

ON THE OPTIMAL DESIGN OF ELLIPTICAL SUPERCONDUCTING CAVITIES

G. Costanza, Electrical and Information Technology, Lund University, Lund, Sweden

Abstract

In this paper a linear regression analysis is used to analyze the behavior of the inner cell of an elliptical cavity. The aim is to understand how the RF parameters are correlated to each other and how they are affected by the change of the geometric parameters. This is done by fitting the RF data to a linear model. The data is obtained by simulating a set of different inner cells automatically by the use of a script. The results are useful in several ways: first of all the analysis sheds light on the behavior of elliptical cavities, in particular on its limitations. The analysis is carried out in the framework of optimal design so it is useful for the cavity designer since it allows to choose the geometry at an early stage of the design. It is also possible to make predictions on the performance of the cavity which are in very good agreement with the simulations. Such predictions facilitate the design of the accelerator when choosing the type and number of cavities and when writing the specifications for the cavities to be used in the accelerator.

the largest and smallest geometric parameters. Simulating the dumbbell is convenient because with one simulation it is possible to calculate both the zero and the π mode, necessary for the calculation of the cell-to-cell coupling.

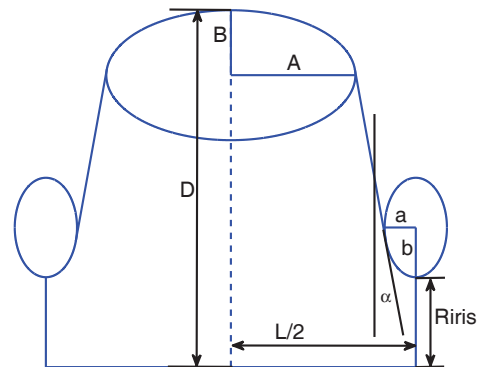


Figure 1: Elliptical cell with its geometric parameters.

INTRODUCTION

During the design of the medium β elliptical cavity for the ESS [1], a large number of geometries have been simulated and the RF parameters (R/Q , Geometric factor (G), E_{pk}/E_{acc} , B_{pk}/E_{acc} and the cell-to-cell coupling (K_{cc})) have been calculated. This set of data can be used to derive a model for the RF parameters that can be useful not only for the design of the cavity itself but also to study the electromagnetic behaviour of the elliptical structure. In what follows the author presents part of that analysis: first a brief overview on the predictors effect is given, then an example of the correlation between RF parameters is presented by considering E_{pk}/E_{acc} and K_{cc} and, in the third section, a linear model is applied to the dataset which allows to calculate the geometric parameters that give the chosen RF parameters.

The following results refer to the inner cell of an elliptical cavity shown in Fig. 1 with the parameters that define the geometry. The dataset has been obtained simulating the elliptical cells automatically with a script that uses COMSOL. The script allows to organize and simulate a large number of cells. To limit the number of geometric parameters the cells are designed for a geometric beta $\beta_g = 0.67$ with a radius of the iris of 47 mm. All the cells are tuned at 704.42 MHz.

The target of the linear modeling is to be able to obtain a good fit to the dataset without having to spend much time in the generation of the dataset itself. This is why the geometric parameters **A** and **B** vary in steps of 5 mm while **a** and **b** vary in steps of 1 mm. The insets in Figure 6 shows the “limiting” geometries that have been simulated, the ones with

PREDICTORS EFFECTS

In this section a brief overview of the effects of the geometric parameters (also *predictors*) on the RF parameters is given.

- B_{pk}/E_{acc} is affected by the change in **A** and **a** to the first degree but very little by a change in **b** and **B** (Fig. 2).
- E_{pk}/E_{acc} is affected almost only by **a** and **b** (Fig. 3). An optimal **b** can be found for which E_{pk}/E_{acc} has a minimum. **A** has little influence and **B** almost none.
- K_{cc} increases with increasing **A**, decreases by increasing **a** and increases by increasing **b**. All these dependencies are almost linear.

CORRELATIONS

It is known to the cavity designer that a compromise has to be made during the cavity design process in order to respect all the specifications. It is then interesting then to quantify and understand how and when the RF parameters are correlated. As an example the correlation between the cell-to-cell coupling and the normalized electric peak field, E_{pk}/E_{acc} is considered. It is desirable in fact, to have a low peak electric field but this usually leads to a non optimal or low cell-to-cell coupling.

Figure 3 shows the scatter plot of K_{cc} vs E_{pk}/E_{acc} and Figure 4 shows their variation against the cell number. It is possible to draw some conclusions:

Content from this work may be used under the terms of the CC BY 3.0 licence (© 2014). Any distribution of this work must maintain attribution to the author(s), title of the work, publisher, and DOI.

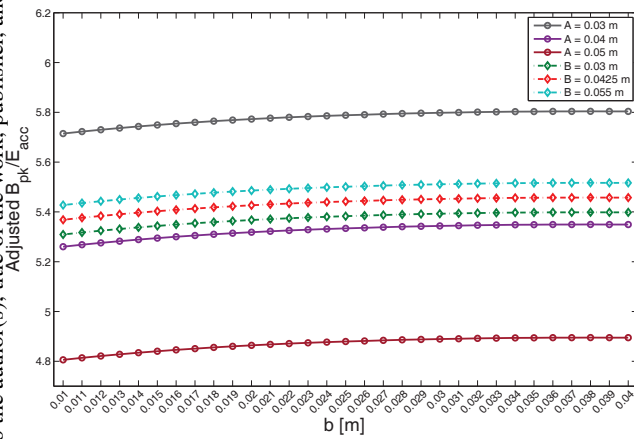


Figure 2: Interaction of **A** and **a** vs. **b**. Effect on B_{pk}/E_{acc} for three levels of **B** (diamonds) and **A** (circles). The effect of **A** is much more important.

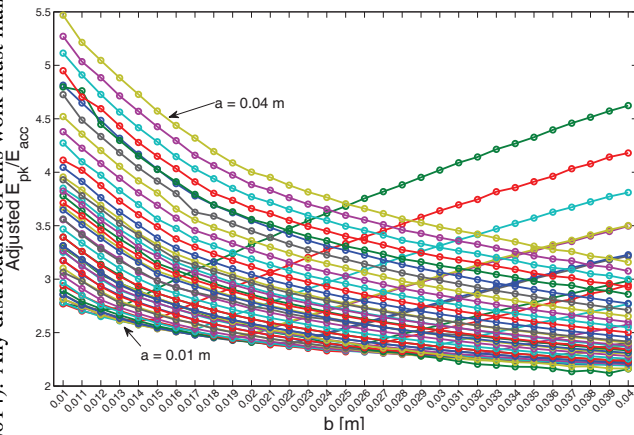


Figure 3: Interaction of **a** and **b**. Effect on E_{pk}/E_{acc} .

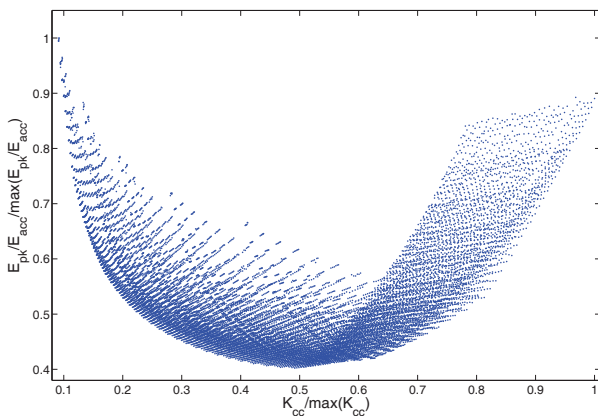


Figure 4: Scatter plot of K_{cc} vs E_{pk}/E_{acc} .

- Figure 3 is the scatterplot between K_{cc} and E_{pk}/E_{acc} . The relationship between these two quantities is non-linear but this doesn't mean that there is no correlation between the two parameters. The plots in Figure 5, 6 and 7, which show the variation of K_{cc} and E_{pk}/E_{acc}

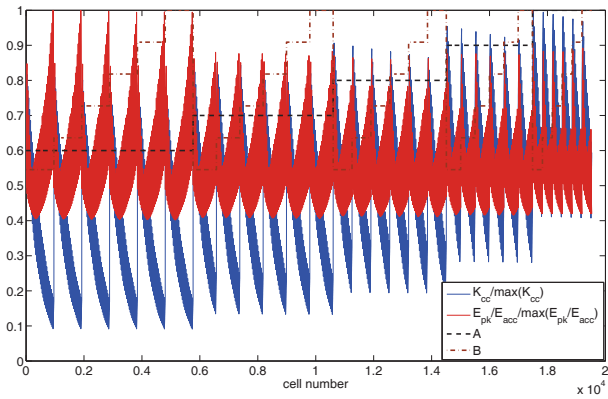


Figure 5: Variation of K_{cc} and E_{pk}/E_{acc} vs. cell number.

against the cell number, help in investigating their relationship further. Figure 5 is a magnified version of the first and last part of the plot in Figure 4; in Figure 6 a low pass filter (moving average) has been applied to the plots of K_{cc} and E_{pk}/E_{acc} . For a fixed value of **B** (Fig. 6 and 7) the correlation between K_{cc} and E_{pk}/E_{acc} is positive in the beginning (they both trend down), it becomes negative the larger **a** is (E_{pk}/E_{acc} increases after its local minimum while the cell-to-cell coupling keeps decreasing). In Fig. 6, when **a** is small, the peaks of K_{cc} and E_{pk}/E_{acc} are aligned but the more **a** increases, the more the peaks shift until the positive peak of K_{cc} coincides with the negative peak of E_{pk}/E_{acc} ; at this point the two parameters start to be negatively correlated. To verify this explicitly the correlation between K_{cc} and E_{pk}/E_{acc} is plotted in Figure 8; in that plot, **A** = 0.3 m and **B** = 0.03 and the plot can be compared with the left side of Figure 6. The behavior remains the same for any value of **A** and **B**.

- The minimum of E_{pk}/E_{acc} is obtained when the correlation between K_{cc} and E_{pk}/E_{acc} reaches a stable value of ≈ -1 . This is true for every fixed value of **A** and **B**. Figure 8 shows an example. It is possible to say that concerning E_{pk}/E_{acc} this is the optimal part of the parameter space but the cell-to-cell coupling might not be sufficiently high. The overall correlation between **A** and K_{cc} is strongly positive: a large **A** leads in general to a higher K_{cc} . This means that **A** cannot be too small otherwise the cell-to-cell coupling will be too low, but it cannot be too high in order not to have a very small **a**. Once **A** has been chosen it is possible to choose **a** to be in the optimal part of the parameter space but in this situation the slope of the side wall of the cavity will probably be small, and this is problematic during the cleaning of the cavity. A possible cure for this is to use a high value for **B** since it has a negligible effect on both K_{cc} and E_{pk}/E_{acc} , and possibly a high value of **b**. Increasing these two parameters increases **a** slightly. Notice also that in the “negative correlation” part of the parameter space a high **b** leads to a high K_{cc} and a

low E_{pk}/E_{acc} which is advantageous (cfr. Fig 6 and 8).

- If the cell-to-cell coupling is unsatisfactory it is necessary to use a smaller \mathbf{a} . In this case E_{pk}/E_{acc} won't be at its minimum and the optimal \mathbf{b} has to be found.

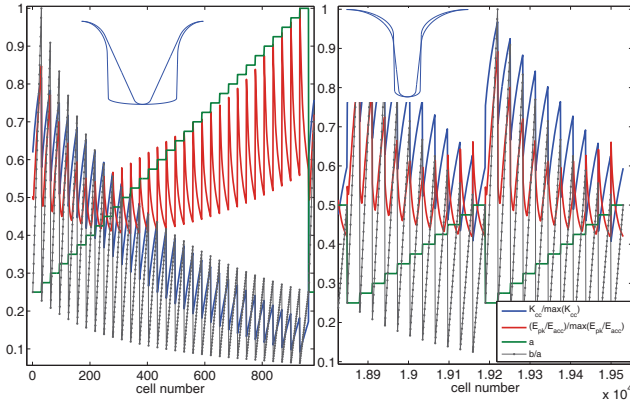


Figure 6: Detail of the variation of K_{cc} vs E_{pk}/E_{acc} vs cell number. With $\mathbf{A} = 0.3$ m and $\mathbf{B} = 0.03$ m on the left, $\mathbf{A} = 0.5$ m and $\mathbf{B} = 0.05, 0.055$ m on the right. The range of \mathbf{a} is limited when \mathbf{A} is large to avoid reentrant geometries.

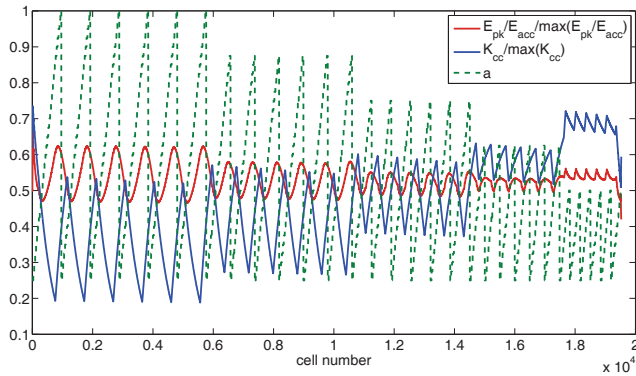


Figure 7: Moving average with a span of 400 values applied to K_{cc} and E_{pk}/E_{acc} . This plot is a smoothed version of the plot in Figure 5.

LINEAR MODEL

In this section a linear model is applied to the RF parameters (also *response variables*) K_{cc} , E_{pk}/E_{acc} and B_{pk}/E_{acc} to design an elliptical cell without performing simulations. Once the specifications are set, it is possible to use the required K_{cc} and E_{pk}/E_{acc} and B_{pk}/E_{acc} as input to the model which gives the geometric parameters, \mathbf{A}, \mathbf{B} , \mathbf{a} and \mathbf{b} , as an output. The missing parameter is the radius of the

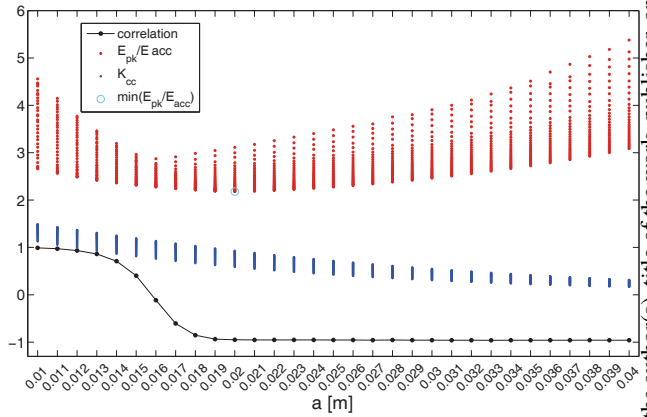


Figure 8: Plot of K_{cc} and E_{pk}/E_{acc} and their correlation coefficient against \mathbf{a} for $\mathbf{A} = 0.3$ m and $\mathbf{B} = 0.03$, to be compared with the left side of Figure 6.

cell (since we keep $\mathbf{Riris} = 0.047$ m, $\mathbf{L} = 0.1426$ m), which is automatically calculated with a tuning routine to let the accelerating mode resonate at 704.42 MHz. At this point the cell is ready, but to verify the quality of the fitting, that is, to verify the correctness of the result, a simulation is performed.

For example let us consider then the design of an elliptical cell. Table 1 summarizes the specifications and the results obtained with the linear model.

Table 1: Design Example

RF parameter	Specs.	Simulation results	Prediction bounds ¹
E_{pk}/E_{acc}	2.4	2.385	[2.370, 2.394]
$B_{pk}/E_{acc} \left[\frac{mT}{MV/m} \right]$	5	5.015	[4.933, 5.107]
$K_{cc} [\%]$	1.2	1.132	[1.074, 1.174]

The model reproduces quite accurately the behaviour of the cell and the predicted RF parameters match the simulated ones quite well. Diagnostic plots help in identifying and eventually excluding outliers, to improve the accuracy of the model. An analysis of the residuals is also advised. Concerning diagnostics tools for data analyses based on linear models it is possible to consult [2].

REFERENCES

- [1] G. Devanz et al., *ESS Elliptical Cavities and Cryomodules*, Proceedings of SRF2013, Paris, France.
- [2] R. D. Cook, S. Weisberg, *Residuals and Influence in Regression*,

¹ non-simultaneous.