fig. 2 : Stochastic and electron cooling time  $\tau$  as a function of beam emittance.

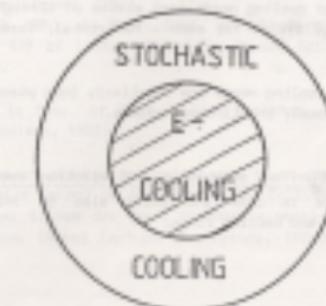


Fig. 3 : Sketch of beam cross-section to indicate regions where one of the two cooling methods is most efficient. The combination of "beam cooling" by electrons with "beam cooling" by the stochastic method is capable of taking advantage of both of them.



## ESR at GSI, by M. Steck

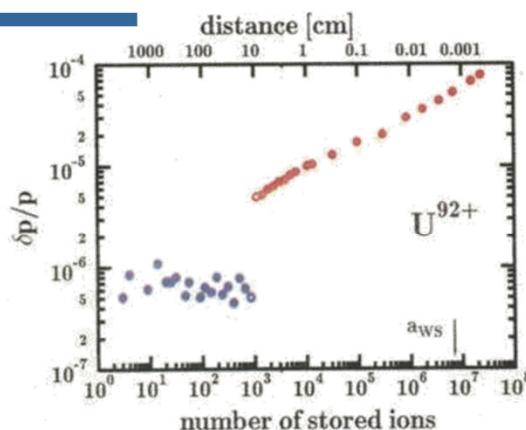


Figure 2. Experimental momentum spreads from Schottky signals vs. number of stored ions in the ESR for electron cooled  $\text{U}^{92+}$  ions at 240 MeV/u. aws indicates the Wigner-Seitz radius of eq.(3). (after ref. <sup>9</sup>)

## ESR at GSI, by M. Steck

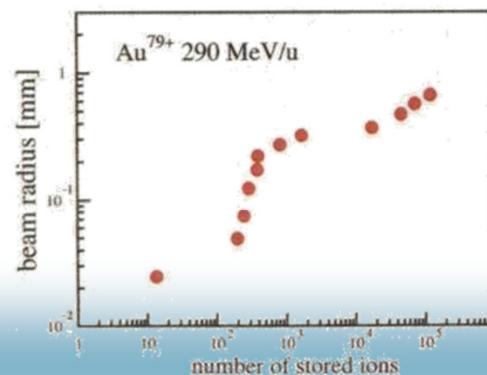


Figure 3. Beam radius measured with a beam scraper vs. number of stored ions in the ESR for electron cooled  $\text{Au}^{79+}$  ions at 290 MeV/u (from ref. <sup>10</sup>).

## CRYRING at Stockholm, by H. Danared

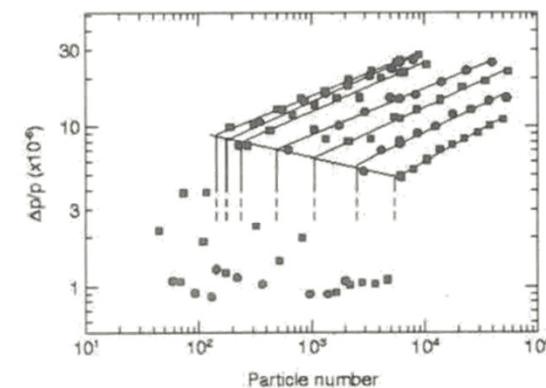


Fig. 5: Relative momentum spread as a function of particle number for the lowest seven electron densities represented in Fig. 2. The density increases from the upper left to the lower right. For each density, a line is fitted to the data points. A line is also drawn through the points corresponding to the transition to the ordered state. (The use of different symbols is just to help identifying which points belong to same electron density.)

## NAP-M at BINP, Novosibirsk by V.V. Parkhomchuk

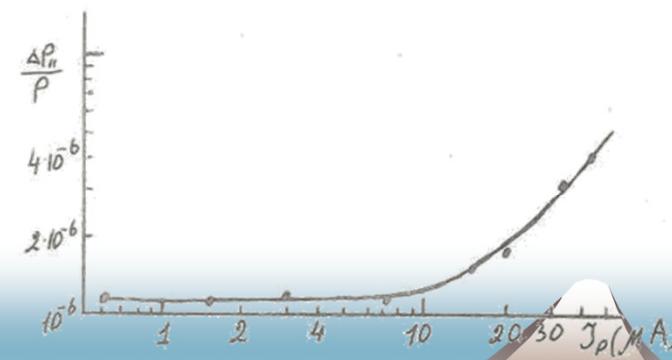
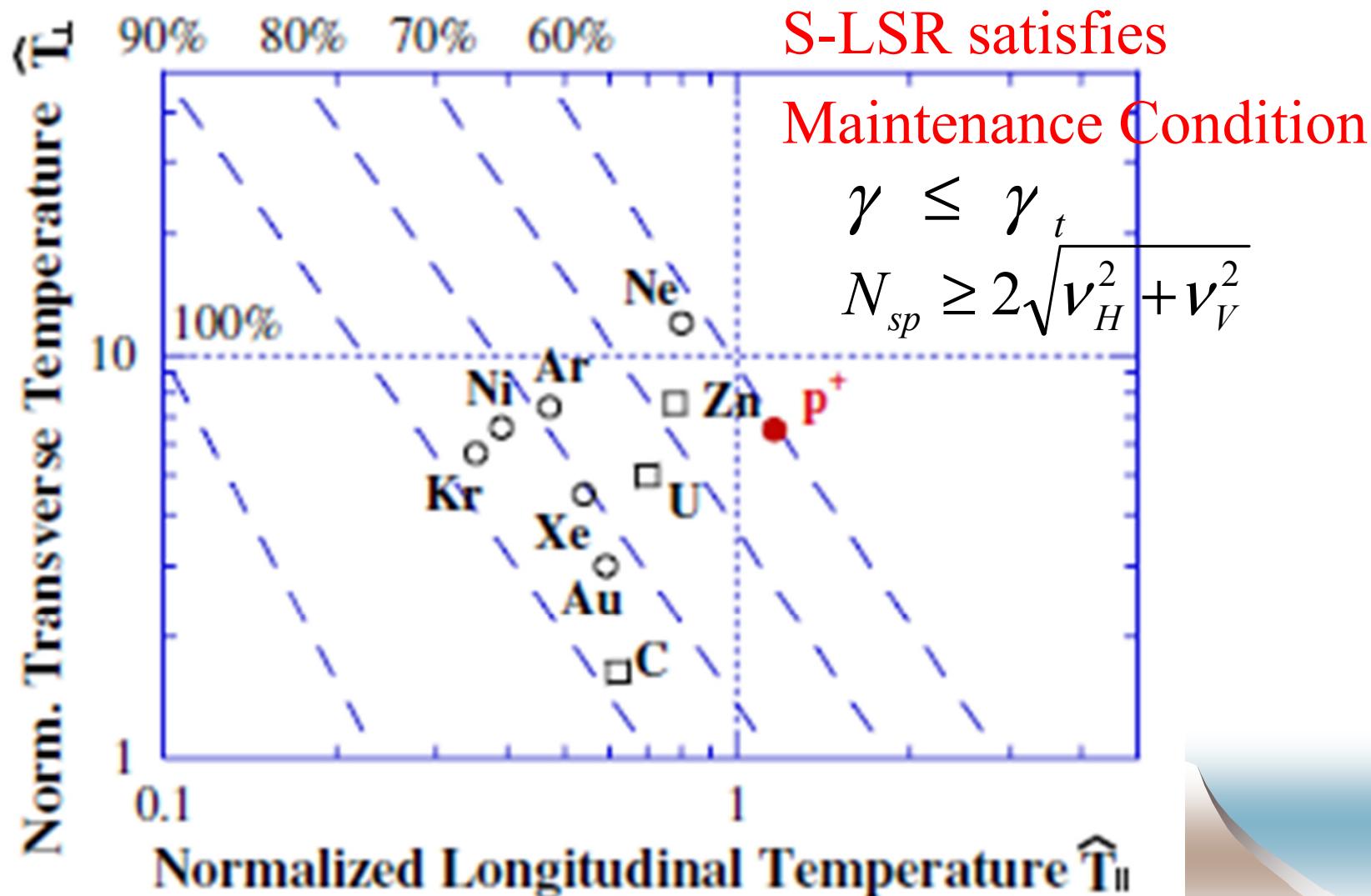
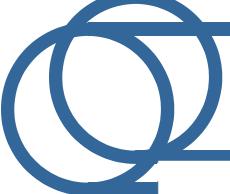


Fig. 6. The momentum spread of proton beam versus current  $J_p$ .



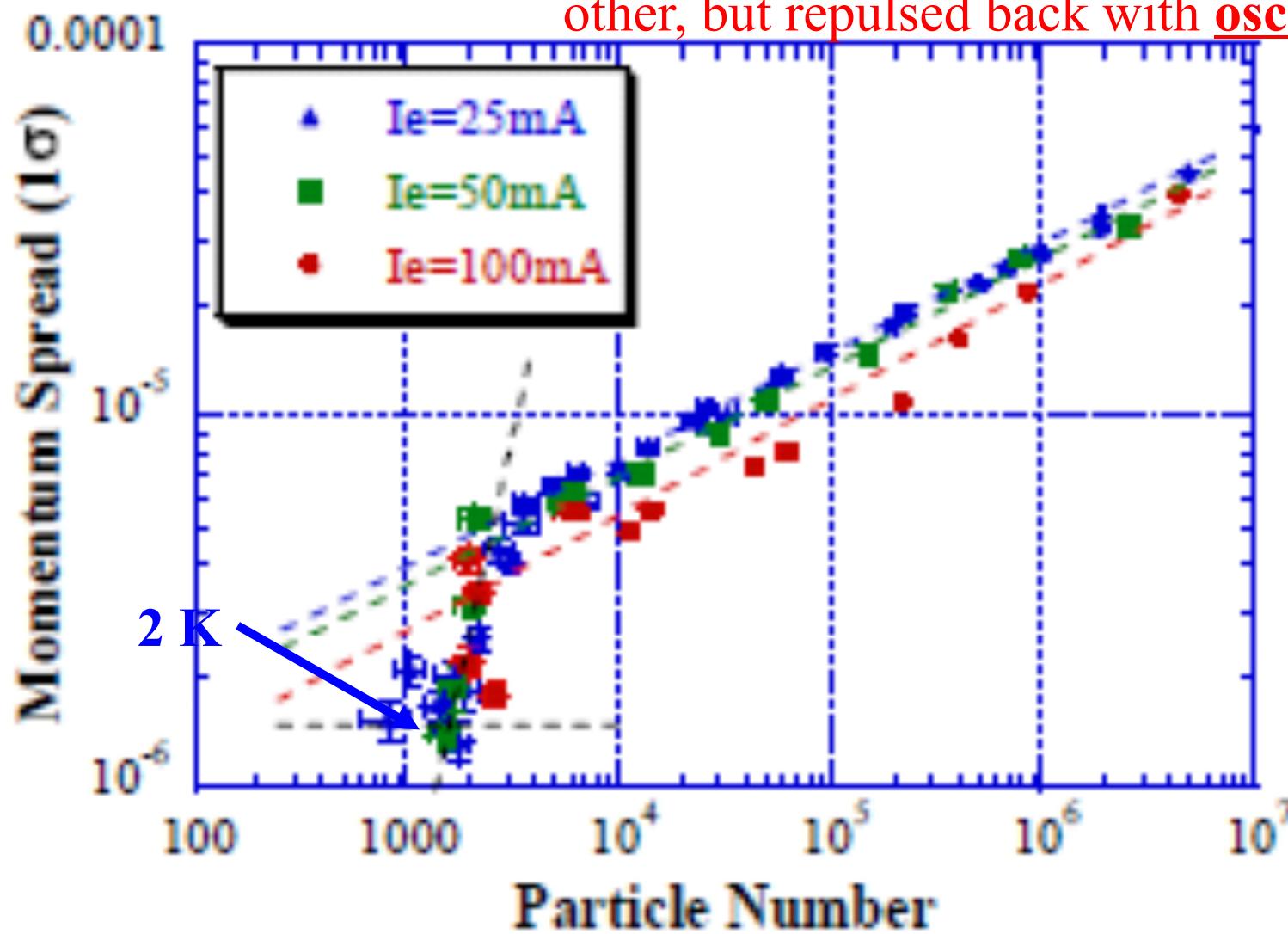
# Reflection Probability of Ions made Phase Transition



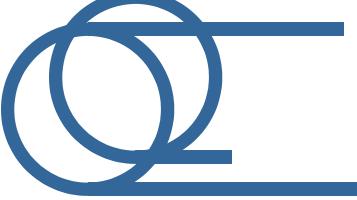


# Phase Transition to 1D Ordered State

Adjacent particles never take over each other, but repulsed back with oscillation

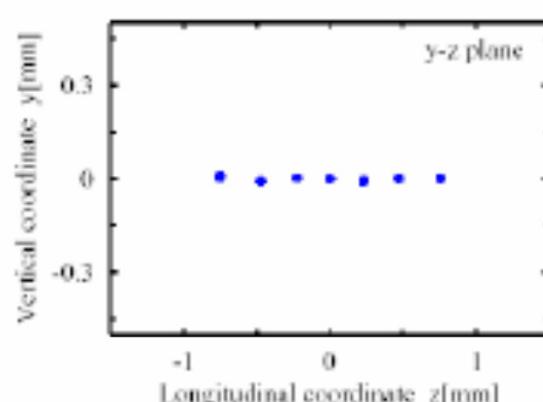
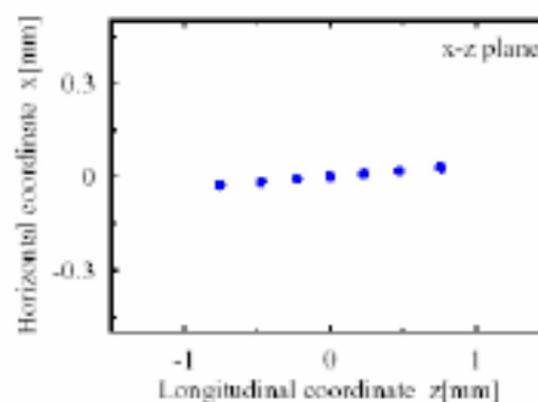
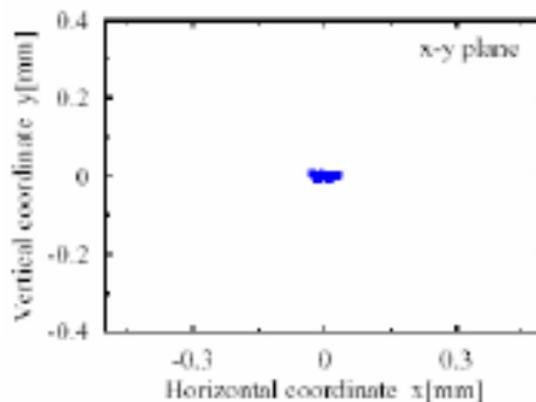


T. Shirai et al., PRL, 98 (2007) 204801

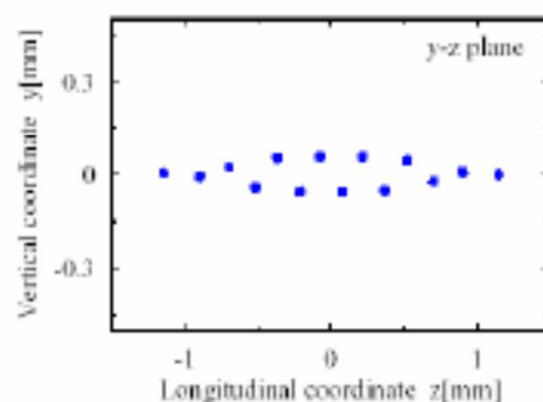
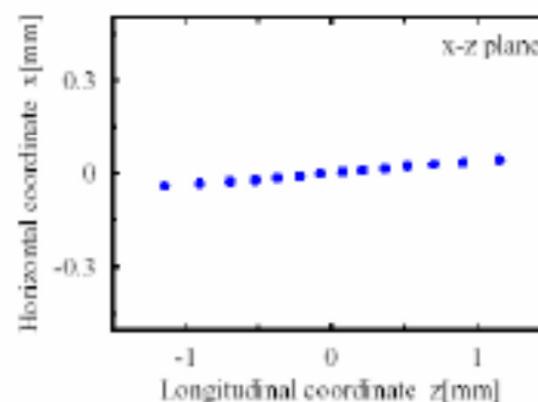
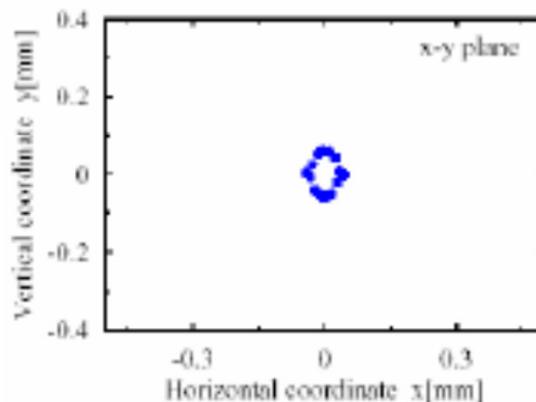


Simulation by H. Okamoto et al.

# Expectation from Simulation



(a) 1 D String



(b) 2D zigzag structure

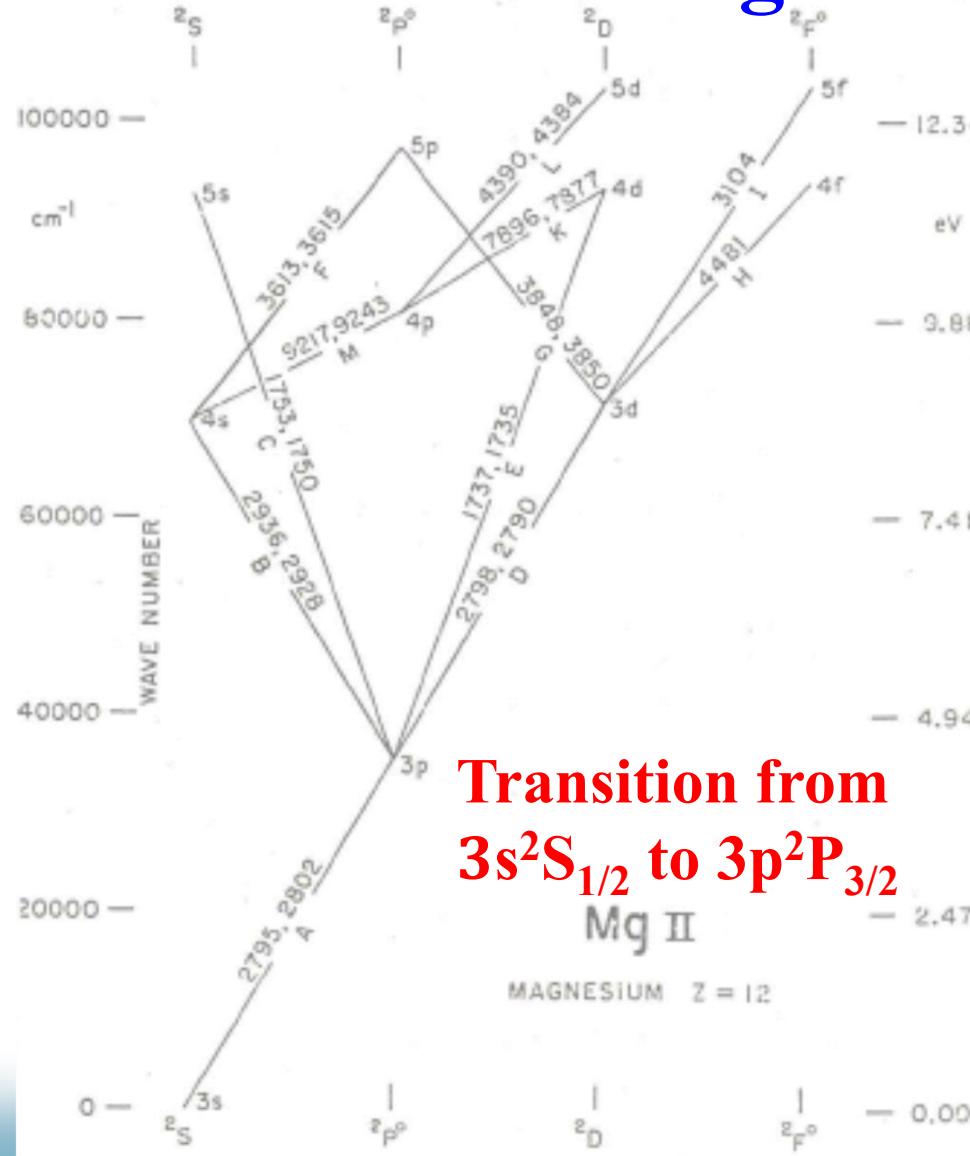


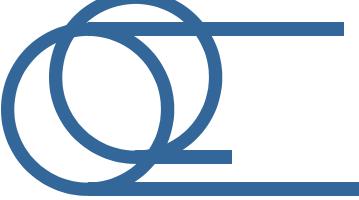
# Doppler Laser Cooling of Ion Beam Circulating in a Storage Ring



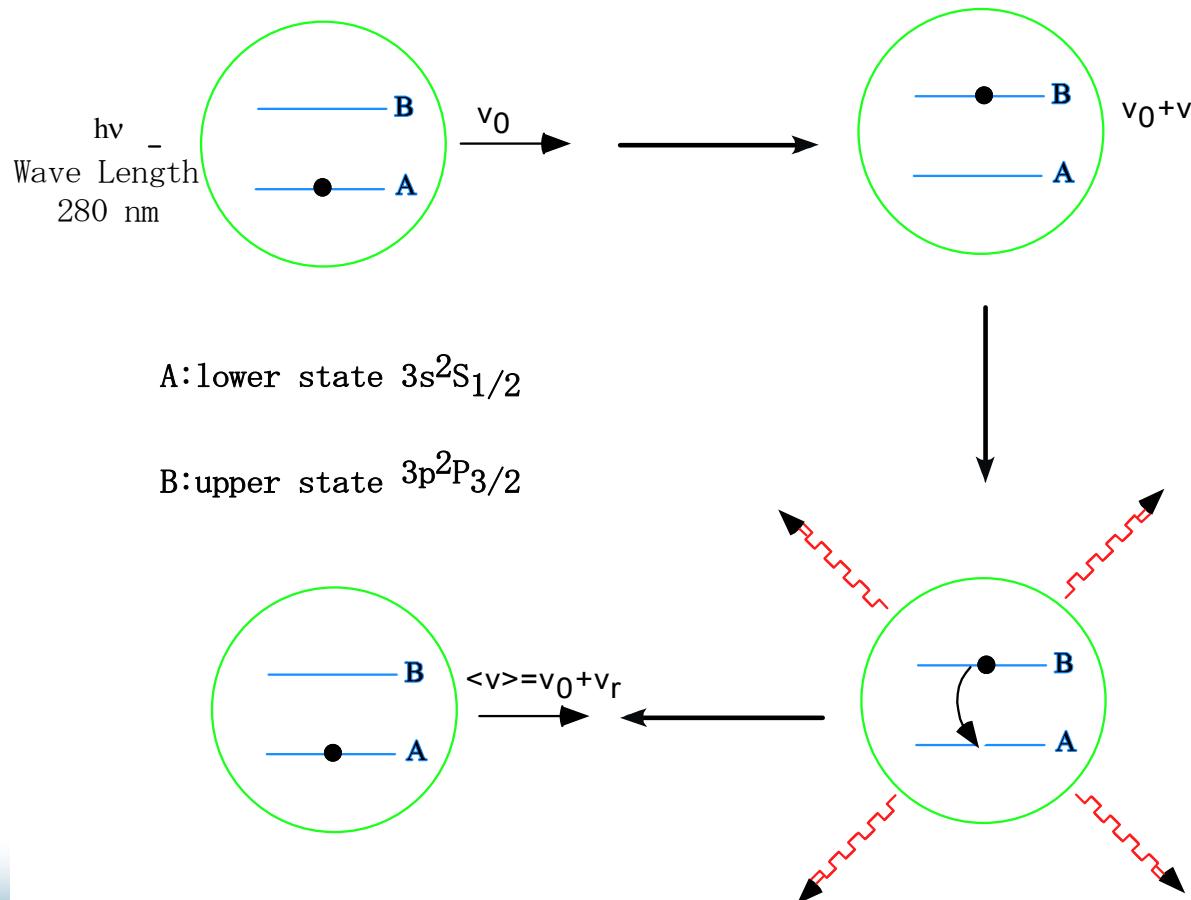


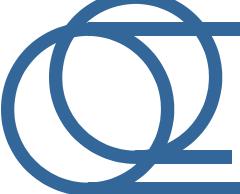
# Excited States of Mg Ion





# Principle of Laser Cooling (Longitudinal)



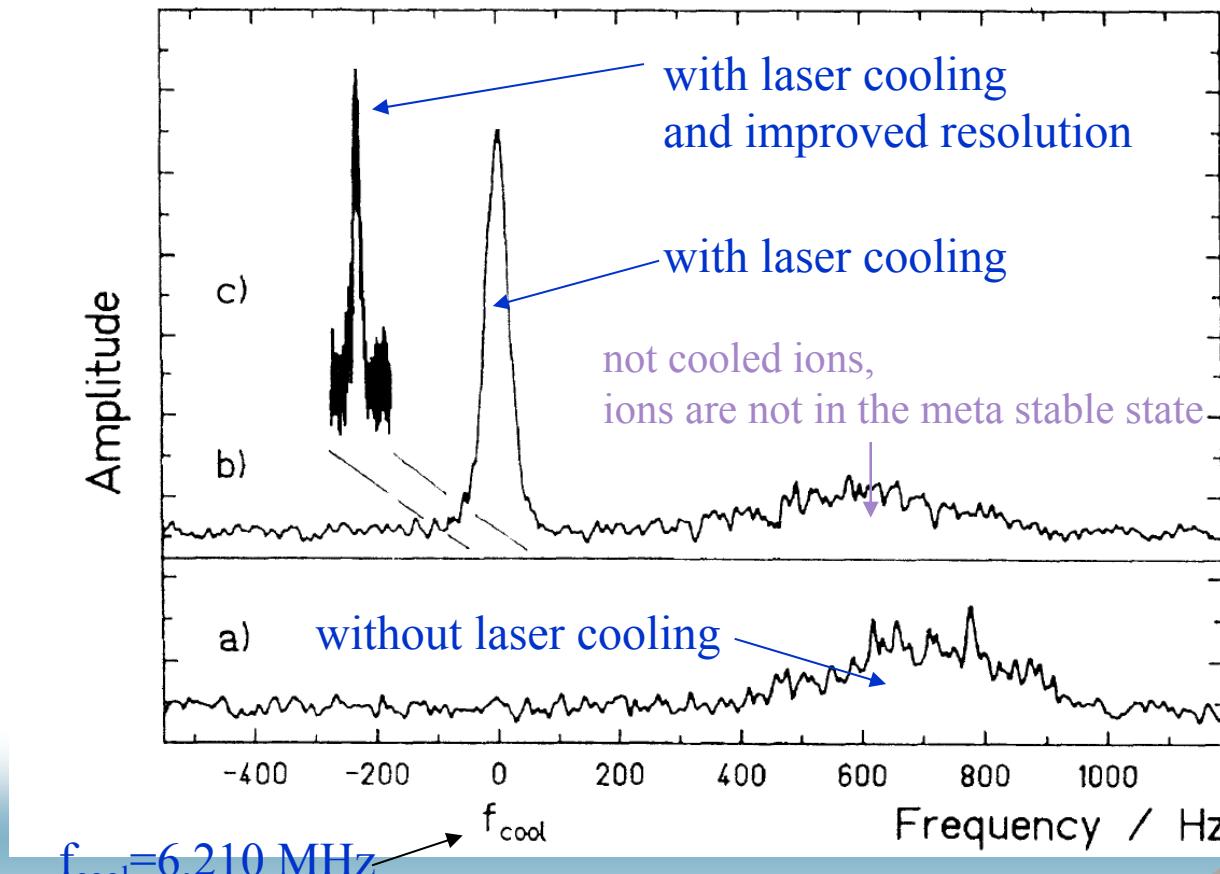


# First laser cooling of ions in a storage ring

S. Schröder et. al. physical review letters, volume 64 No. 24 (1990) p. 2901-2904

Experiments done at TSR storage ring with  ${}^7\text{Li}^+$  ( $E=13.4$  MeV) ions.

Schottky noise signals with and with out laser cooling



Only effective  
in the beam  
direction  
(1D)

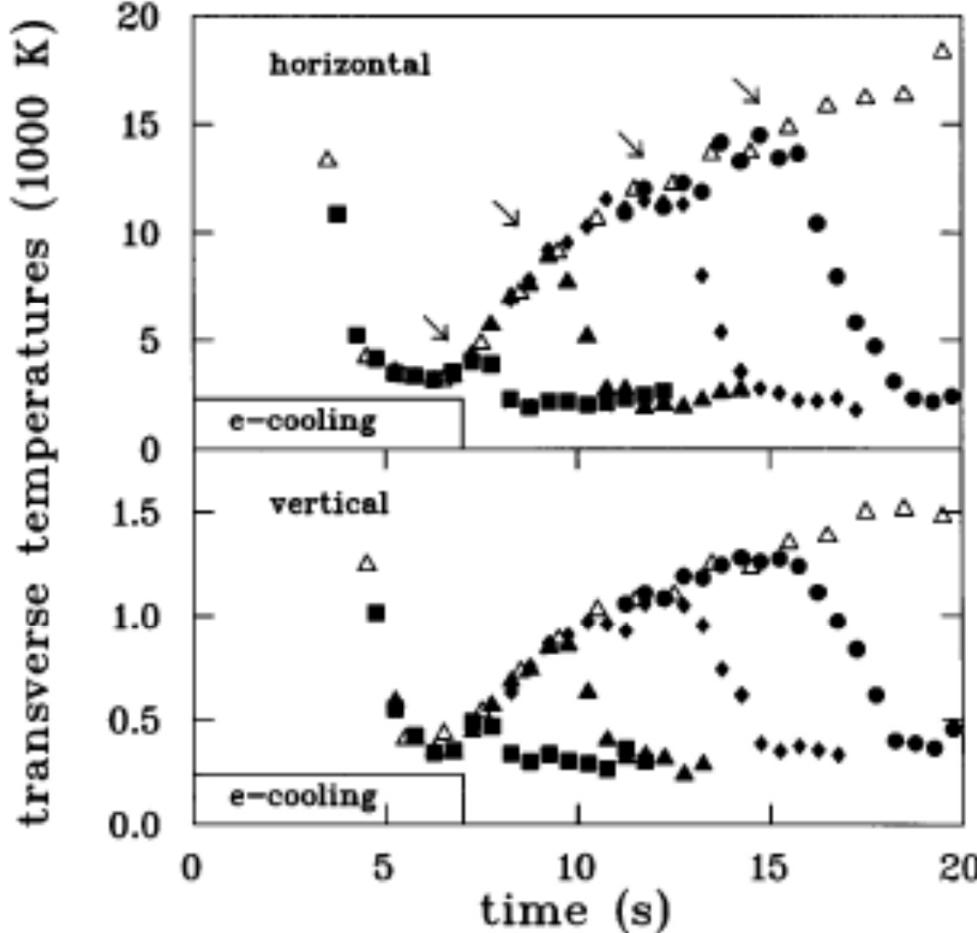
final longitudinal temperature:  $T_{||} \leq 2.9 \text{ K}$



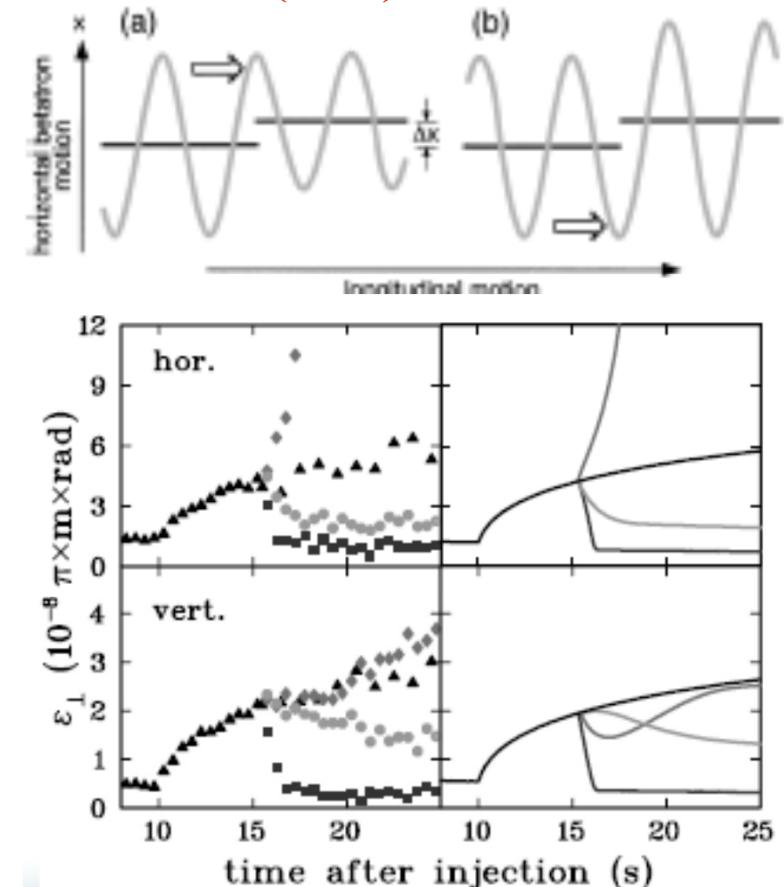
# Transverse Laser Cooling attained at TSR

## Utilize IBS

H.J. Miesner et al., Phys. Rev. Lett. 77, 623 (1996)



Dispersive Coupling  
I. Lauer et al., Phys. Rev. Lett. 81, 2052 (1998)



Longitudinal : 15 K, H: ~4000 K, V: 500 K

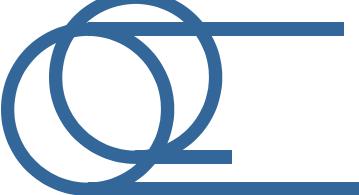
ASTRID Longitudinal : 2.5 K, H: 17 K, V: 21 K

Intensity  $7 \times 10^5$ , 100 keV, Mg Ion

16, June, 2014

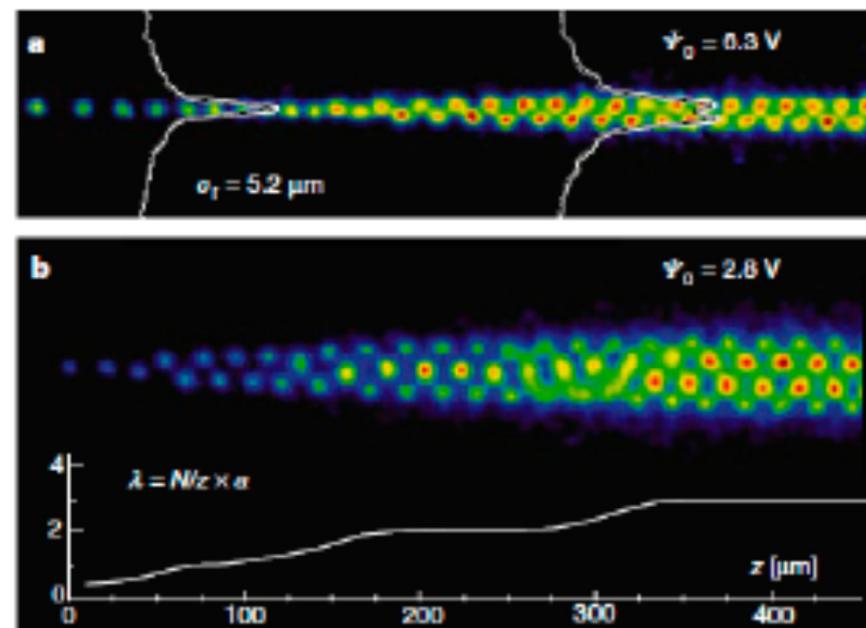
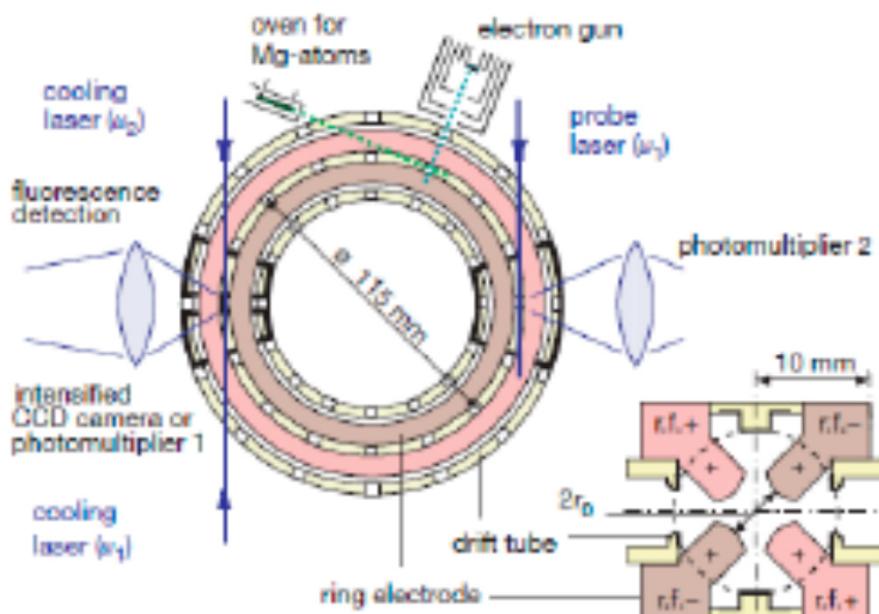
Longitudinal : few tens K, H: ~500 K,  
V: ~150 K

Akira Noda, NIRS at IPAC'14, Dresden, Germany



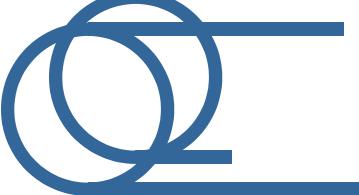
# Crystalline Beam in Circular RFQ, PALLAS

T. Schatz, U. Schramm, D. Habs: Nature, 412, 717 (2001)



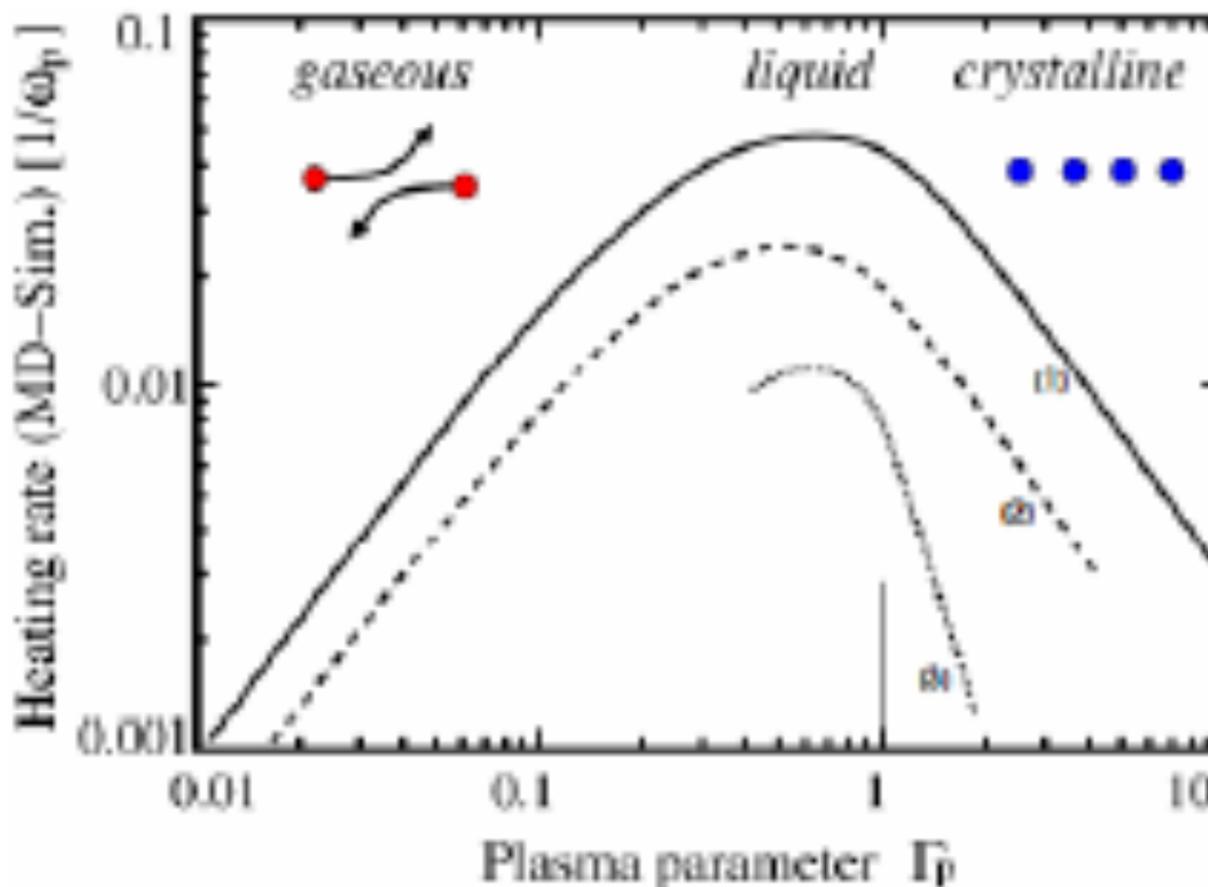
Structure of Circular RFQ, PALLAS

Images of ion crystals at rest in PALLAS



By M. Bussmann U. Schramm and D. Habs et al., SPARC07

# Plasma Parameter Dependence of Heating Rate



$$\Gamma_p = \frac{E_{\text{Coulomb}}}{E_{\text{thermal}}} = \frac{Z_{\text{ion}}^2 e^2}{4\pi\epsilon_0 a_{\text{ws}} \cdot k_B T_{\text{ion}}}, \quad a_{\text{ws}} = \left(\frac{4}{3}\pi n_{\text{ion}}\right)^{\frac{1}{3}}$$

- (1): Q. Spreiter et al., NIM A 364 (1995), 239  
(2): linear ion density reduced by a factor of 10  
(3): string, J. Wei et al., PRL 80 (1998), 2606



# Compact Cooler Ring S-LSR

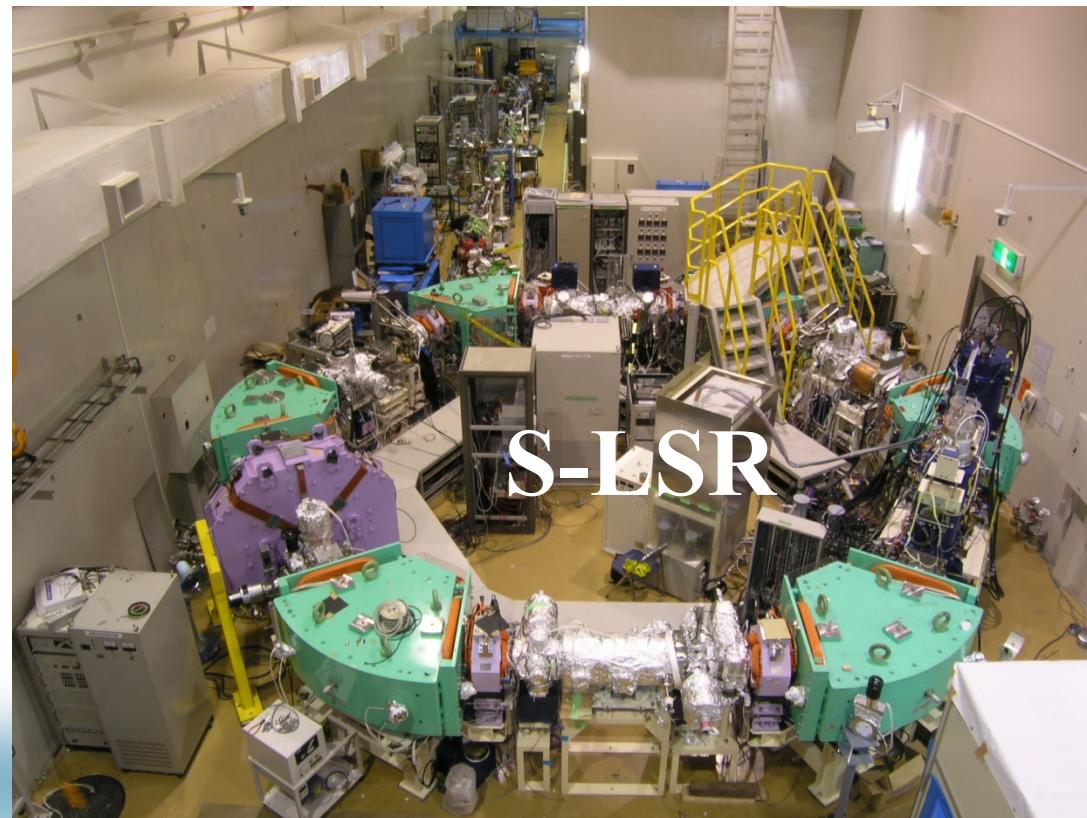
- Circumference 22.56m
- Straight Section Length 1.86m

## E-cooling modes

- Protons 7MeV  
( $E_e=3.8\text{keV}$ )

## Laser cooling

- $^{24}\text{Mg}^+$  40 keV  
( $\lambda=282\text{ nm}$ )



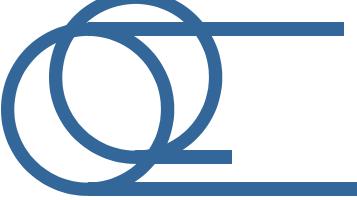
Operation: Since October, 2005 to March, 2013, Now in rest

16, June, 2014 Akira Noda, NIRS at IPAC'14, Dresden, Germany



# Main Parameters of S-LSR

<b>Circumference</b>	<b>22.557 m</b>
<b>Average radius</b>	<b>3.59 m</b>
<b>Length of straight section</b>	<b>1.86 m</b>
<b>Number of periods</b>	<b>6</b>
<b>Betatron Tune</b>	
<b>Crystalline Mode</b>	<b>Normal Operation Mode</b>
<b>1.45 (H) , 1.44 (V)</b>	<b>1.872(H), 0.788 (V): EC</b>
	<b>2.068(H), 1.105, 1.070 (V): LC</b>
	<b>(H-type)</b>
<b>Bending Magnet</b>	<b>0.95 T</b>
<b>Maximum field</b>	<b>1.05 m</b>
<b>Curvature radius</b>	<b>70 mm</b>
<b>Gap height</b>	<b>Rogowski cut+Field clamp</b>
<b>Pole end cut</b>	<b>60°</b>
<b>Deflection Angle</b>	
<b>Weight</b>	<b>4.5 tons</b>
<b>Quadrupole Magnet</b>	
<b>Core Length</b>	<b>0.20 m</b>
<b>Bore radius</b>	<b>70 mm</b>
<b>Maximum field gradient</b>	<b>5 T/m</b>



# Ring Layout

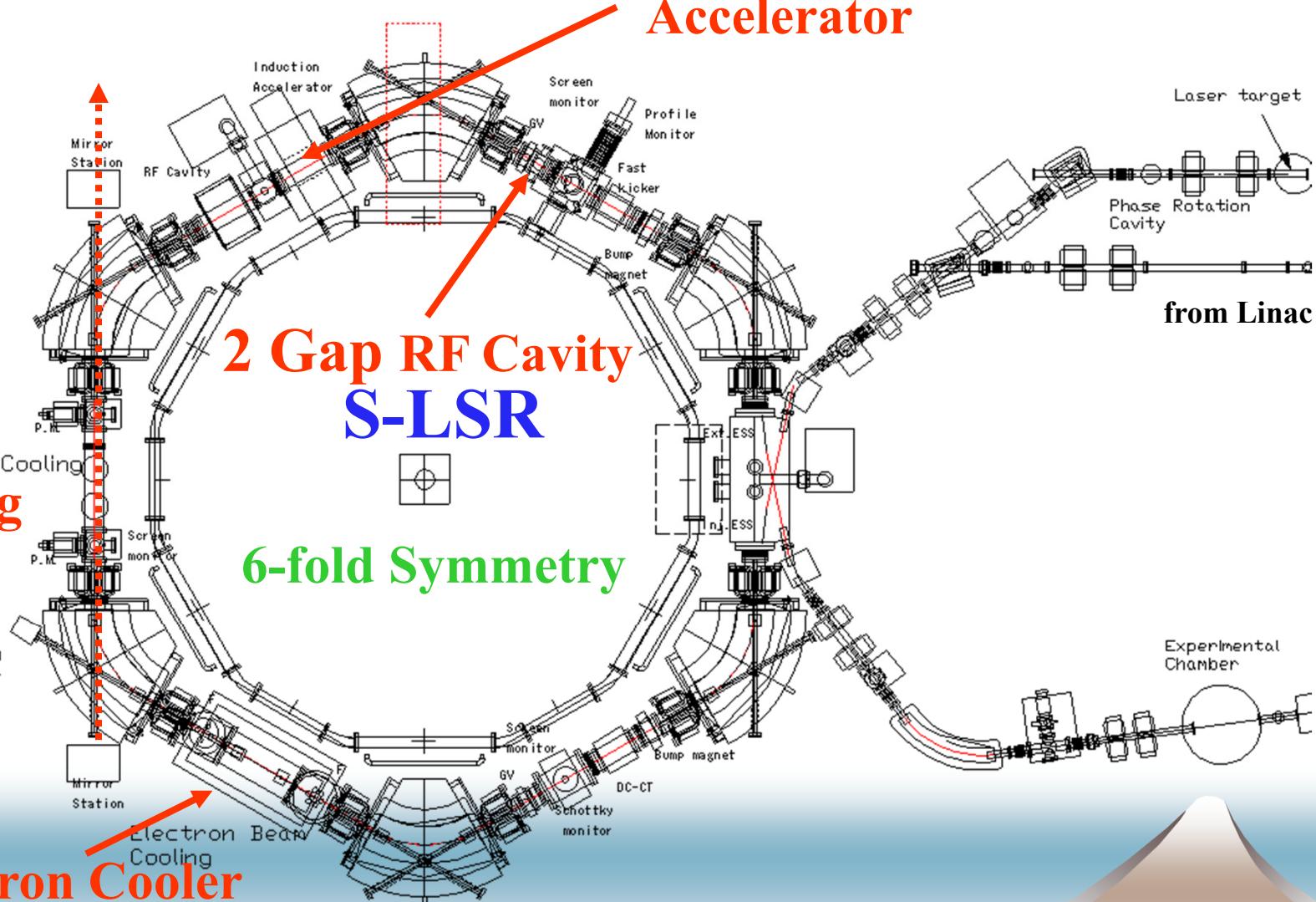
Induction Accelerator

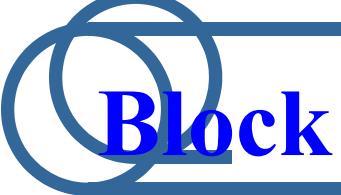
2 Gap RF Cavity  
S-LSR

Laser  
Cooling

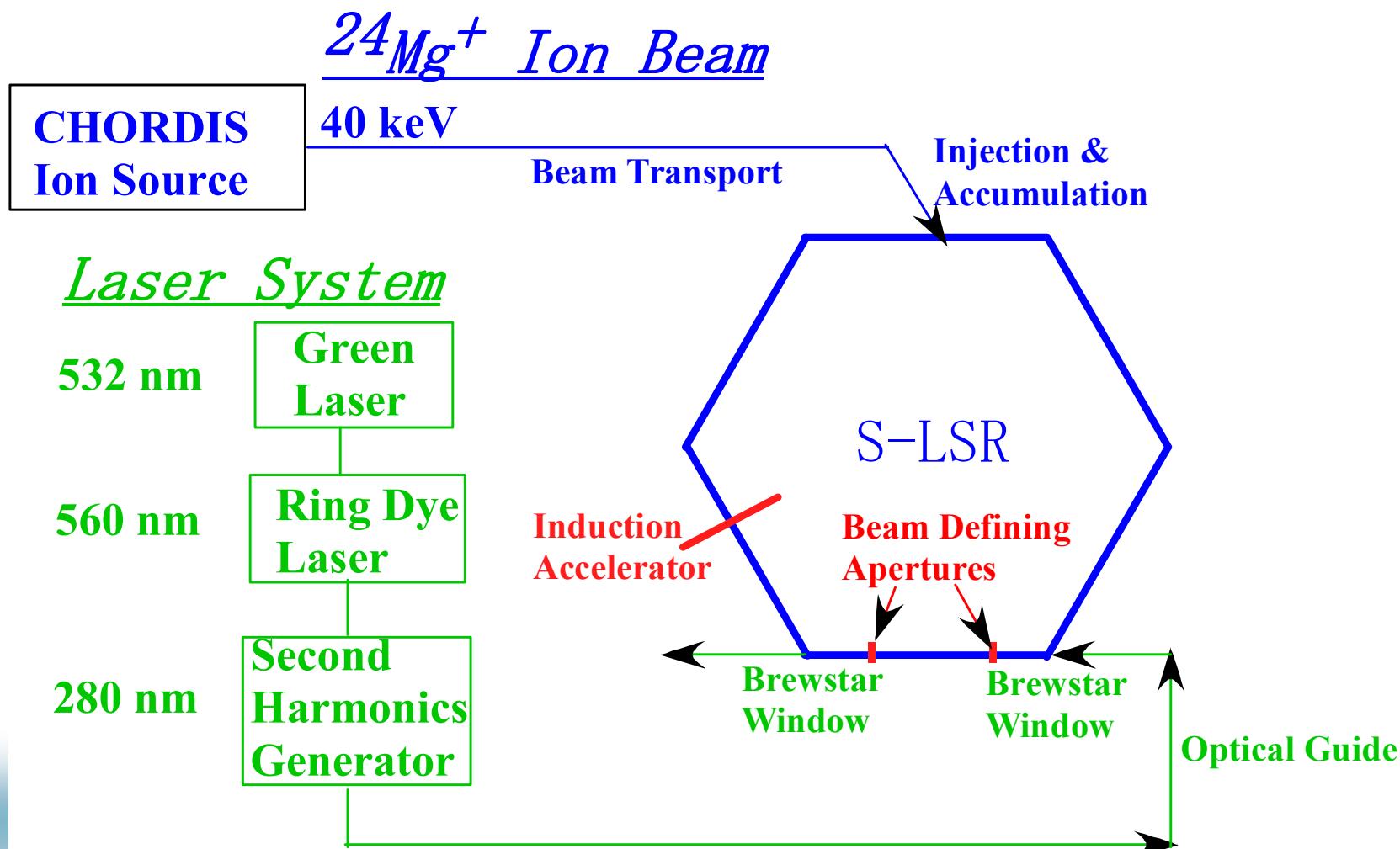
6-fold Symmetry

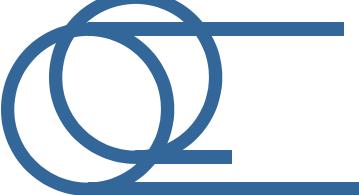
Electron Cooler



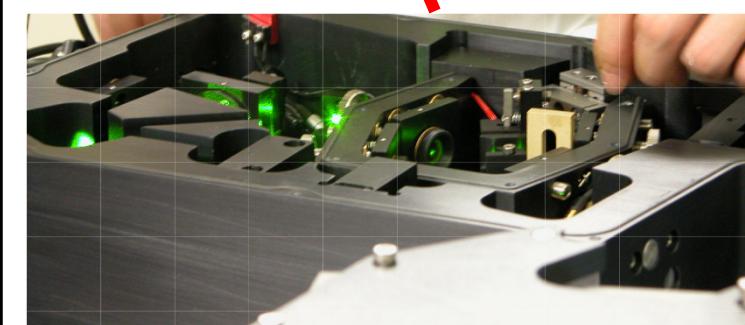
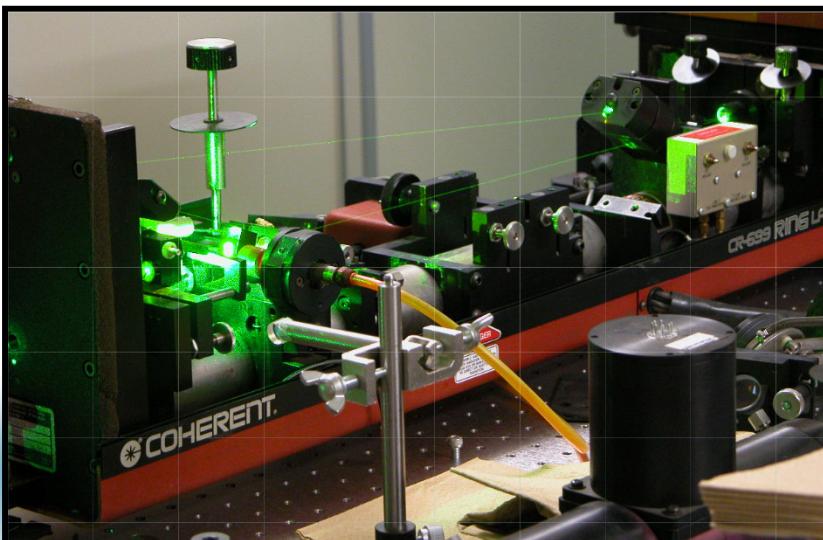
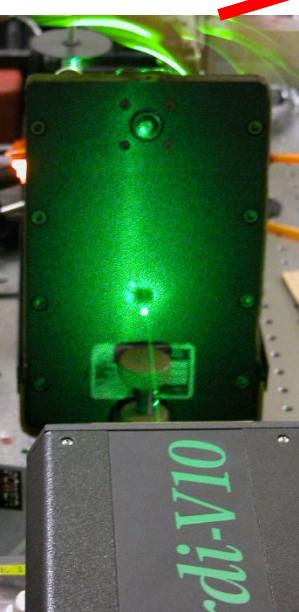
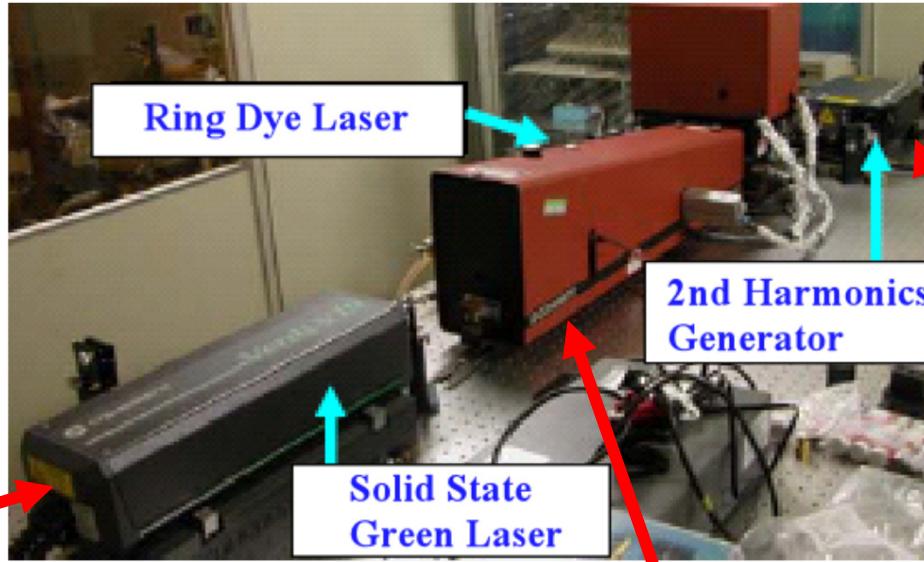


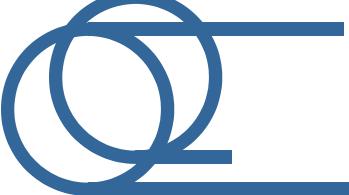
# Block Diagram of Laser Cooling at S-LSR





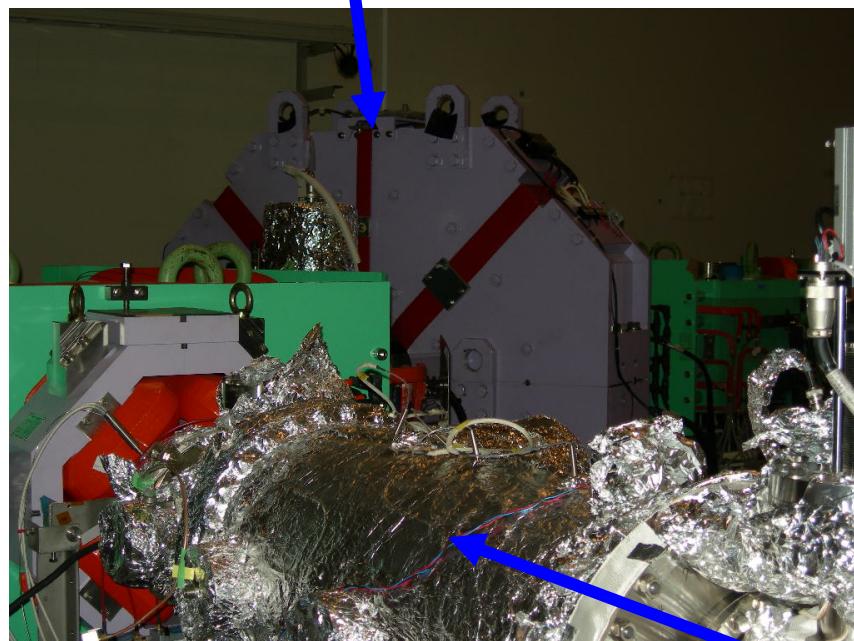
# Laser System for Cooling



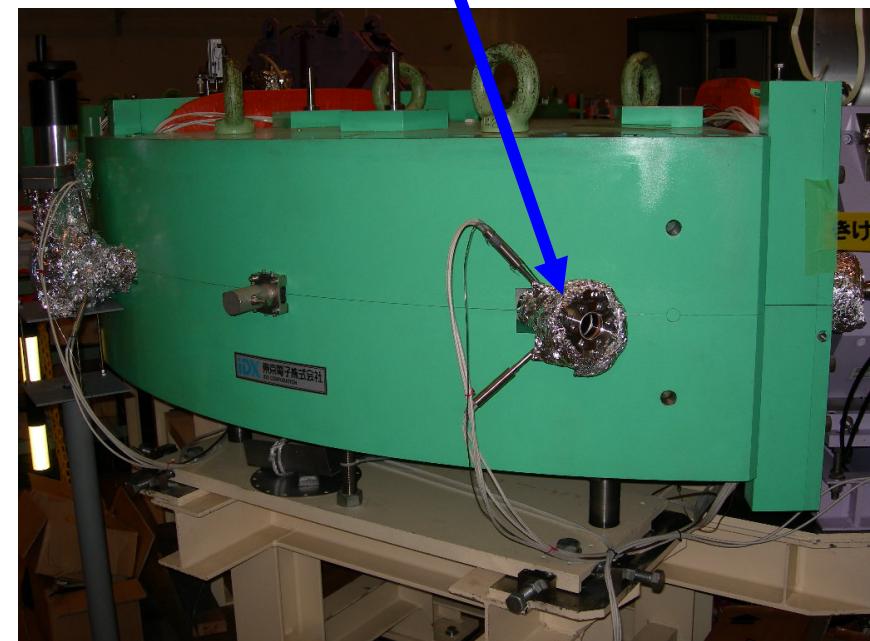


# Laser Cooling Section of S-LSR

Induction Accelerator



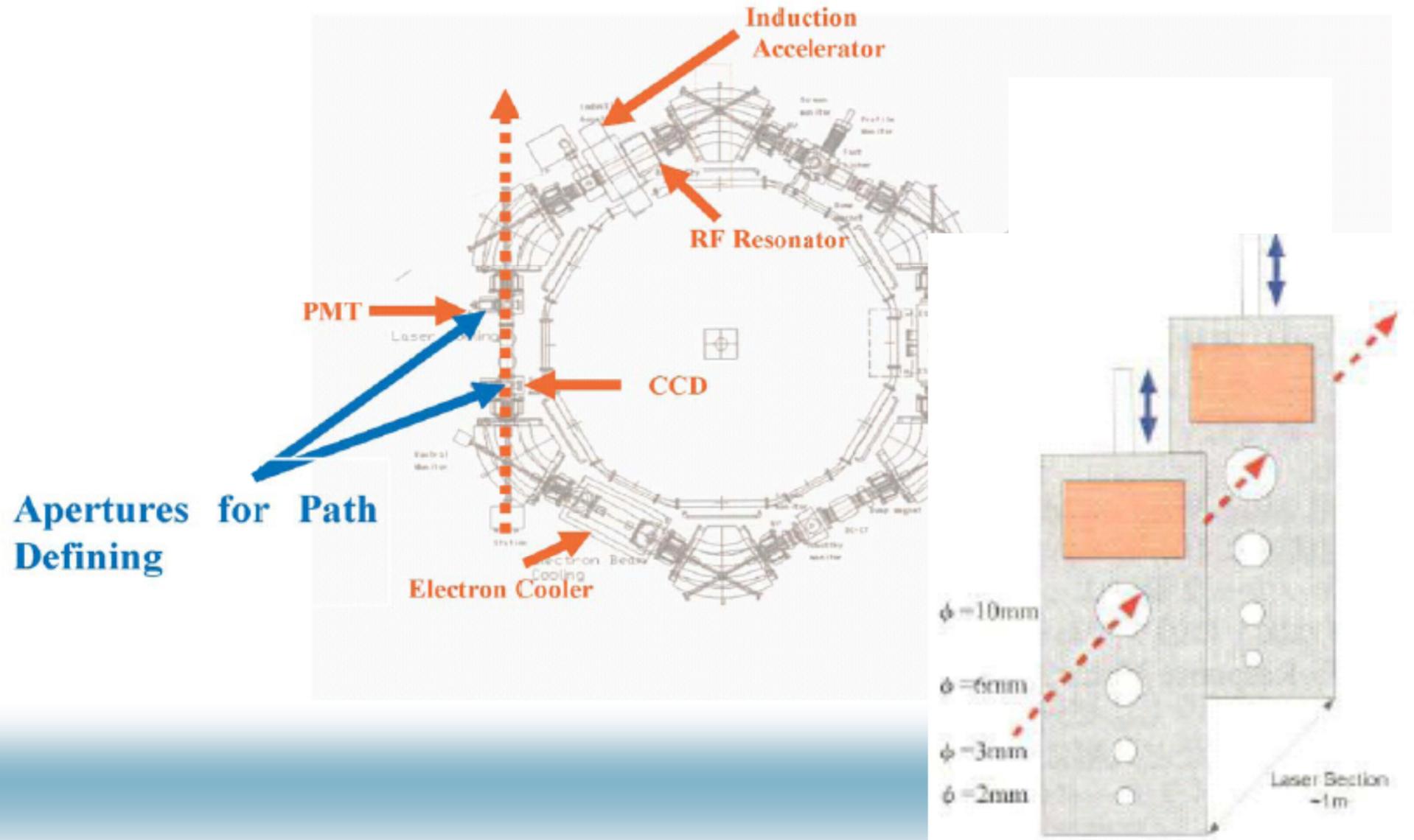
Window for Laser port  
(with Brewster Angle)

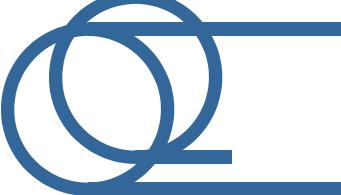


Helical Schottky Pick-up for 7 MeV proton is installed here.



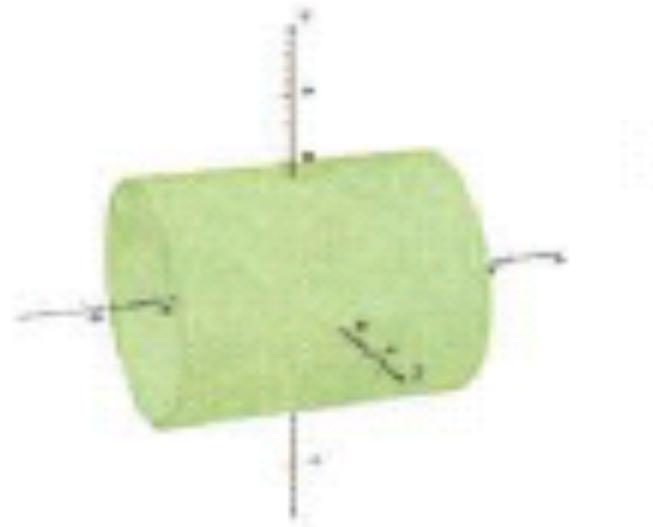
# Overlapping of Ion and Laser Beams





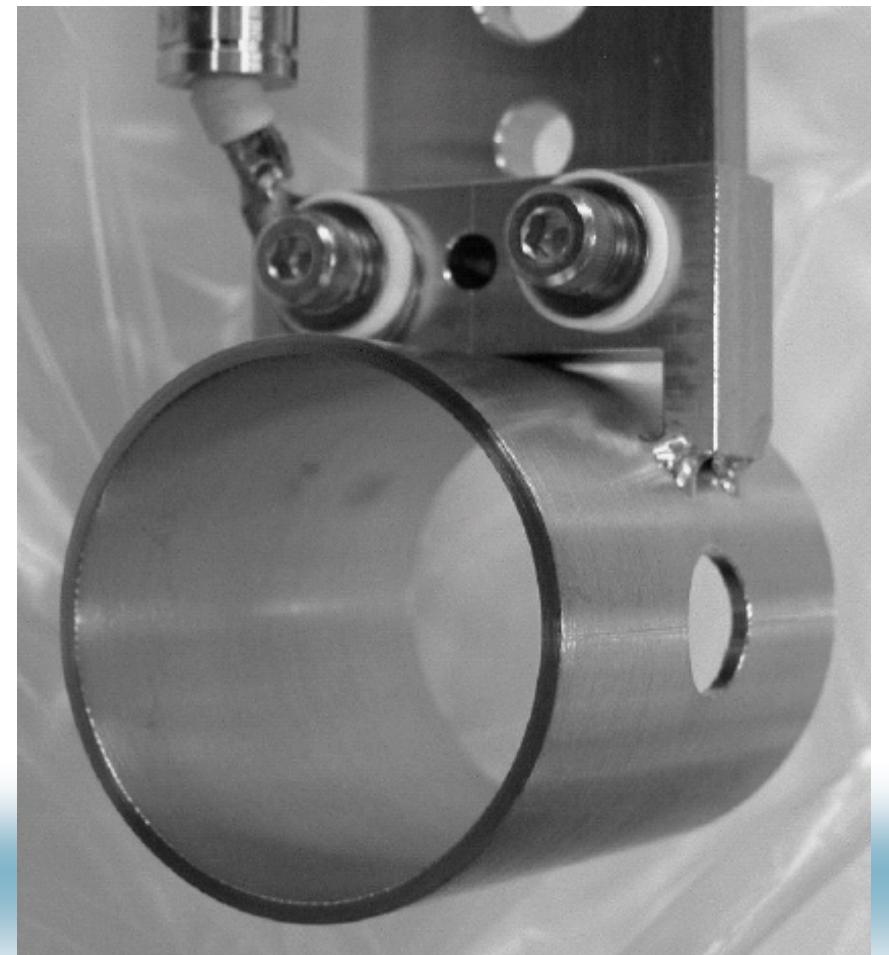
# Post Acceleration Tube (PAT)

-Energy Sweep is applied for Distribution Measurement-

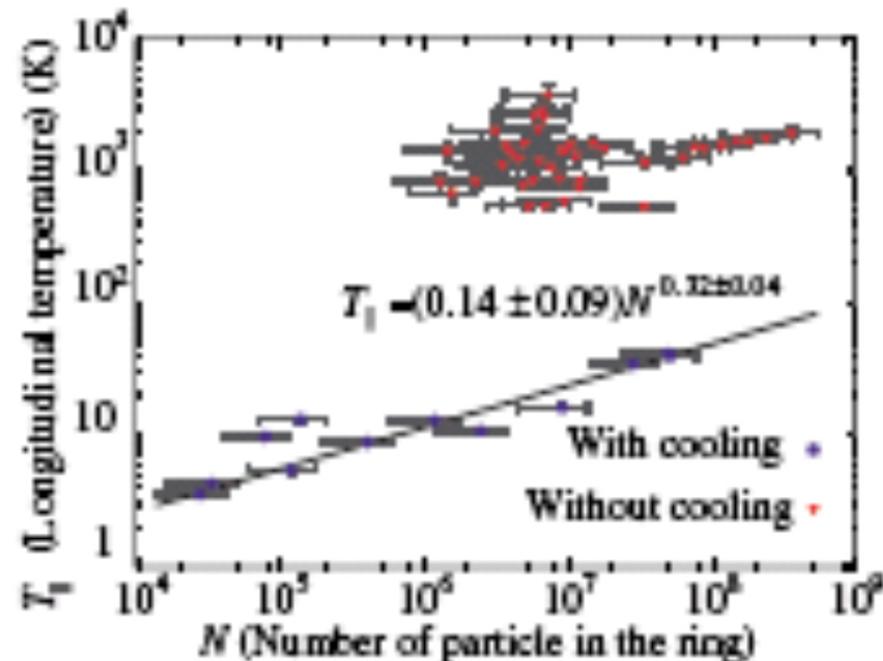
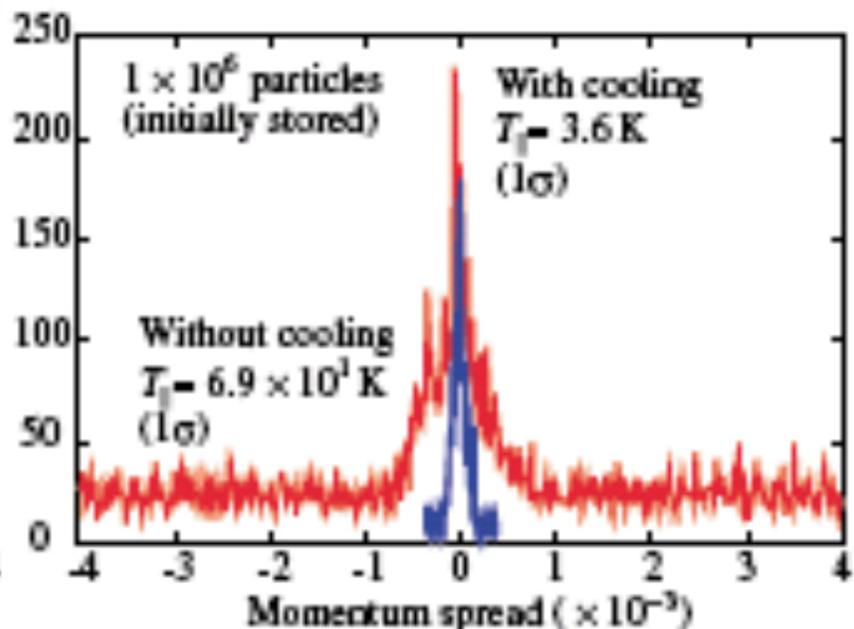


## Specification of PAT

Inner Diameter	$\phi 35$ mm
Outer Diameter	$\phi 38$ mm
Length	44 mm
Observation Hole	$\phi 10$ mm



# Laser Cooling of Coasting Beam at S-LSR



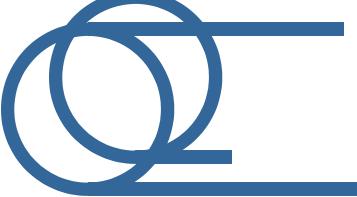
$$k_B T_L = m v_0^2 \left( \frac{\Delta p}{p} \right)^2$$

$$\Lambda_{IBS} \propto \frac{N}{T_H T_V \sqrt{T_L}}$$

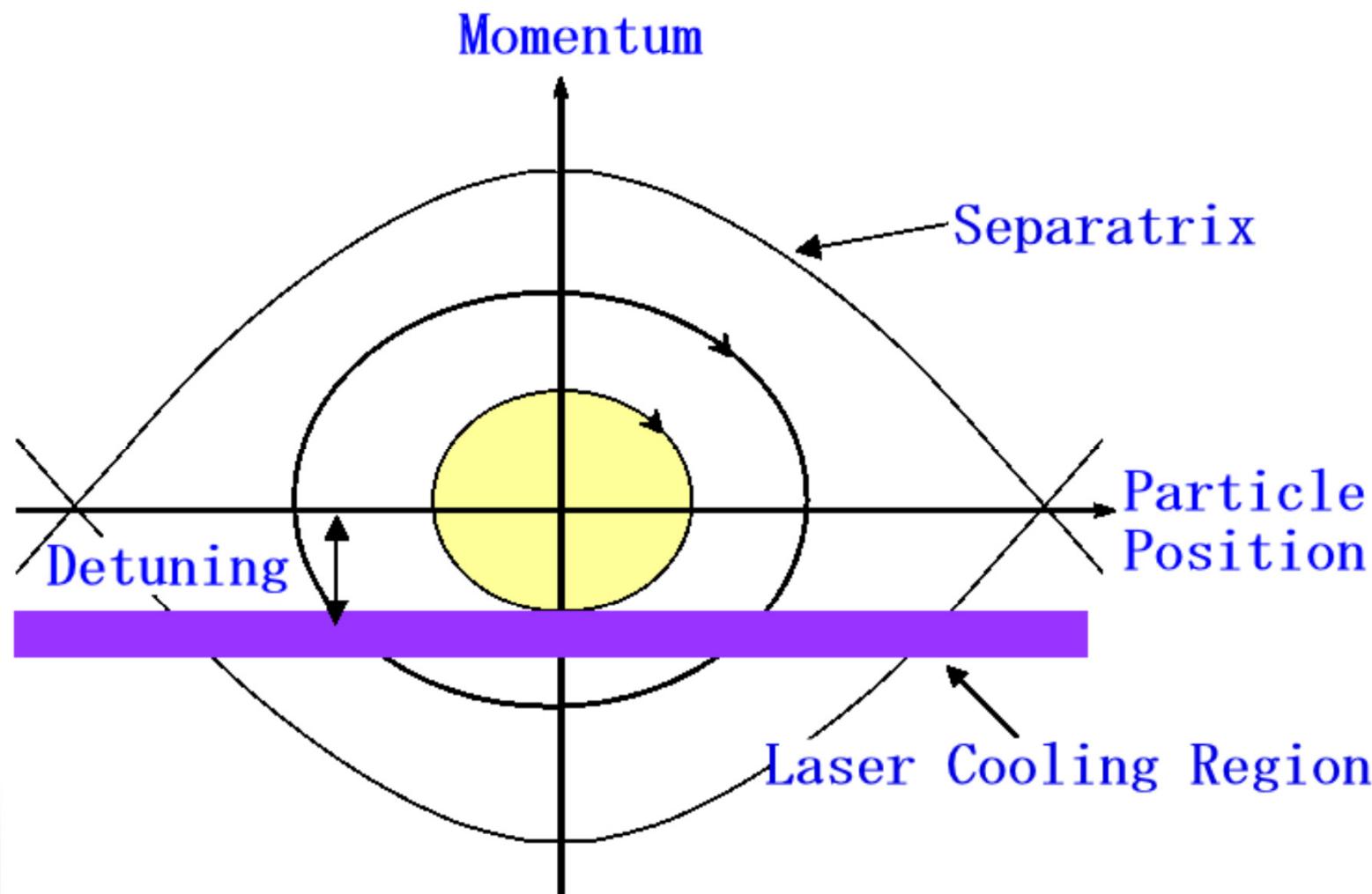
$$T_L \propto N^{0.32 \pm 0.04}$$

$$T_L = 0.02 T_\perp$$

$$T_L \propto N^{0.4}$$



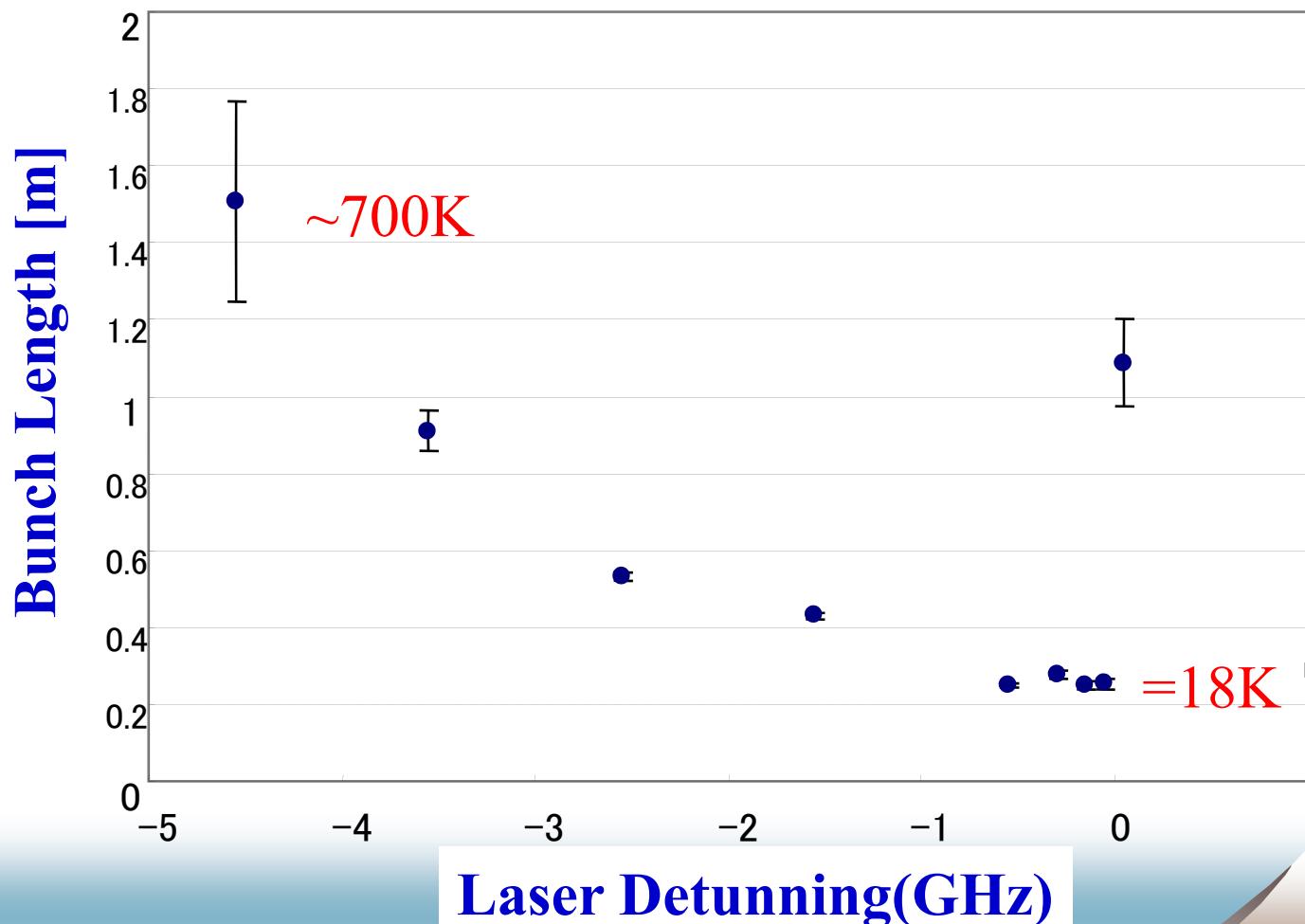
# Bunched Beam Cooling

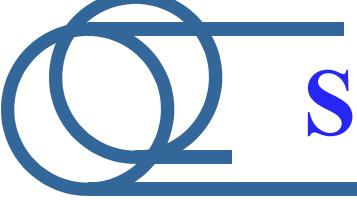




# Result of Bunched Beam Cooling

$N=6 \times 10^6$ , RF Freq=125.96kHz( $h=5$ ), Voltage=3.06V





# Synchro-Betatron Resonance Coupling (SBRC) Scheme

$$H = \underbrace{\frac{1}{2}(p_x^2 + \kappa_x^2 x^2)}_{\text{Betatron oscillation}} + \underbrace{\frac{1}{2}(p_x^2 + \kappa_y^2 y^2)}_{\text{Synchrotron oscillation (laser cooled)}} + \underbrace{\frac{1}{2}(p_x^2 + \kappa_z^2 z^2)}_{\text{Controllable coupling potential}} + \psi_c$$

- When three degrees of freedom are independent of each other ( $\psi_c = 0$ ), nothing takes place in  $x$  and  $y$  directions even if we strongly cool the  $z$  direction.
- Switch on the coupling potential to correlate the harmonic motions in the three directions. Linear coupling potentials should be employed for this purpose:

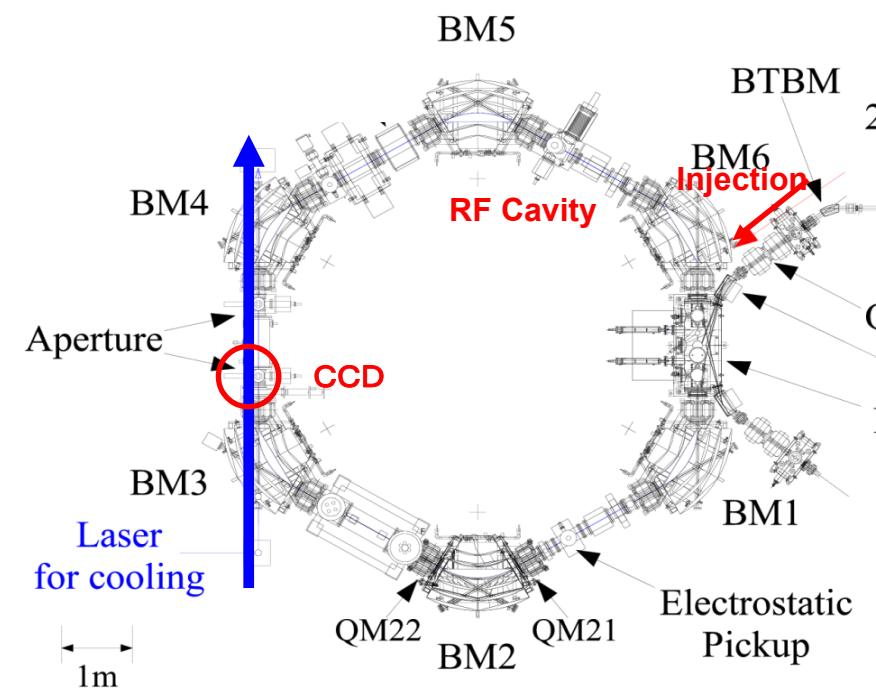
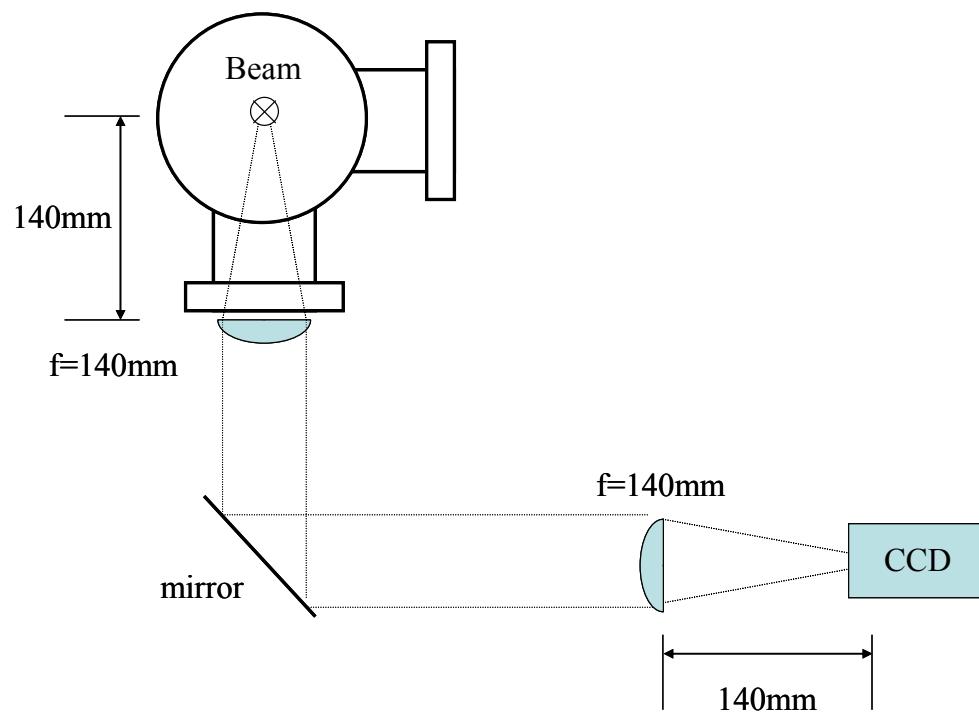
$$\psi_c = g_1 xy \cdot \delta_p(s - s_1) + g_2 xz \cdot \delta_p(s - s_2)$$

- Move the operating point onto coupling resonance:

$$\nu_x - \nu_y = \text{integer}, \quad \nu_x - \nu_z = \text{integer}$$



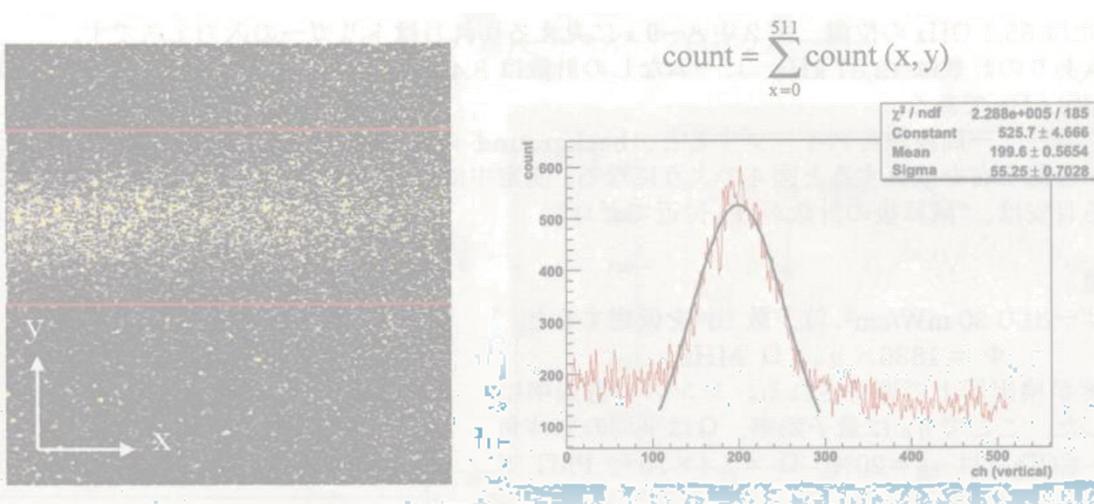
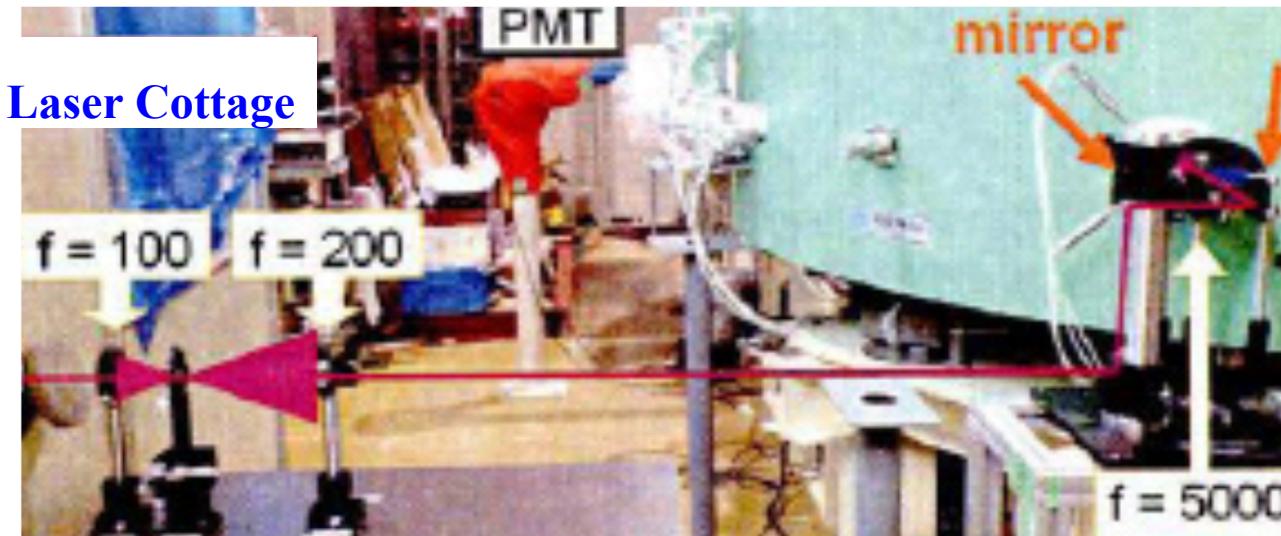
# Observation of Transverse Beam Size by CCD Camera



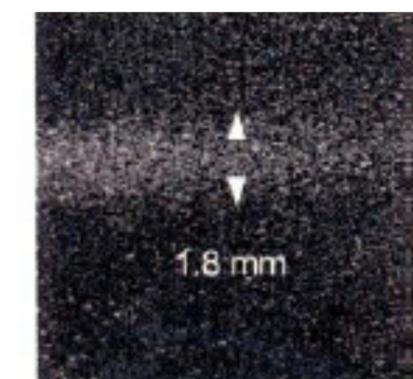
Cooled CCD Camera  
(Hamamatsu Photonics C7190-11W)



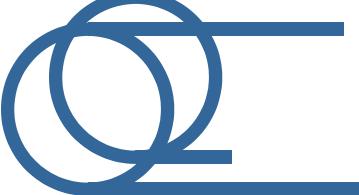
# Ion Observation with Emitted Light



Laser Profile



Fluorescent light  
from the ion beam

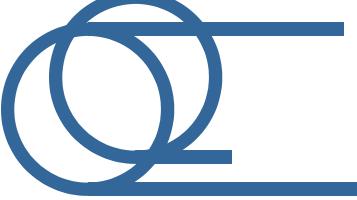


# L-H Coupling

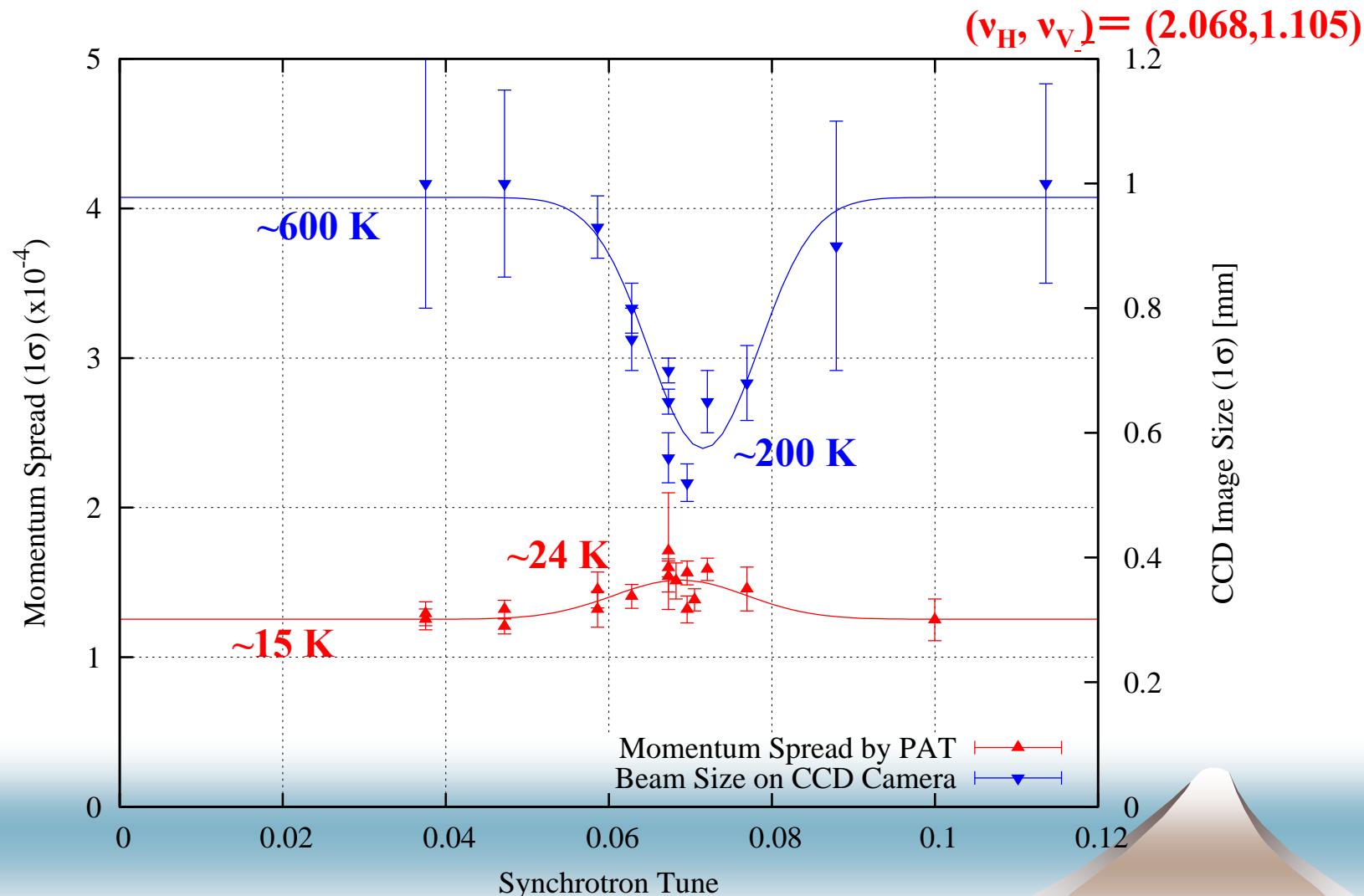
Only the relation:

$v_H - v_s = \text{integer}$

is satisfied!



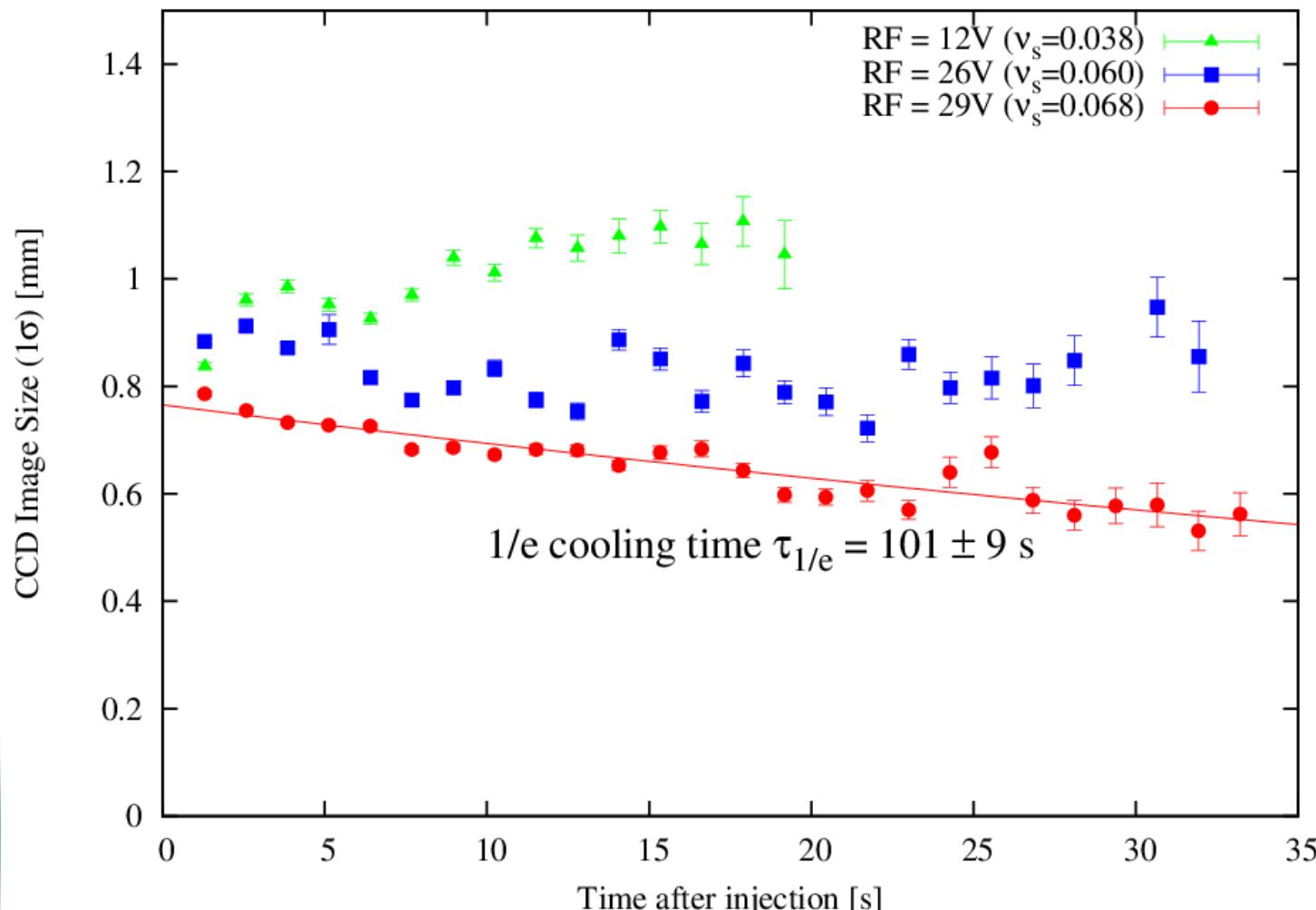
# Transverse Laser Cooling by Synchro-Betatron Coupling

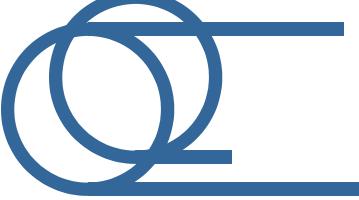




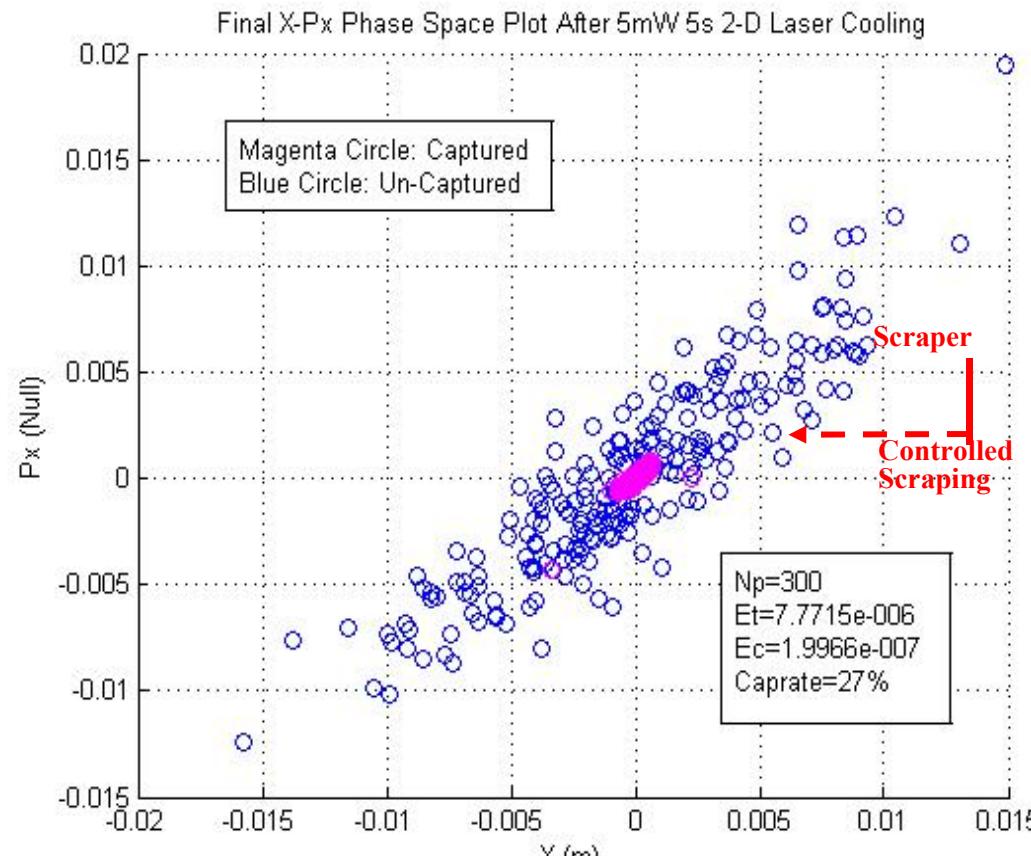
# Time Variation of Transverse Beam Size for Various Synchrotron Tune (Beam Intensity $1 \times 10^7$ )

$(v_H, v_V) = (2.068, 1.105)$



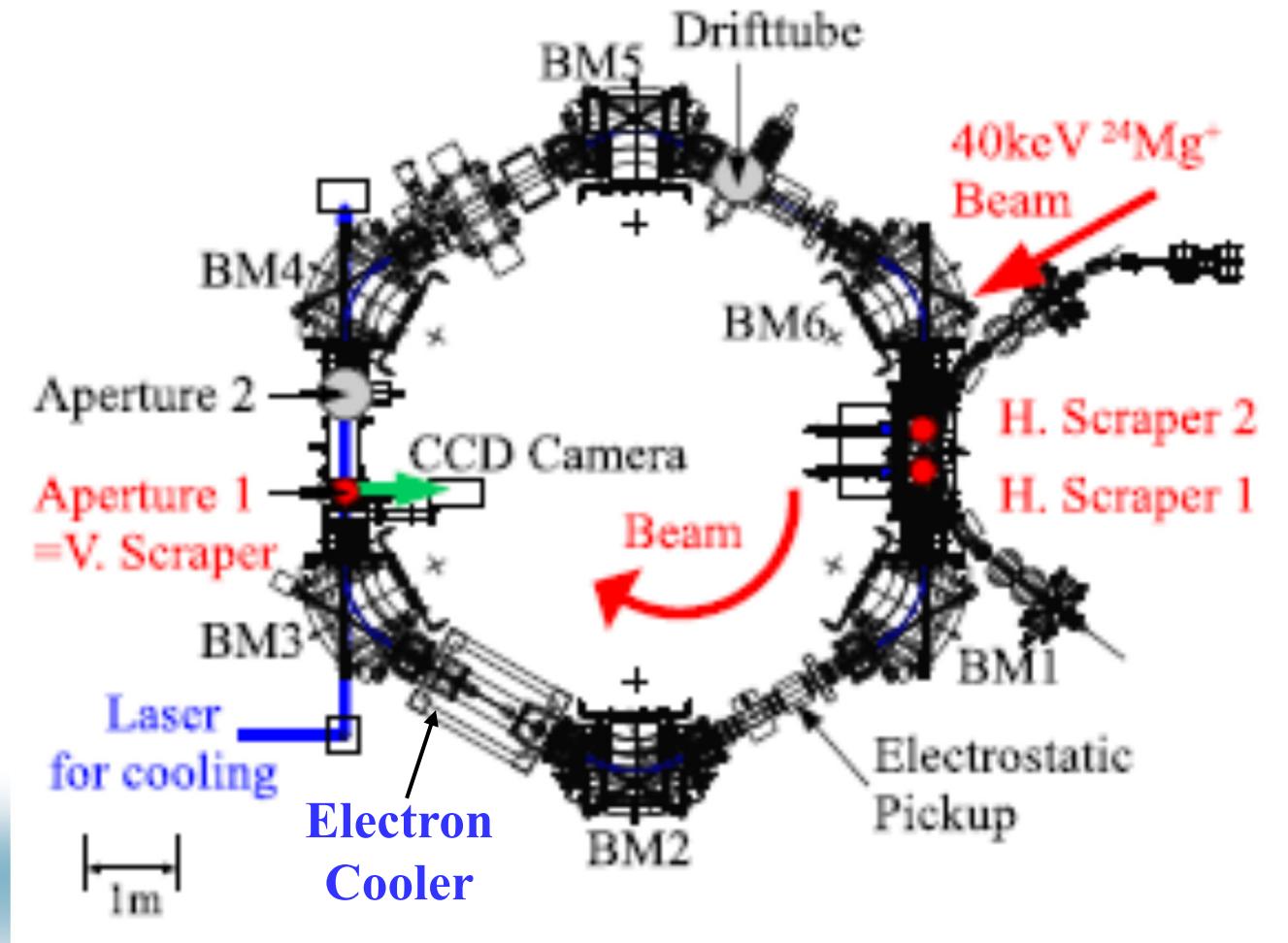


# Controlled Scraping to Suppress IBS Effects



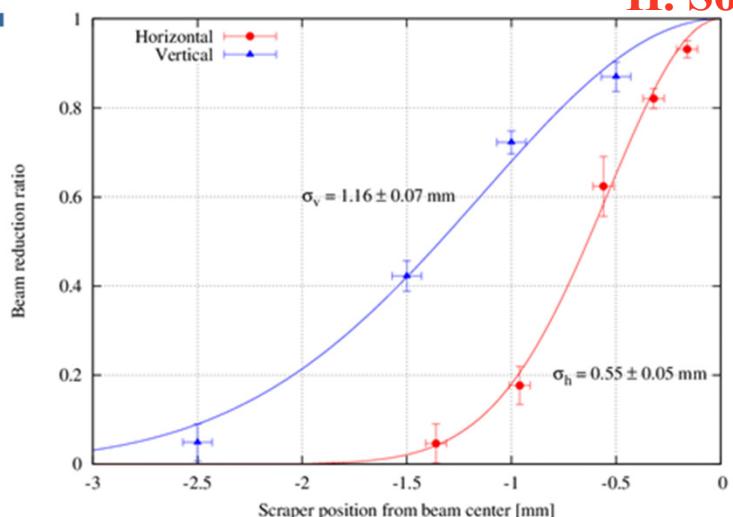


# Scraping System for Intensity Reduction and Beam Size Measurement

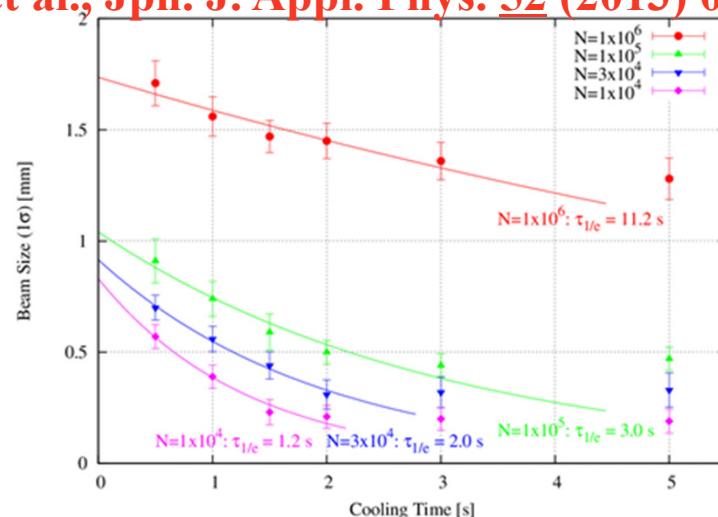


# Efficiency Increase of Transverse Laser Cooling by SBRC

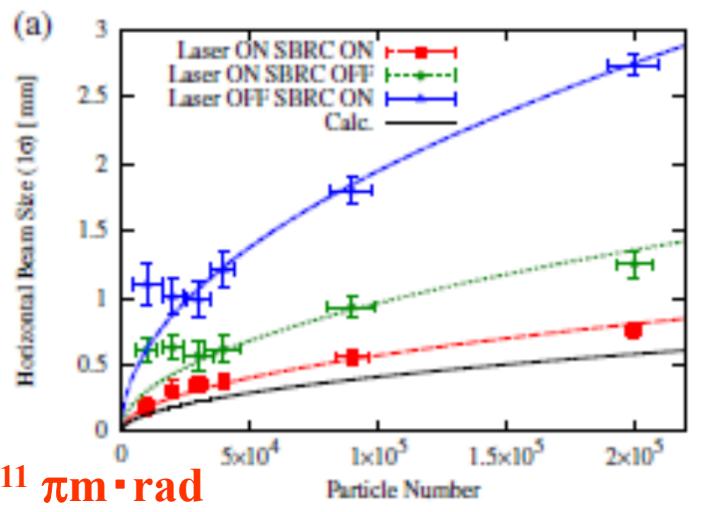
H. Souda et al., Jpn. J. Appl. Phys. 52 (2013) 030202



Beam size measurement by scraping

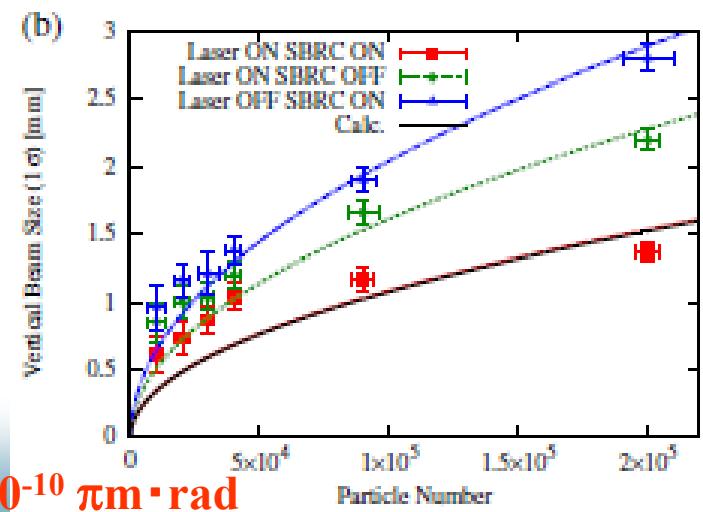


Time dependence of the cooled beam size



20 K  
 $7.0 \times 10^{-11} \pi m \cdot rad$

Horizontal beam size

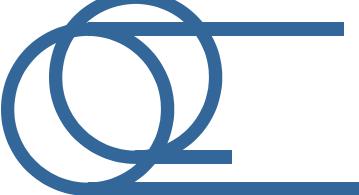


29 K  
 $1.9 \times 10^{-10} \pi m \cdot rad$

Vertical beam size

Ion Number Dependence of the Indirectly Laser cooled Beam Size

16, June, 2014 Akira Noda, NIRS at IPAC'14, Dresden, Germany



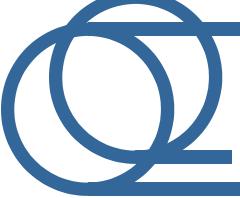
# H-V Coupling is added

**Relations:**

$v_H - v_s = \text{integer}$ ,

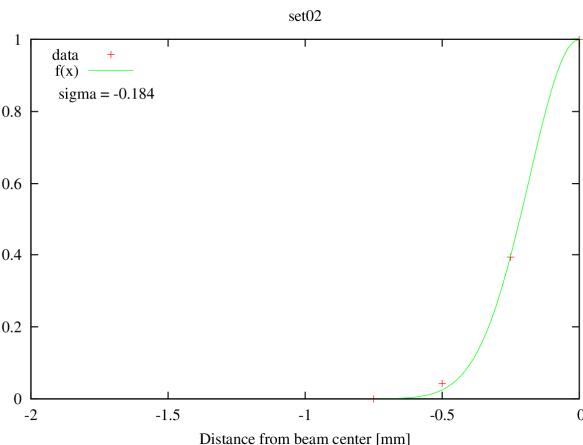
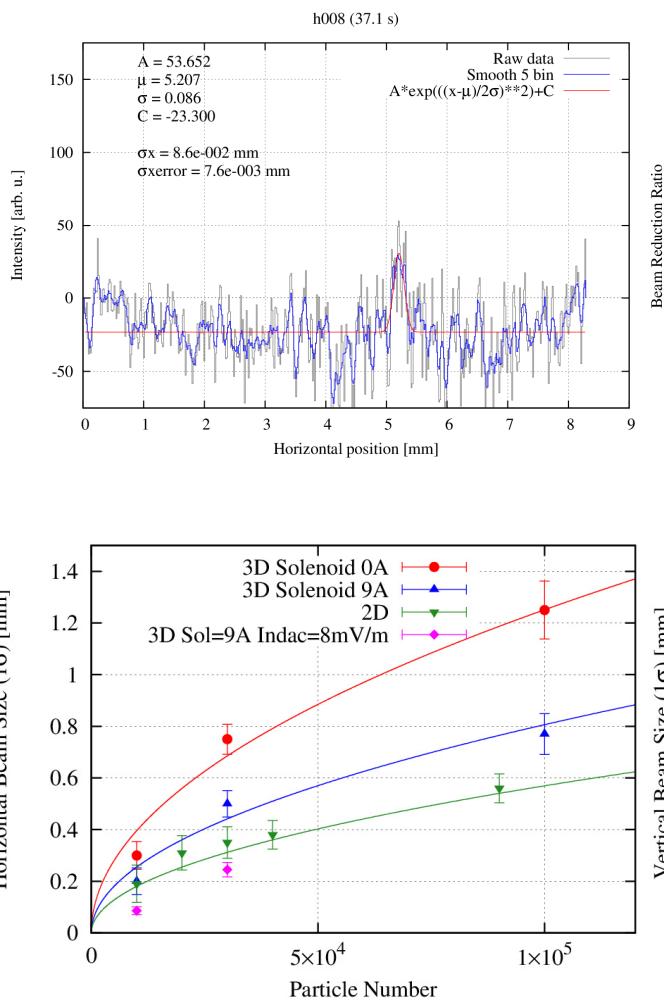
$v_H - v_V = \text{integer}$

**are satisfied together with  
the use of a Solenoid**

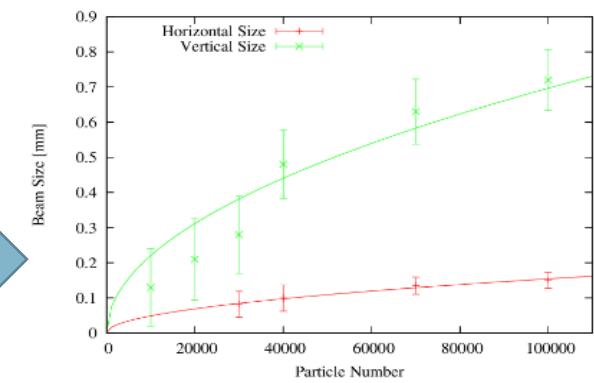
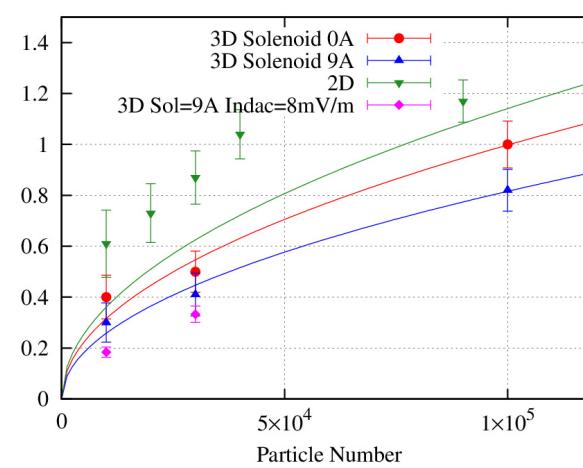
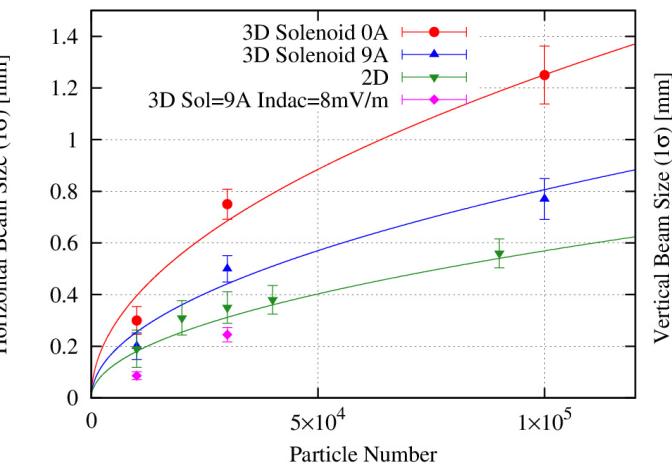


# 3 D Laser Cooling + INDAC

Operation Point = (2.068, 1.07)



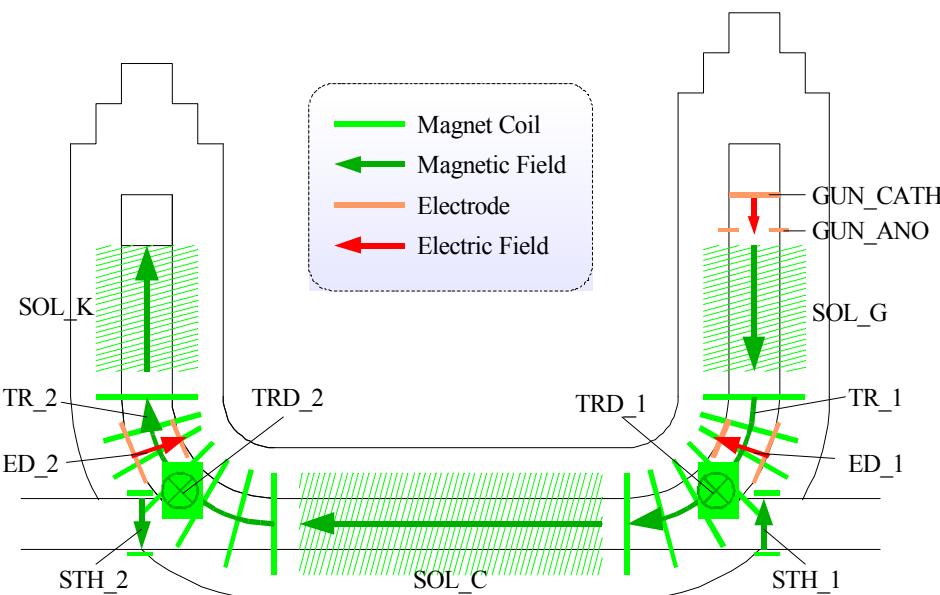
Optimization →



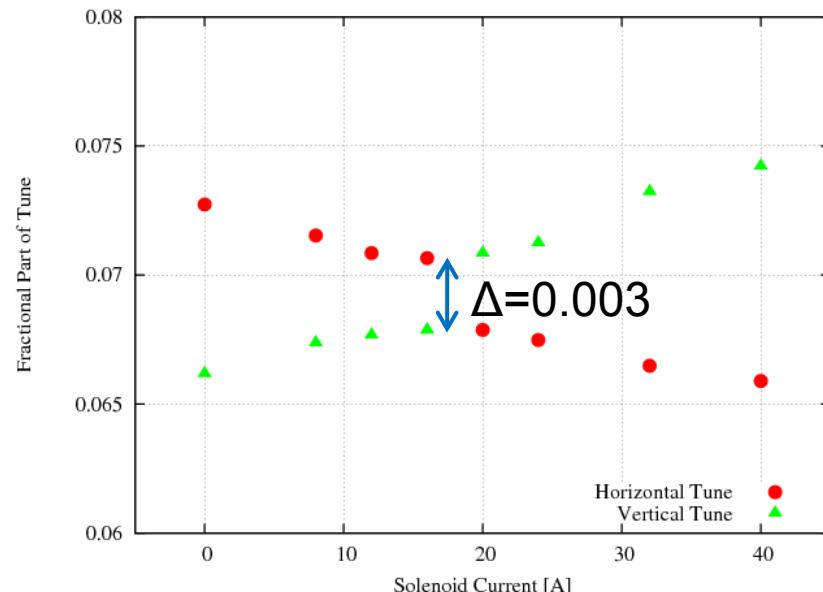
Horizontal  
6.4 K  
 $1.3 \times 10^{-11} \pi \text{m} \cdot \text{rad}$   
Vertical  
2.1 K  
 $8.5 \times 10^{-12} \pi \text{m} \cdot \text{rad}$



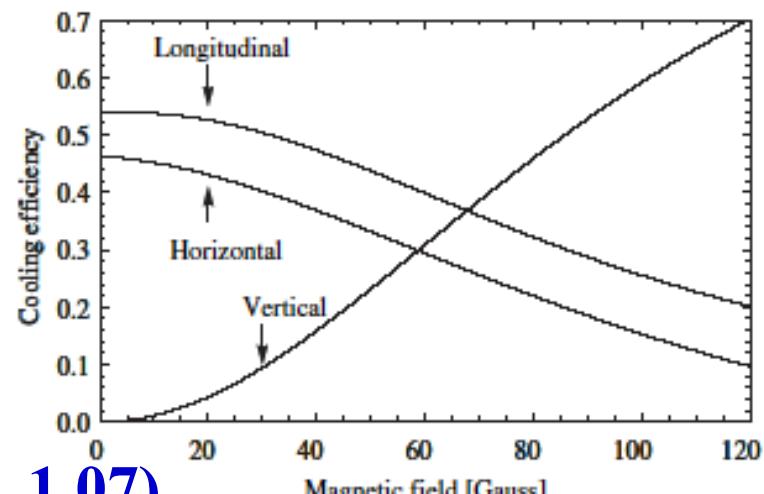
# H-V Coupling with a Solenoid Field



Solenoid Field of Electron Cooler  
(Effective Length=1.2 m)



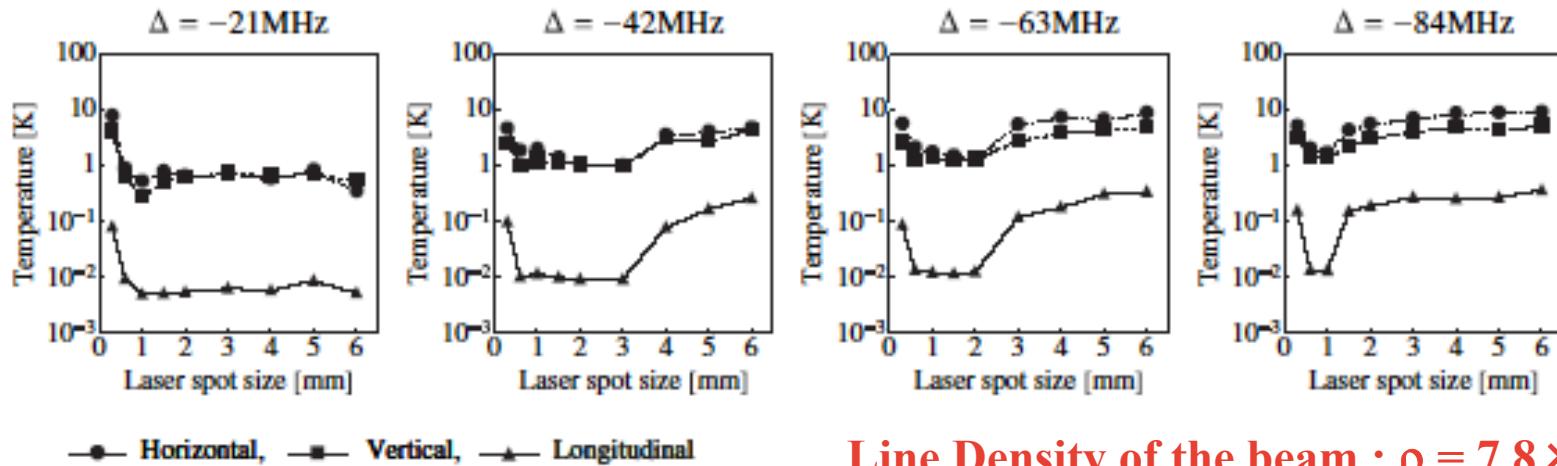
Operating Point  $\sim (2.073, 1.067)$



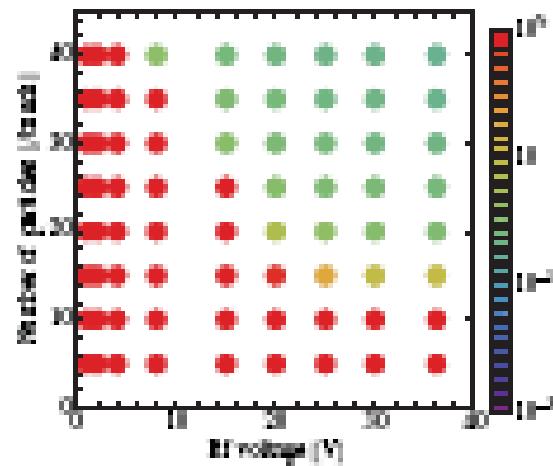
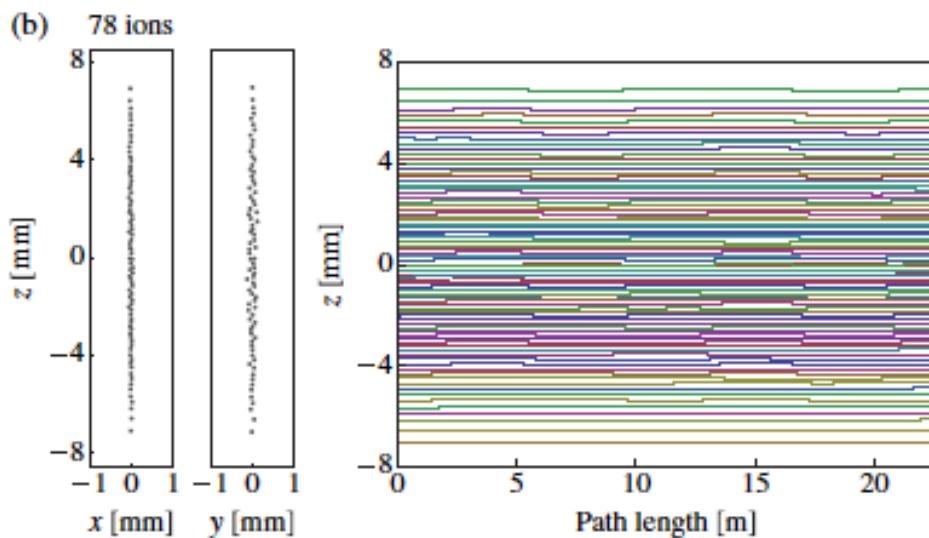
Operating Point  $\sim (2.07, 1.07)$

# Prediction of 1D Longitudinal String (Bunched)

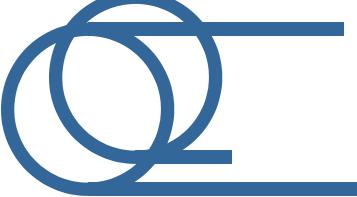
Laser spot size : 1.5 mm, detuning -42MHz, with RF Voltage Ramping



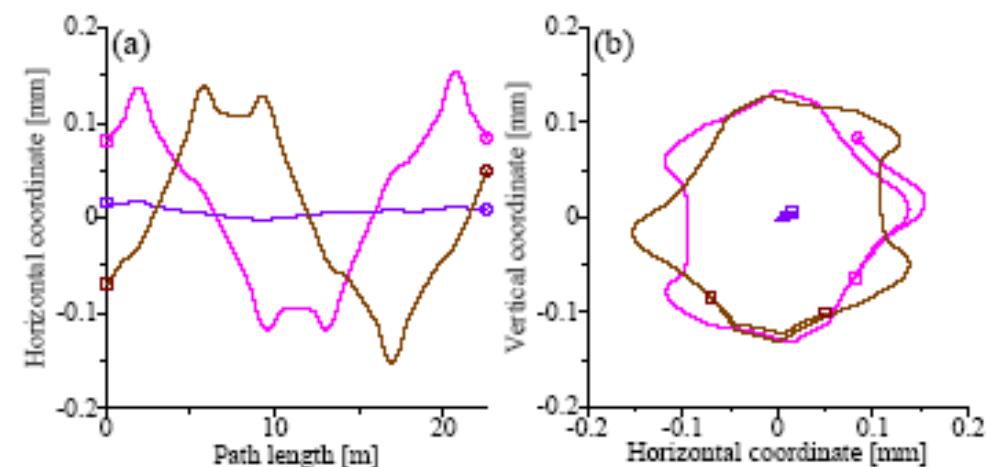
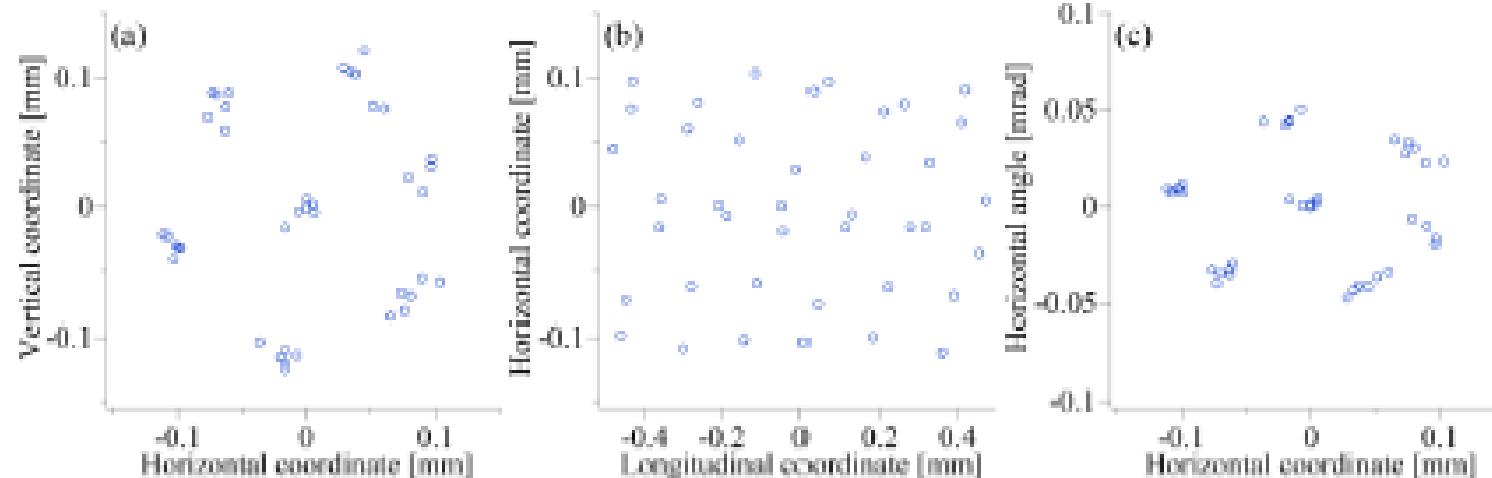
Line Density of the beam :  $\rho = 7.8 \times 10^3 \text{ 1/m}$



Longitudinal  $0.001 \text{ K} : 10^{-13} \pi\text{m} \cdot \text{rad}$ , Transverse  $0.1 \text{ K} : 10^{-12} \pi\text{m} \cdot \text{rad}$



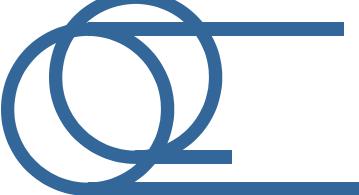
# MD Simulation of 3D Ordered State



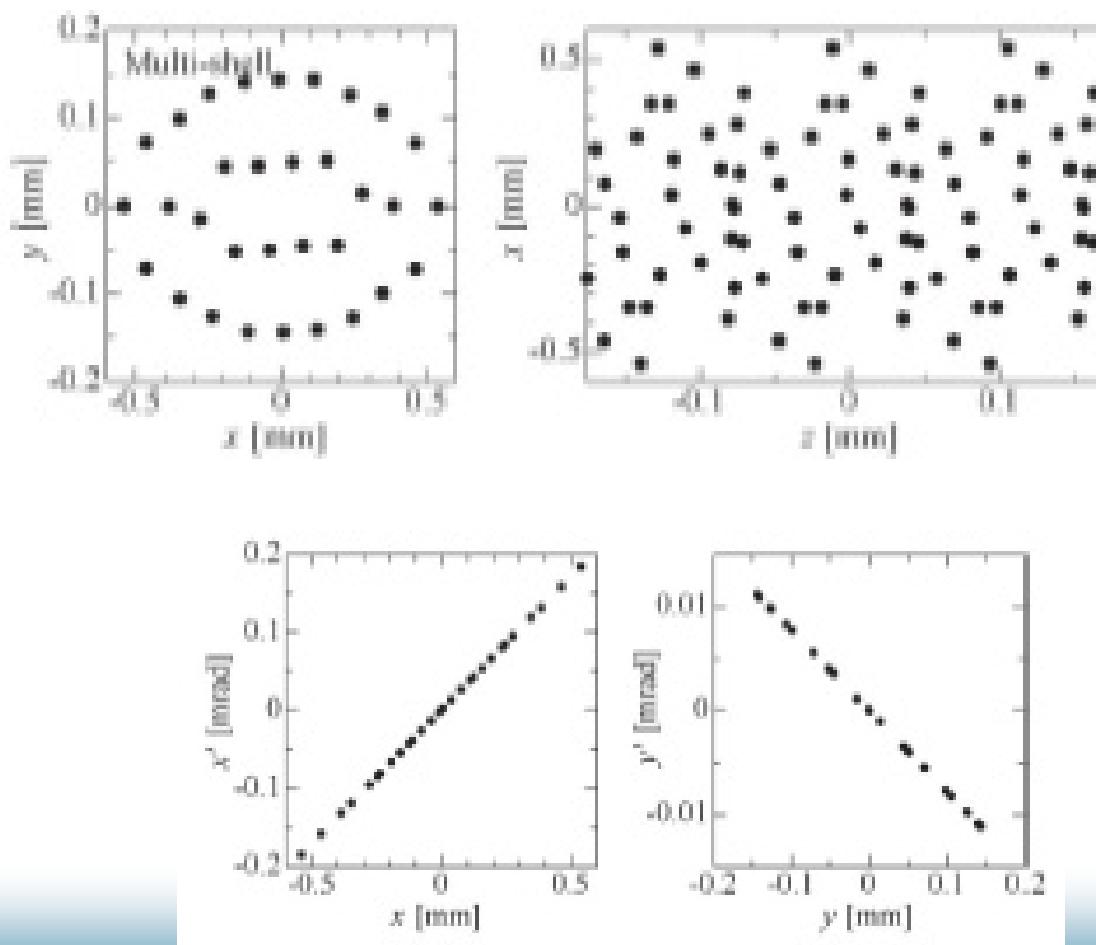
Operation Point = (1.44, 1.44)  
Laser Waist Size  $\sim 2.5$  mm ( $1\sigma$ )  
3 mm shifted in parallel to  $^{24}\text{Mg}^+$  beam  
Ramped final laser detuning  $\sim -61$  MHz

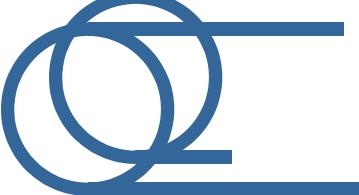
Line Density of the beam :  $\rho = 4 \times 10^4$  1/m

Predicted Temperature  
= 6mK (longitudinal )  
= 0.6K (Transverse)



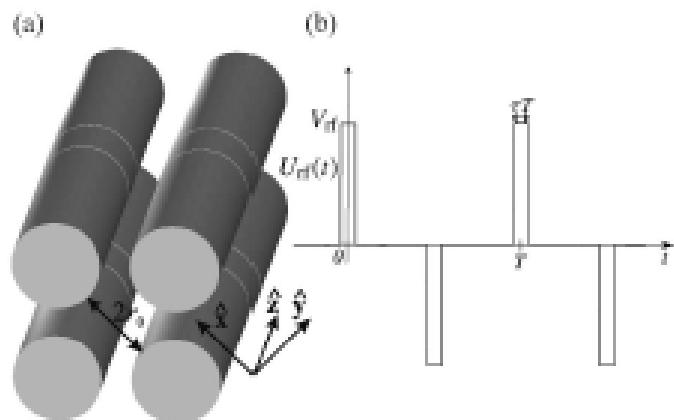
# Double-Shell Crystalline Beam



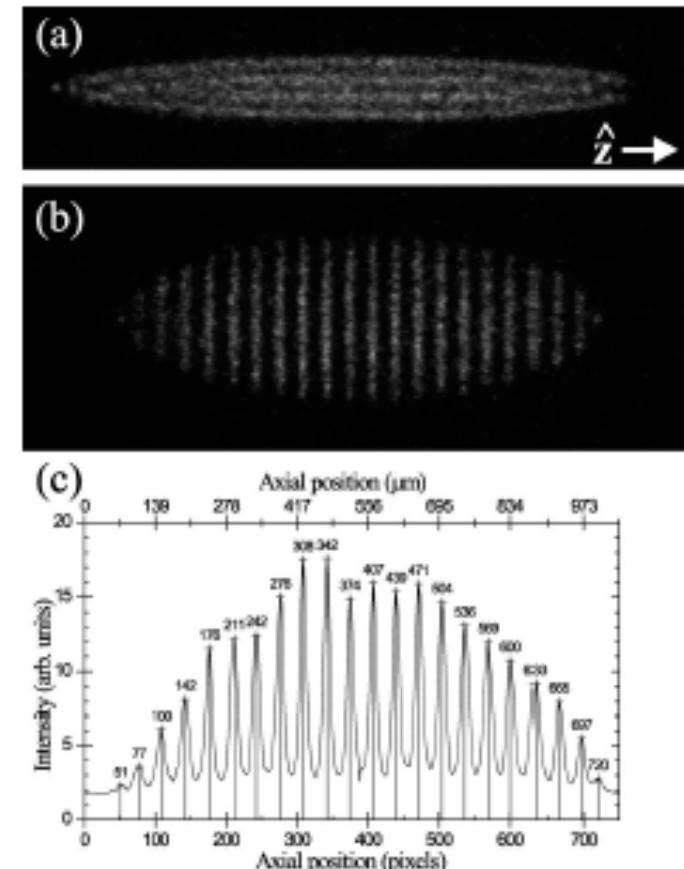


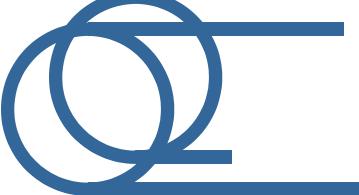
N. Kjærgaard and M. Drewsen, Phys. Rev. Lett., 91, 095002 (2003)

# ”String of Discs” observed at Linear Paul Trap



$$\Psi(x, y, z, t) = \frac{U_d(t)}{2r_0^2}(x^2 - y^2) + \frac{U_{dc}\eta}{2z_0^2}[2z^2 - (x^2 + y^2)],$$

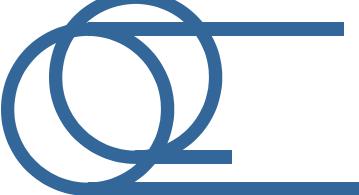




# Comparison of Experimental Data and MD Simulations

Table 2 Comparison of laser cooled ion beam temperatures. ( Shaded columns represent the recent MD simulations.)

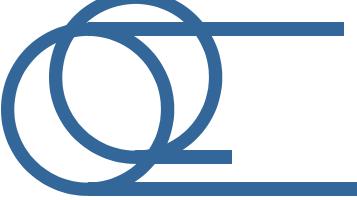
Year Ring	Method	Ion	Kinetic Energy	Intensity	Beam Density	$T_{\parallel}$ (K)	$T_H$ (K)	$T_V$ (K)	Ref.
1996 TSR	IBS	$^{9}_{\Lambda}\text{Be}^+$	7.3 MeV	$2.0 \times 10^7$	$3.6 \times 10^5$	15	4000	500	[15]
1998 TSR	Dispersive cooling	$^{9}_{\Lambda}\text{Be}^+$	7.3 MeV	$1.0 \times 10^7$	$1.8 \times 10^5$	few tens	$\sim 300^\pm$	$\sim 150^\pm$	[16]
1999 ASTRID	IBS	$^{24}_{\Lambda}\text{Mg}^+$	100 keV	$7 \times 10^6$	$1.8 \times 10^5$	2-3	17	21	[17]
2001 PALLAS	RFQ	$^{24}_{\Lambda}\text{Mg}^+$	1 eV	$1.8 \times 10^4$	$5.0 \times 10^4$	$\sim 0.003$	$T_{\perp} < 0.4$		[28]
2006 S-LSR	IBS	$^{24}_{\Lambda}\text{Mg}^+$	40 keV	$1.0 \times 10^7$	$4.4 \times 10^5$	11	-	500	[22]
2009 S-LSR	W SBRC (2D)	$^{24}_{\Lambda}\text{Mg}^+$	40 keV	$1.0 \times 10^7$	$4.4 \times 10^5$	27	220 <sup>±</sup>		[25]
2009 S-LSR	WO SBRC	$^{24}_{\Lambda}\text{Mg}^+$	40 keV	$1.0 \times 10^7$	$4.4 \times 10^5$	16			[25]
2012 S-LSR	W SBRC (2D)	$^{24}_{\Lambda}\text{Mg}^+$	40 keV	$1 \times 10^4$	$4.4 \times 10^2$	(0.4)	20	29	[26]
2013.2.1 S-LSR	W SBRC (3D)	$^{24}_{\Lambda}\text{Mg}^+$	40 keV	$1 \times 10^4$	$4.4 \times 10^2$	-	40	11	[27]
2013.3.7 S-LSR ( $\Delta f = -190$ MHz)	W SBRC (3D) (INDAC ON)	$^{24}_{\Lambda}\text{Mg}^+$	40 keV	$1 \times 10^4$	$4.4 \times 10^2$	-	8.1	4.1	[27]
2013.3.22 S-LSR ( $\Delta f = -26$ MHz)	W SBRC (3D) (INDAC ON)	$^{24}_{\Lambda}\text{Mg}^+$	40 keV	$1 \times 10^4$	$4.4 \times 10^2$	-	6.4 ( $3 \times 10^4$ )	2.1	[27]
Simulation with MD ( $\Delta f = -42$ MHz)	W SBRC (3D) (RF trapping)	$^{24}_{\Lambda}\text{Mg}^+$	40 keV	$7.8 \times 10^3$	$6 \times 10^3$	$\sim 0.001$	$\sim 0.1$	$\sim 0.1$	[30]
Simulation with MD ( $\Delta f = -61$ MHz)	W SBRC (3D) (W Dispersive cooling)	$^{24}_{\Lambda}\text{Mg}^+$	40 keV	$9 \times 10^3$	$4.0 \times 10^4$	0.003	0.6	0.6	[31]



# Conclusions

1. Lowest temperature (smallest normalized emittance) was realized for moving ion beam of non-negligible speed by Doppler Laser Cooling with SBRC.
2. Its normalized temperature comes close to the one of 1D Longitudinal String predicted by MD simulation although the momentum spread has not yet measured for such a small beam intensity. →Urgent Technical Issue (Improvement of Observation Sensitivity is needed!!)
3. 3D Ordered State is also predicted by Dispersive Cooling of Coasting Beam following the one at an Ion Trap.

=>These prediction is desired to be realized experimentally as soon as possible!!



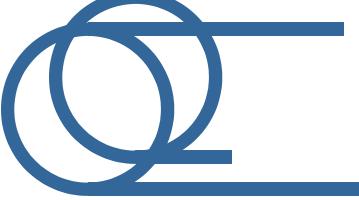
# Thank you for your attention!!



Vielen Dank für Ihre  
Aufmerksamkeit!!

Borrowed from Dr. M. Grieser

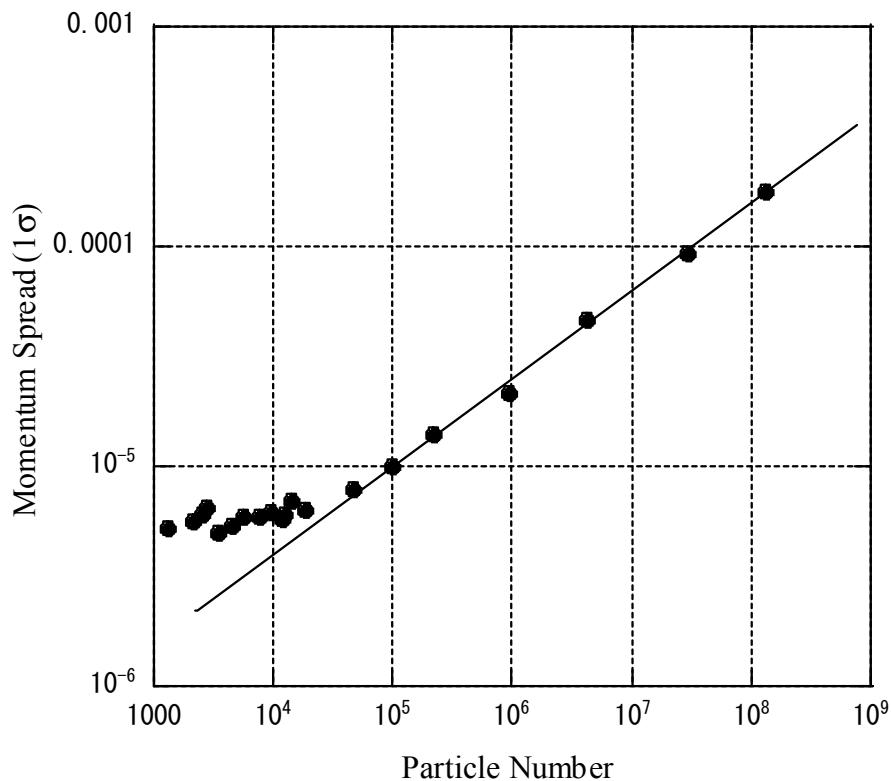
Akira Noda, NIRS at IPAC'14, Dresden, Germany



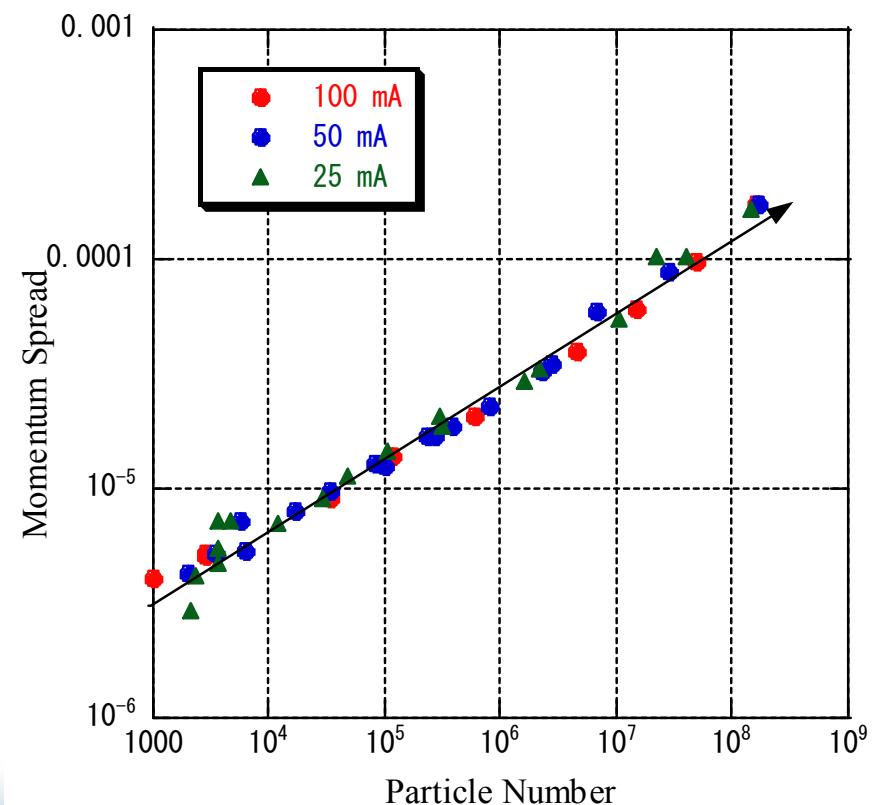
# Back Up Slides



# Fractional Momentum Spread vs Particle Number



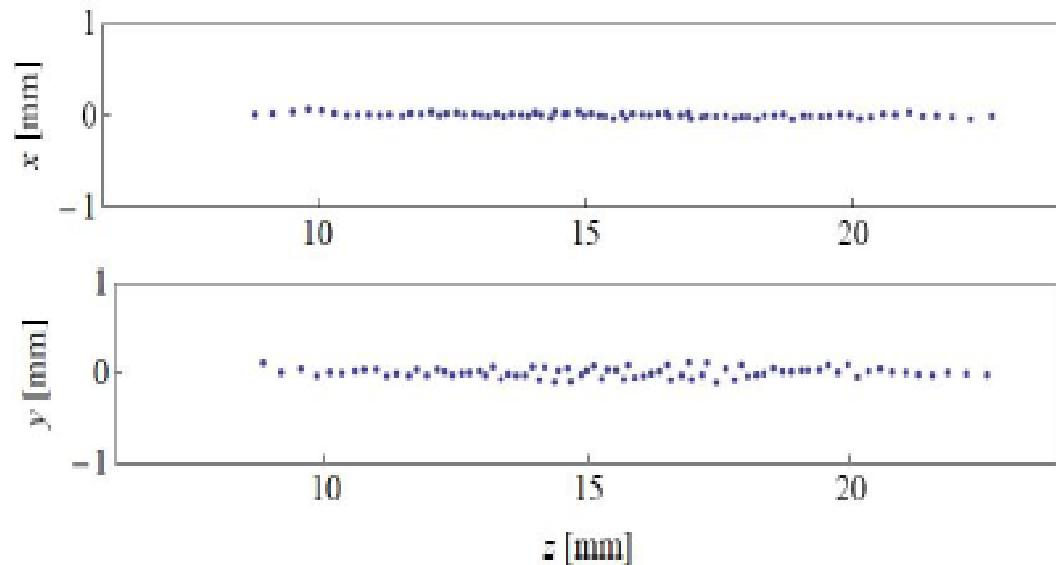
Data in Feb., 2006



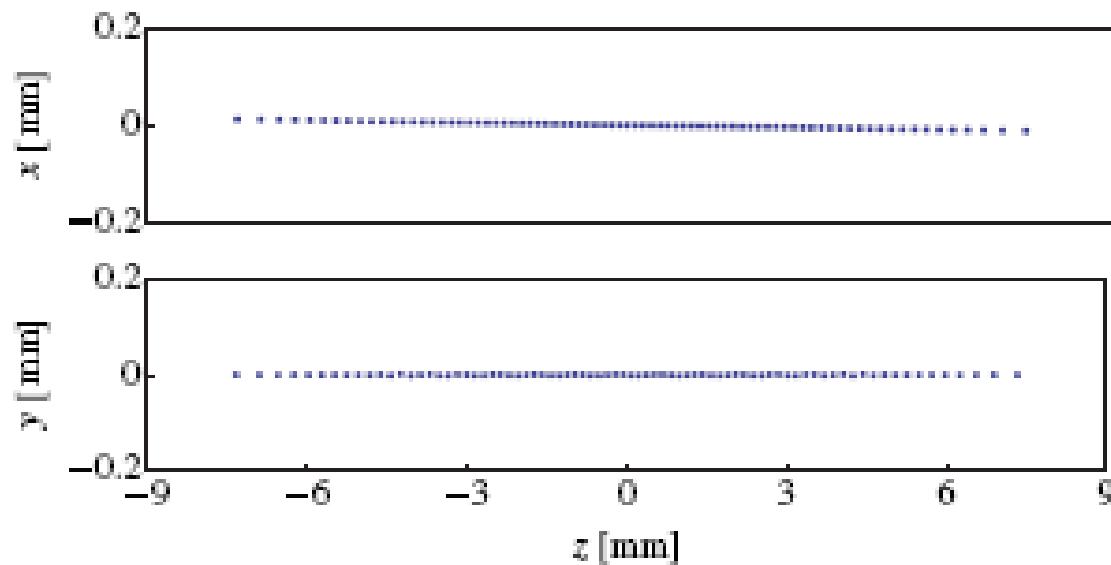
Data on the 8<sup>th</sup>, June, 2006

# Comparison of 1 D Longitudinal String Structure

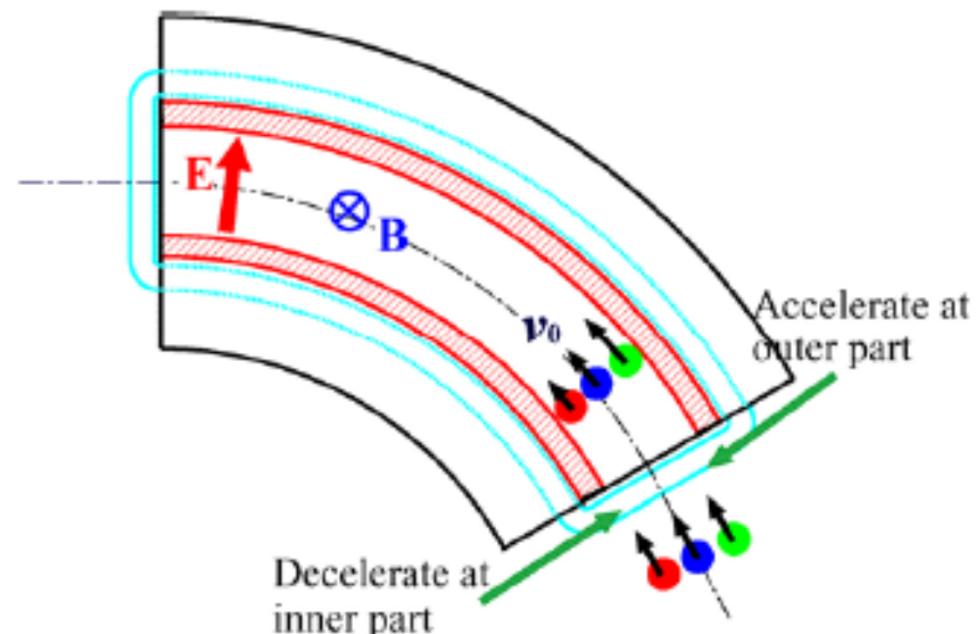
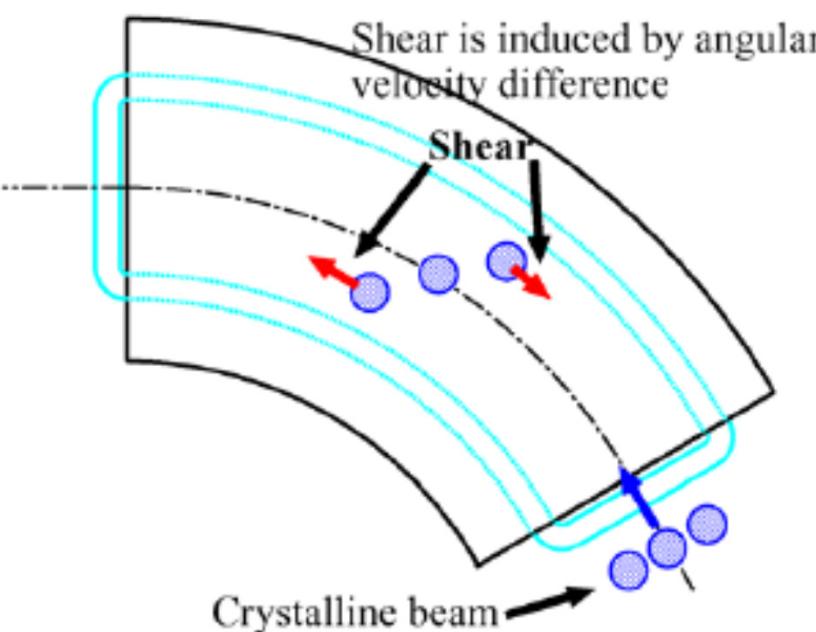
Assume real Laser used at S-LSR Experiment



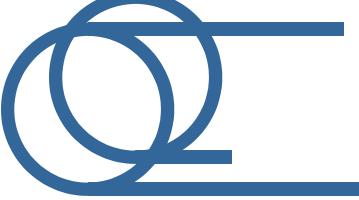
Linear dissipative force in 3 dimensions are assumed



# Shear Heating and Dispersion Suppressor



This idea was first proposed by W. Henning: Anns. Phys. Lpz. 19, 335 (1934) and W.E. Millet :Phys. Rev. 74, 1058 (1948) and is also recently claimed by R.E. Pollock, Z. Phys. :A. Hadrons and Nuclei. 341, 95 (1991)



# Dispersion Suppressor

$$\frac{d^2x}{ds^2} + \frac{3-n}{\rho^2}x = \frac{1}{\rho} \frac{\Delta W}{W}$$

**Electric Field**

$$\frac{d^2x}{ds^2} + \frac{1-n}{\rho^2} = \frac{1}{\rho} \frac{\Delta p}{p}$$

**Magnetic Field**

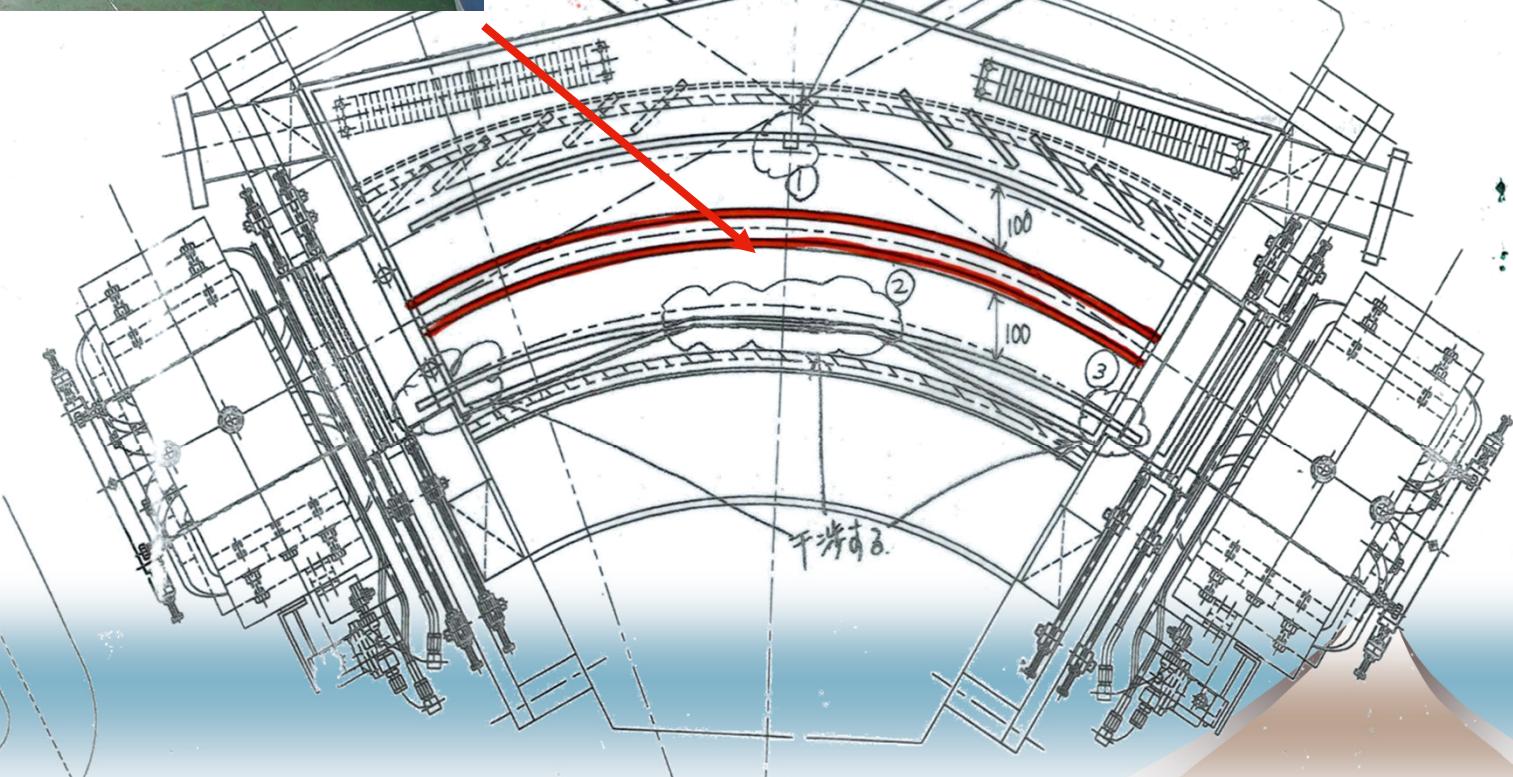
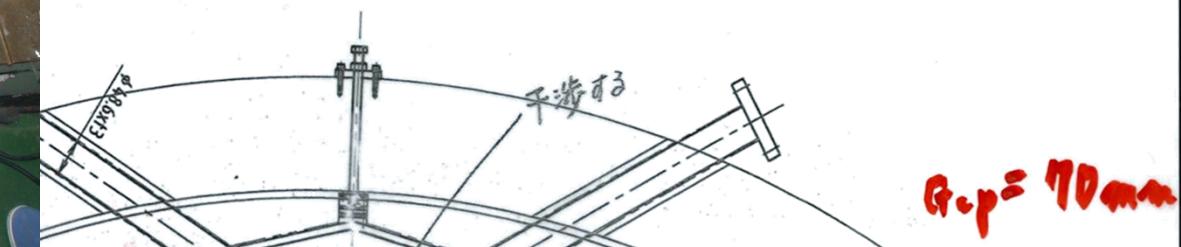
$$\frac{\Delta W}{W} = 2 \frac{\Delta P}{P}$$

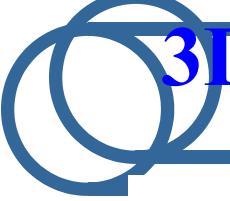
**Non-relativistic Case**



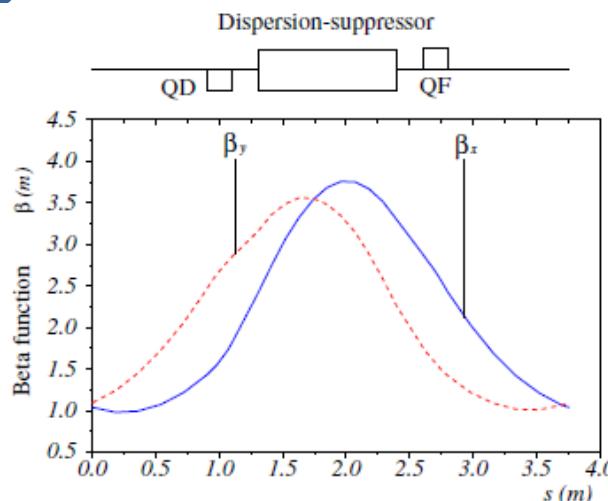
$$2\vec{E} = -(\vec{v} \times \vec{B})$$

# Vacuum Chamber in the Magnet Section (includes the Electrodes)

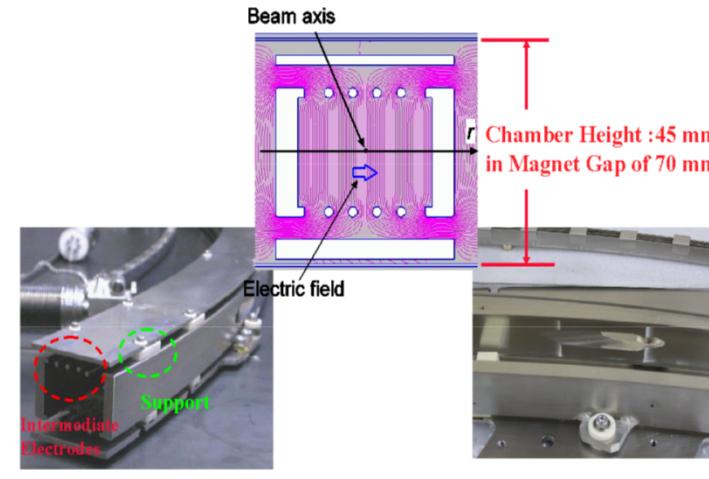
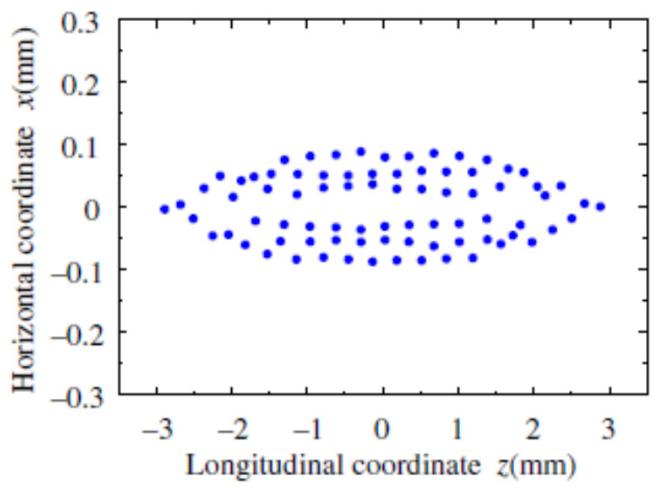




# 3D Laser Cooling with Dispersion Free Lattice



$$(v_x, v_y) = (2.07, 2.07)$$



$$2\vec{E} = -(\vec{v} \times \vec{B})$$

A coupling cavity is needed  
To couple the longitudinal  
and transverse degrees of  
Freedom.  
A few layers 3D crystalline  
beam is expected.

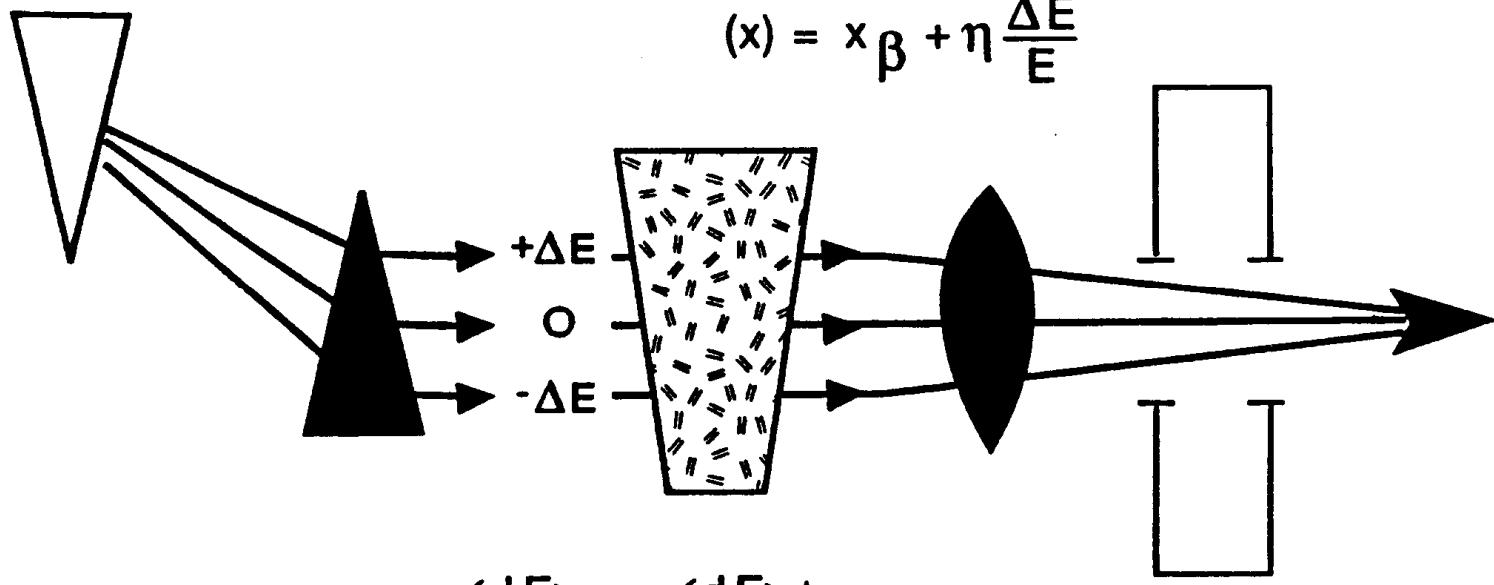


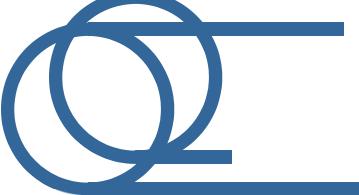
# Ionization Cooling

**USE WEDGE ABSORBER AT  $\eta \neq 0$   
TO INCREASE ENERGY-COOLING**

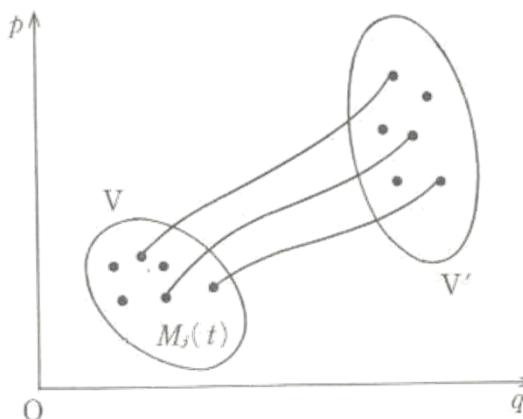
$$\text{Width: } \delta(x) = \delta_0 + \delta' x$$

$$(x) = x\beta + \eta \frac{\Delta E}{E}$$





# Liouville's Theorem



Trajectories in the 6 dimensional phase space does not cross  
Laminar Flow-velocity distribution at a point is single valued.

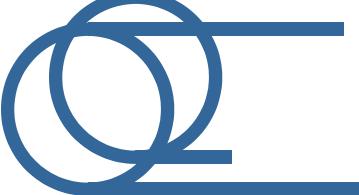
Phase Space Density  $f(x,p)$  does not change

$$\frac{df}{dt} = 0$$

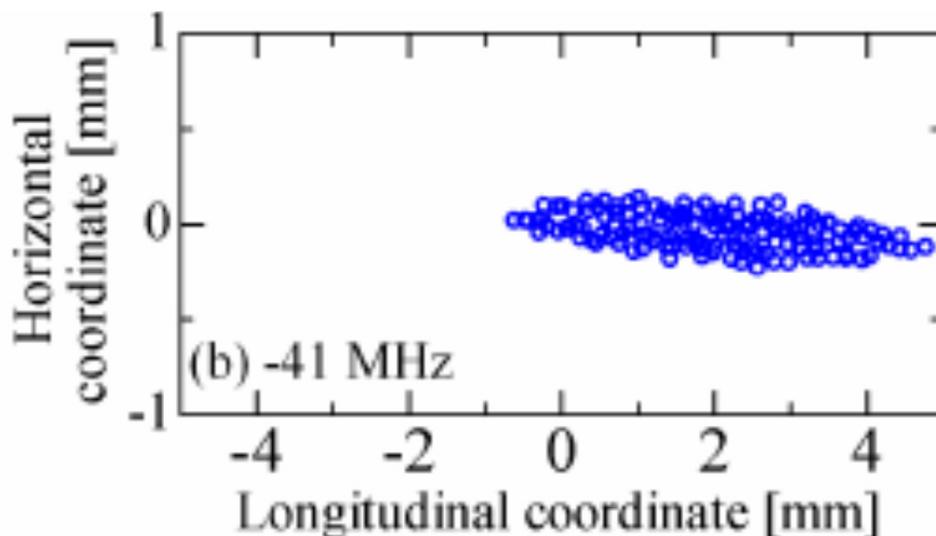
Phase Space Volume occupied by the beam does not change

In case, inter-particle interaction exists, phase space volume increases → Entropy Increases (2-nd law of Thermodynamics)

Beams in the accelerators basically follows Liouville's Theorem (without inter-particle interaction)



# MD Simulation

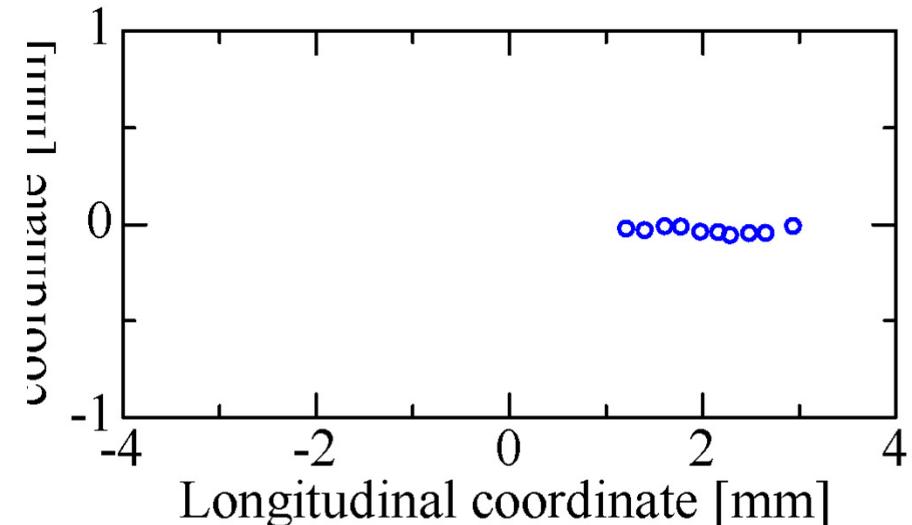


$N = 10^4$ , detuning is -41 MHz

Laser Power is 8 mW

Experimentally attained!!

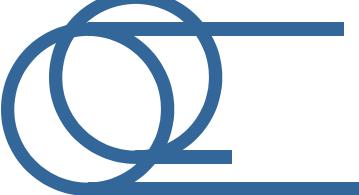
$T_H = 6.4 \text{ K}$ ,  $T_V = 2.1 \text{ K}$



$N = 10^3$ , detuning is -41 MHz

Laser Power is 8 mW

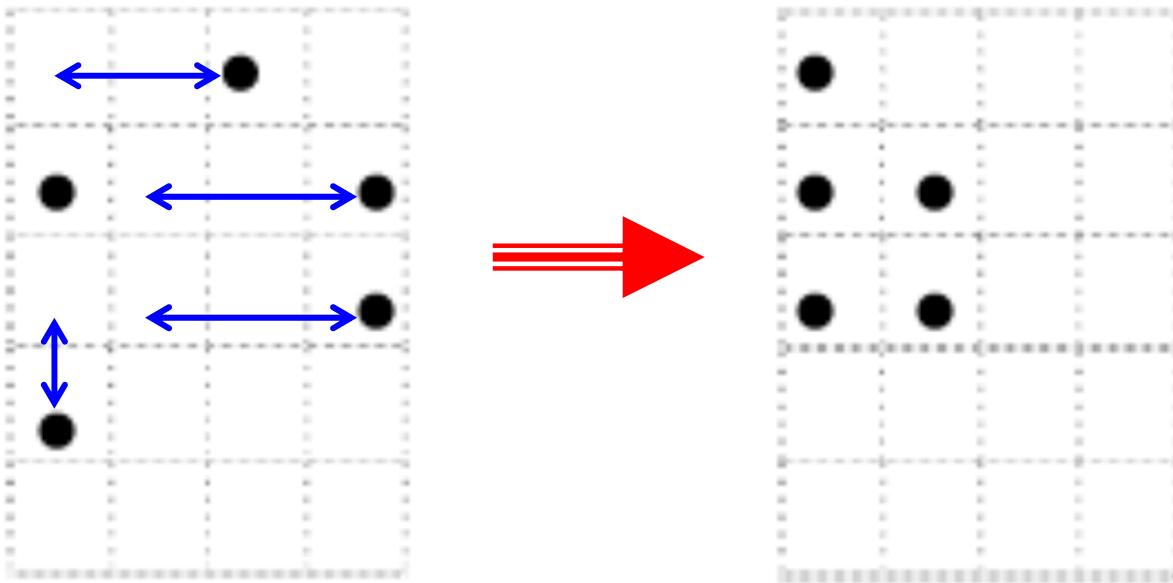
↓  
1 D Crystalline string の形成



# Stochastic Cooling

# Principle of Stochastic Cooling

by Simon van der Meer

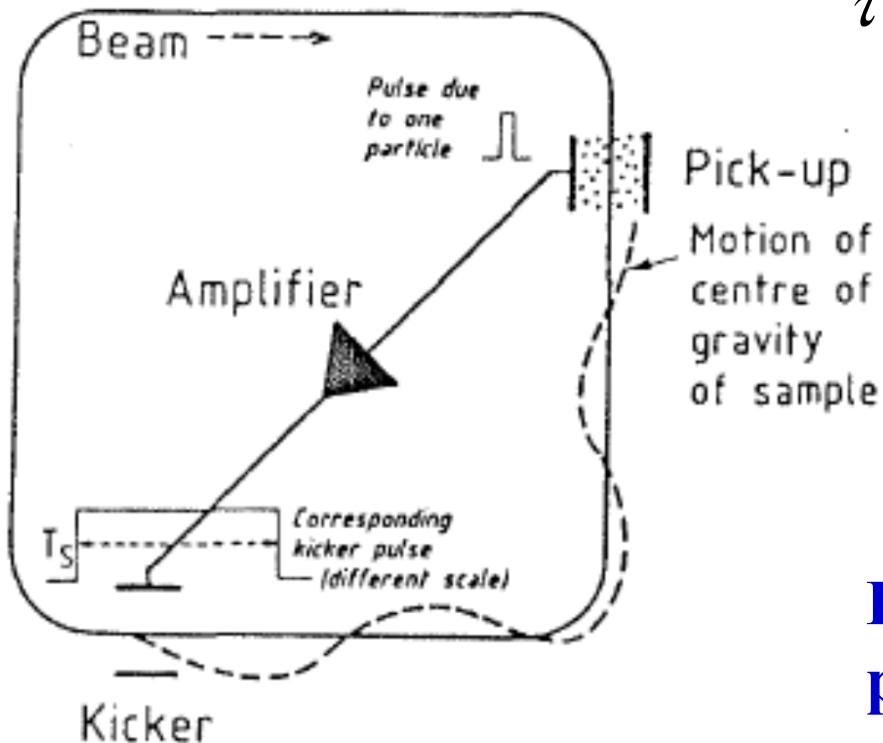


Replace an empty sub-ensemble with the one containing particle(s) → reduce the volume of phase space containing particles → reduce beam emittance ( if a single charge:  $1.6 \times 10^{-19}$  Coulomb can be detected by Maxwell's demon, it is negative feedback)

# Transverse Stochastic Cooling System

D. Möhl: CERN/PS/85-8(LEA)

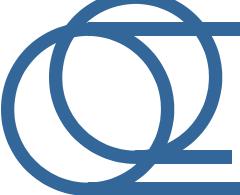
$$\frac{1}{\tau} = \frac{W}{2N} \left[ 2g(1 - \tilde{M}^{-2}) - g^2(M + U/Z^2) \right]$$



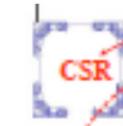
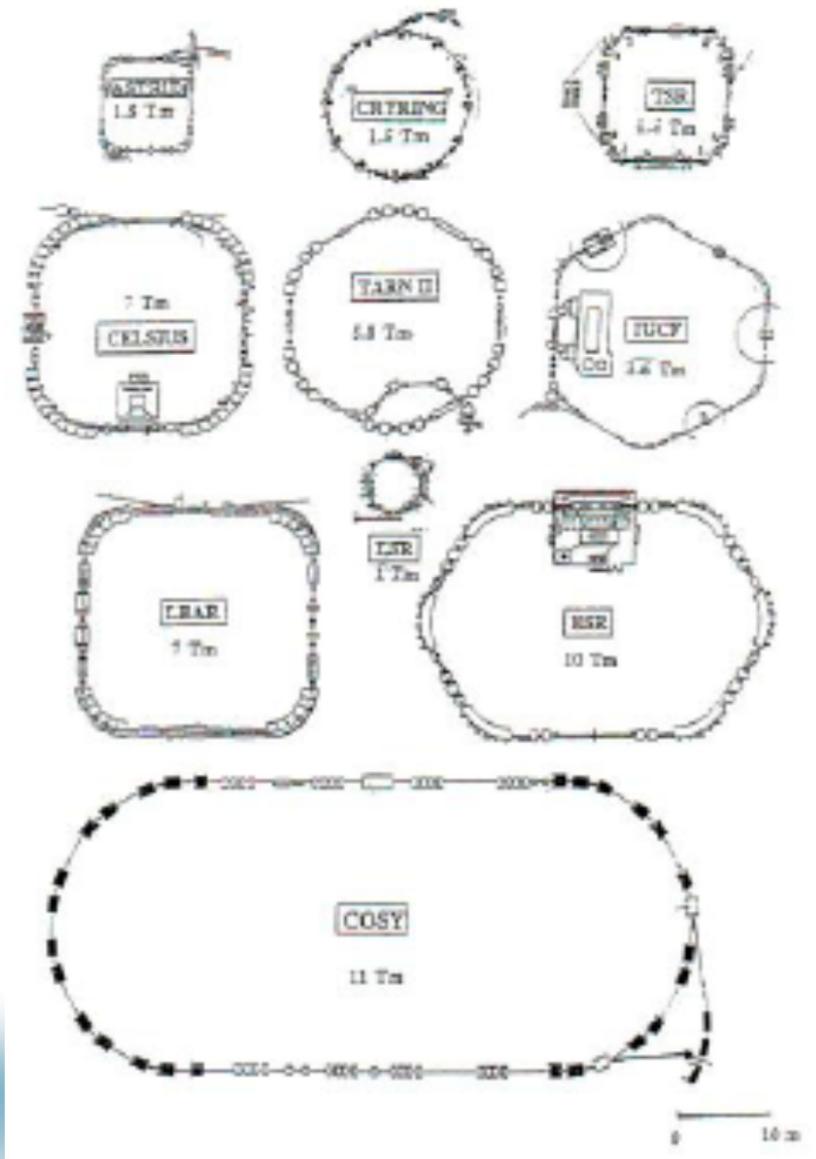
$$T_s = 1/(2W)$$

N: No. of Particles in the beam  
W: Bandwidth of cooling system  
g: “gain” parameter (<1)  
 $M$ : Desired Mixing from kicker to PU  
 $\tilde{M}$ : Undesired Mxing from PU to Kicker  
U: Noize to signal ratio ( $U > 0$ )  
Z: charge number of the particle

Due to momentum spread,  
particles migrate between samples  
and this mixing continuously  
exchanges the sample populations.



# Cooler Storage Rings in the World



CSR (C=35 m) to be  
operated at 2 K has  
stored  $^{40}\text{Ar}^+$  ion beam  
(17, March, 2014)