# BEAM-DRIVEN TERAHERTZ SOURCE BASED ON OPEN ENDED WAVEGUIDE WITH A DIELECTRIC LAYER * 

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## Abstract

Electromagnetic waves with frequencies from 0.1 THz to 10 THz are of great importance for a number of scientific and practical applications. Different techniques are known allowing generating these frequencies. However, a current trend of physics and industry is to fill this gap with more powerful and efficient sources. For example, recent experiments have shown promising THz generation in dielectric loaded structures. Developing this area, we consider the THz emitting scheme where an ultrarelativistic charge exits the open end of a cylindrical waveguide with a dielectric layer and produces THz waves in a form of Cherenkov radiation. The end of the waveguide is supposed to be either orthogonal to the structure axis or skewed. To obtain THz frequencies from waveguides with centimeter or millimeter radii, we consider high order modes. We present typical field patterns (in the Fraunhofer zone) and show that the aperture of the vacuum channel gives, as a rule, the main contribution.

## INTRODUCTION

Filling so-called "Terahertz gap" (usually supposed to be the range $0.1-10 \mathrm{THz}$ ) with efficient sources of radiation is an important problem for series of applications [1,2]. Here we consider the generation of THz waves in a form of the wakefield left behind by a charged particle bunch (Cherenkov radiation). This mechanism allows generation of a high peak power, narrow bandwidth and high energy THz pulse. Several experiments have demonstrated generation of THz radiation in dielectric loaded structures [3-5].

Recently, a few techniques allowing production of subpicosecond electron bunch trains have been developed and demonstrated (see [6] and references therein). Such a bunch train can selectively drive a mode in the wakefield structure with the frequency corresponding to the periodicity of bunches. This fact allows efficient generation of THz radiation via high order modes in waveguide with millimeter diameter.
Here we consider the problem of extraction of high order modes ( $\mathrm{TM}_{0 m}$ - like) from a dielectric loaded waveguide [7]. We take into account that the end of the waveguie can be

[^0]made at some arbitrary angle with respect to the waveguide axis.

Note that electromagnetic fields in open-ended waveguides have been actively investigated in the literature. However, the methods used in such problems are not applicable for the complex situation under consideration where we deal with a layered structure in waveguide (dielectric layer and vacuum channel). Therefore we have to use certain approximate techniques.

## METHOD OF INVESTIGATION

We consider a cylindrical waveguide of radius $a$ with an axisymmetric vacuum channel of radius $b$ and a cylindrical layer of nondispersive dielectric (described by permittivity $\varepsilon$ and permeability $\mu$ ) of thickness $d=a-b$. One end of the waveguide is open and skewed at an angle $\alpha$ (see Fig. 1). We introduce the Cartesian coordinate frame $x, y, z$ where the $z$ axis directed along the waveguide axis, the corresponding cylindrical frame $r=\sqrt{x^{2}+y^{2}}, \varphi, z$, the Cartesian frame $x^{\prime}, y^{\prime}, z^{\prime}$ where the $x^{\prime} y^{\prime}$-plane coinciding with the plane of the waveguide end, and the corresponding spherical frame $R^{\prime}=R, \theta^{\prime}, \phi^{\prime}$. We consider a single "incident" waveguide mode with frequency $\omega_{m}$. For example, this mode can be generated by a point charge $q$ moving along the $z$ axis on the trajectory $x=y=0, z=v t$, where $v=\beta c$ and $c$ is the light speed in vacuum. Note that in the case of an infinite waveguide (without an open end), the rigorous expressions for the field components are known [8].

We use approach known in antenna theory [9]. This method allows calculating the field at any distance from the aperture (including the Fraunhofer area where ray optics fail). According to this method, the radiation field in the Fraunhofer zone is a spherical wave, with its electrical vector being described by the following integral over the surface of the waveguide aperture:

$$
\begin{align*}
\vec{E} & =\frac{i k_{0} g_{0}}{4 \pi} \iint_{S_{\mathrm{a}}} \mathrm{~d} \xi \mathrm{~d} \eta\left\{\left[\vec{e}_{R},\left[\left[\vec{e}_{z^{\prime}}, \vec{H}^{\mathrm{a}}\right], \vec{e}_{R}\right]\right]+\right.  \tag{1}\\
& \left.+\left[\left[\vec{E}^{\mathrm{a}}, \vec{e}_{z^{\prime}}\right], \vec{e}_{R}\right]\right\} \exp \left[-i k_{0}\left(\overrightarrow{\mathrm{r}}_{\mathrm{a}}^{\prime}, \vec{e}_{R}\right)\right],
\end{align*}
$$

where $k_{0}=\omega_{m} / c, g_{0}=\exp \left(i k_{0} R\right) / R, \vec{e}_{R}$ is a radial unit vector, $\vec{r}_{\mathrm{a}}^{\prime}=\vec{e}_{x^{\prime}} \xi+\vec{e}_{y^{\prime}} \eta$ is the radius vector of a point on the aperture, and $\vec{E}^{\text {a }}$ and $\vec{H}^{\text {a }}$ are given electric and magnetic fields on the aperture $S_{\mathrm{a}}$ ( $S_{\mathrm{a}}$ is a system of two confocal ellipses).

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Figure 1: Geometry of the problem.


Figure 2: The electric field (in normalized units) in the Fraunhofer zone for $m=10$ in the case of an orthogonal flange $(\alpha=\pi / 2) . E^{v}$ is generated by the aperture of the vacuum channel, $E^{d}$ is generated by the dielectric part of the aperture, and $E$ is a total field.

Formula (1) is valid under the following conditions:

$$
\begin{array}{r}
R \gg 1 / k_{0}, \quad R \gg a / \sin \alpha, \\
R \gg \frac{a^{2}}{\lambda_{m} \sin ^{2} \alpha} \quad\left(\lambda_{m}=\frac{2 \pi}{k_{0}}\right) . \tag{2}
\end{array}
$$

For example, if $a \approx 2 \mathrm{~mm}, b \approx d \approx 1 \mathrm{~mm}, \alpha \approx 45^{\circ}, \varepsilon \mu \approx 10$, than one can obtain for the 10 -th mode $(m=10) \lambda_{10} \approx$ $0.6 \mathrm{~mm}, v_{10}=c / \lambda_{10} \approx 500 \mathrm{GHz}, R \gg 15 \mathrm{~mm}$.

To use formula (1), we should know $\vec{E}^{\text {a }}$ and $\vec{H}^{\text {a }}$ on the outer (with respect to the waveguide) surface $S_{\text {a }}$. To find $\vec{E}^{\text {a }}$ and $\vec{H}^{\text {a }}$ we utilize the following approximation [7]. The field on the vacuum channel aperture $(r<b)$ is supposed to be equal to the field in the unfinite homogeneuos waveguide. The field on the dielectric aperture $(b<r<a)$ is calculated by decomposing the waveguide mode into two cylindrical waves (quasi-plane convergent and divergent waves). Each of them is pesented as sum of waves of two polarisations, and Fresnel transmission coefficients are applied for obtaining the field on outer surface of dielectric.

## NUMERICAL RESULTS

To present numerical results, we use the angle $\theta_{1}$ which can be both positive and negative, as well as the angle WEPRO006
$\theta_{1}^{\prime}=\theta_{1}+\alpha-\pi / 2$ (Fig.1). Figure 2 shows the dependence of the electric field $E_{\theta}$ on the angle $\theta_{1}=\theta_{1}^{\prime}$ in the case of an orthogonal waveguide end ( $\alpha=\pi / 2$ ). As one can see, the vacuum channel aperture gives the main contribution to the field while radiation from dielectric part of the aperture is negligible. As simple calculations show, this fact is connected with the total internal reflection of waves at the interface of the dielectric layer aperture. It is interesting that radiation for $\theta_{1}=0$ is absent. Radiation pattern is symmetrical with respect to the $z$-axis and exhibits maxima for $\theta_{1} \approx \pm 15^{\circ}$.

For the case of non-orthogonal waveguide end, the electric field pattern is presented in Fig. 3 which shows the electric field in the $x z$ - and $y z^{\prime}$-planes. For $\alpha=60^{\circ}$ and $\alpha=45^{\circ}$ the radiation patterns are generally similar to those for the case of the orthogonal waveguide end, and the main contribution to the field comes from the vacuum channel aperture. However, the two main lobes of the radiation pattern differ in magnitude, and a number of narrow lobes appear. The essential change arises for $\alpha=30^{\circ}$, where radiation from the dielectric aperture become comparable with that from the vacuum channel aperture. The main lobe situated in the region $\theta_{1}>0$ become essentially larger compared with the lobe located in the region $\theta_{1}<0$. In the plane $y^{\prime} z^{\prime}$ radiation patterns for component exhibit a main maximum for $\theta_{1}^{\prime}=0$. This projection of the diagram changes from the pattern with three expressed lobes ( $\alpha=60^{\circ}$ and $45^{\circ}$ ) to the wide-angle pattern with singe lobe $\left(\alpha=30^{\circ}\right)$.

## REFERENCES

[1] G. P. Williams, Rep. Prog. Phys. 69, 301 (2009).
[2] S. V. Garnov and I. A. Shcherbakov, Physics-Uspekhi 54, 91 (2011).
[3] A. M. Cook, R. Tikhoplav, S. Y. Tochitsky, G. Travish, O. B. Williams, and J. B. Rosenzweig, Phys. Rev. Lett. 103, 095003 (2009).
[4] G. Andonian, O. Williams, X. Wei, P. Niknejadi, E. Hemsing, J. B. Rosenzweig, P. Muggli, M. Babzien, M. Fedurin, K. Kusche, M. R., and V. Yakimenko, Appl. Phys. Lett. 98, 202901 (2011).
[5] S. Antipov, C. Jing, A. Kanareykin, J. E. Butler, V. Yakimenko, M. Fedurin, K. Kusche, and W. Gai, Appl. Phys. Lett. 100, 132910 (2012).
[6] S. Antipov, M. Babzien, C. Jing, M. Fedurin, W. Gai, A. Kanareykin, K. Kusche, V. Yakimenko, and A. Zholents, Phys. Rev. Lett. 111, 134802 (2013).
[7] S. N. Galyamin, A. V. Tyukhtin, S. Antipov, and S. S. Baturin, Optics Express 22, 8902 (2014).
[8] B. M. Bolotovskii, Physics-Uspekhi 4, 781 (1962).
[9] A. Z. Fradin, Microwave Antennas (Pergamon, 1961).


Figure 3: The electric field (normalized units) in the Fraunhofer zone for $m=10$ generated by the aperture of the vacuum channel ( $E_{\theta^{\prime}, \phi^{\prime}}^{v}$ ), the dielectric aperture ( $E_{\theta^{\prime}, \phi^{\prime}}^{d}$ ) and the total field ( $E_{\theta^{\prime}, \phi^{\prime}}$ ) as functions of $\theta_{1}^{\prime}$ in $x z$-plane (left and middle column) and $y^{\prime} z^{\prime}$-plane (right column) for cases $\alpha=60^{\circ}$ (top row), $\alpha=45^{\circ}$ (middle row) and $\alpha=30^{\circ}$ (bottom row).


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