# LATTICE CORRECTION MODELING FOR FERMILAB IOTA RING 

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## Abstract

The construction of the Integrable Optics Test Accelerator (IOTA) is underway at Fermilab. Among the main goals of the facility are the proof-of-principle experiments on nonlinear integrable optics and optical stochastic cooling. Both require outstanding quality of the linear lattice and closed orbit. Software was developed to thoroughly test the proposed lattice configurations for error correction performance. The presented analysis is based on a statistical approach on a number of error seeds, such as various alignment, calibration and field errors.

## INTRODUCTION

The first stage of the Integrable Optics Test Accelerator (IOTA) experimental program is feasibility testing of the nonlinear lattices with two integrals of motion [1]. The design of the linear lattice was prepared with two straights for nonlinear insertions (Fig. 1). Numerical simulations predict that the stability of the nonlinear system is very sensitive to the errors of the linear lattice [2]. Table 1 contains restrictions on the imperfections of the main parameters.

To reach a perfectly tuned linear lattice, IOTA will have a wide range of tools:

- Individual main quadrupole corrections
- Precision mechanical alignment design
- 20 combined X, Y and skew-field correctors
- 8 X -correctors in the main dipoles
- 20 electrostatic pickups with the closed orbit measurement precision of $1 \mu \mathrm{~m}$
- 8 beam profile and position measurement monitors based on synchrotron light from the main dipoles

Table 1: Maximum Errors of the IOTA Lattice for the Integrable Optics Experiments

| Parameter | Max error |
| :--- | :---: |
| Betas at the insertion | $1 \%$ |
| Beta beating | $3 \%$ |
| Dispersion | 1 cm |
| Closed orbit at insirtion | 0.05 mm |
| Phase advance between insertions | 0.001 |

## INVERSE TASK SOLVER

Both tasks of the closed orbit and linear lattice correction can be formulated as inverse problems when some set of experimental data $V_{\text {exp }, j}$ is available and the goal is to find


Figure 1: IOTA layout with project and relocated positions of BPMs.
the parameters $P_{i}$ of the model $\mathfrak{M}_{j}\left(P_{i}\right)$ that best describes the measurements. To find the approximate solution, the iterative method is used. The model parameters at the iteration ( $n$ ) are:

$$
\begin{equation*}
V_{m o d, j}^{(n)}=\mathfrak{M}_{j}\left(P_{i}^{(n)}\right) \cdot s_{j}, \tag{1}
\end{equation*}
$$

here $s_{j}$ is normalization coefficients, that can be used to modify weights of some experimental data points. In addition, both $V_{\text {exp }, j}$ and $V_{m o d, j}$ are assumed to be normalized to the statistical error of the $V_{\text {exp }, j}$.

The parameters of the model after iteration $(n)$ are:

$$
\begin{equation*}
P_{i}^{(n)}=P_{i}^{(0)}+\sum_{m=0}^{n-1} \Delta P_{i}^{(m)} \tag{2}
\end{equation*}
$$

The difference between the experimental data and the model is:

$$
\begin{equation*}
D_{j}^{(n)}=V_{\exp , j}-V_{m o d, j}^{(n)} \tag{3}
\end{equation*}
$$

The goal is to find such variation of the parameters $\Delta P_{i}^{(n)}$ that cancels the residual difference between model and experimental data:

$$
\begin{equation*}
\Delta V_{m o d, j}^{(n)}=-\Delta D_{j}^{(n)}=D_{j}^{(n)} . \tag{4}
\end{equation*}
$$

The model can be linearized in case of small parameters variation:

$$
\begin{align*}
\Delta V_{\text {mod }, j}^{(n)} & =s_{j}\left(\mathfrak{M}_{j}\left(P_{i}^{(n)}+\Delta P_{i}^{(n)}\right)-\mathfrak{M}_{j}\left(P_{i}^{(n)}\right)\right) \\
& \left.\simeq s_{j} \frac{\partial \mathfrak{M}_{j}}{\partial P_{i}}\right|_{P_{i}^{(n)}} k_{i} \frac{\Delta P_{i}^{(n)}}{k_{i}}=\mathfrak{M}_{j i}^{(n)} \frac{\Delta P_{i}^{(n)}}{k_{i}} \tag{5}
\end{align*}
$$

where $\mathfrak{M}_{j i}^{(n)}$ is linearized and weighted model at iteration ( $n$ ):

$$
\begin{equation*}
\mathfrak{M}_{j i}^{(n)}=\left.s_{j} k_{i} \frac{\partial \mathfrak{M}_{j}}{\partial P_{i}}\right|_{P_{i}^{(n)}} . \tag{6}
\end{equation*}
$$

The model parameters variation can be obtained from Here by applying pseudo inversion to $\mathfrak{M}_{j i}^{(n)}$. Singular Values Decomposition (SVD) is a powerful method for such calculation. One of the remarkable features of this technique is easy control over the influence of the statistical errors in the experimental data on the output result. Application of SVD $\approx$ gives the parameters variation at the iteration (n):

$$
\begin{equation*}
\Delta P_{i}^{(n)}=k_{i} \sum_{j}\left(\mathfrak{M}_{j i}^{(n)}\right)_{S V D}^{-1} D_{j}^{(n)} \tag{7}
\end{equation*}
$$

Summation over all iterations gives the total correction to the model parameters:

$$
\begin{equation*}
\Delta P_{i}=\sum_{n} k_{i} \sum_{j}\left(\mathfrak{M}_{j i}^{(n)}\right)_{S V D}^{-1} D_{j}^{(n)} \tag{8}
\end{equation*}
$$

In the case of no systematic errors, the $\chi^{2}$ function limit is:

$$
\begin{equation*}
\chi^{2}=\sum D_{j}^{2} \rightarrow J-I, \tag{9}
\end{equation*}
$$

where $J$ is the size of experimental data and $I$ is the number of fitting parameters.

## CORRECTION MODELING

To study the possible issues with closed orbit and linear lattice correction, the "sixdsimulation" software developed for VEPP-2000 was used [3]. Test algorithm is based on a statistical analysis with repeated generation of random errors with Gaussian spread and subsequent attempt to correct it. Figure 2 shows the flow chart of the test procedure. The most important step is pseudo-experimental data fit, that can be done automatically or manually. In the manual mode, there are interactive tools for detailed analysis of the fit procedure.

Table 2: Standard Deviations of Errors for Linear Lattice Correction modeling

| Quads |  | BPMs |  | Corr. calibr. |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| G | rot. | Calibr. | rot. | X in bends | X\&Y short |
| $1 \%$ | $0.1^{\circ}$ | $4 \%$ | $2^{\circ}$ | $1 \%$ | $2 \%$ |

Table 2 lists the error values for linear lattice correction modeling and their standard deviations. The same set of parameters was used in the fit procedure.

Table 3: Standard Deviations of Errors for Closed Orbit Correction Modeling

| Quadrupoles | Bends |  |
| :---: | :---: | :---: |
| X, Y shifts | X, Y, S shifts | X\&Y tilts |
| 0.1 mm | 0.1 mm | $0.06^{\circ}$ |

For orbit correction modeling, the misalignments of the quadrupoles and dipoles presented in Table 3 were used. All

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Figure 2: Correction test flow chart.


Figure 3: Comparison of singular values for lattice correction for project and relocated BPMs.
dipole correctors were involved in the closed orbit correction.

The singular value spectrum of the matrix $\mathfrak{M}_{j i}^{(n)}(6)$ for lattice correction for the project lattice configuration reveals the degeneracy of the 4-th order and another 2 dimensions have very small singular values with respect to others. The reason for such flow are locations of BPMs, which are all outside of doublets and triplets. The latter forms an almost axially symmetrical focusing and thus may be rotated as a whole without noticeable influence to the outer lattice. To solve this problem new locations of the BPMs were tested (see Fig. 1). The comparison of the singular values spectra for the project and relocated BPMs positions for lattice correction is presented in Fig. 3. The same spectra for closed orbit correction do not show significant difference (Fig. 4).

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Figure 4: Comparison of singular values for orbit correction for project and relocated BPMs.

## Closed Orbit Correction Modeling Results

In spite of no significant difference in singular values spectrum, modeling shows much better orbit correction for the lattice with relocated BPMs, see Table 4. It is explained by more even distribution of monitors in terms of betatron phase advance. Relatively soft focusing of the ring requires small corrector fields for realistic alignment precision.

Correction of the orbit even with better BPMs positions may be not enough. Fine tuning of the closed orbit in the nonlinear insertions will be done by manual scan using local bumps.

Table 4: Results of the Orbit Correction Modeling for Two Configurations of the BPMs

| Parameter | Error | Fixed <br> project | Fixed <br> relocated |
| :--- | :---: | :---: | :---: |
| $\left\langle X^{2}\right\rangle^{0.5}, \mathrm{~mm}$ | 5.55 | 0.21 | 0.073 |
| $\left\langle Y^{2}\right\rangle^{0.5}, \mathrm{~mm}$ | 3.02 | 0.28 | 0.11 |
| $\langle \| X_{\text {insertion }}\| \rangle, \mathrm{mm}$ | 2.7 | 0.34 | 0.042 |
| $\langle \| Y_{\text {insertion }}\| \rangle, \mathrm{mm}$ | 1.65 | 0.55 | 0.17 |
| $\left\langle\left(L H_{y, \text { cor }}\right)^{2}\right\rangle^{0.5}, \mathrm{Gs} \mathrm{cm}$ | 0.0 | 65 | 75 |
| $\left\langle\left(L H_{y, \text { cor }}\right)^{2}\right\rangle^{0.5}, \mathrm{Gs} \mathrm{cm}$ | 0.0 | 61 | 63 |

## Linear Lattice Correction Modeling Results

The experimental data set for linear lattice correction modeling was composed of the following values:

- Closed orbit responses to the dipole correctors measured with $1 \mu m$ precision.
- betatron tunes with errors of $10^{-6}$.
- Dispersion measured with precision of 0.1 mm .

The absence of degeneracy in modified IOTA lattice for the used set of adjustable parameters gives about twice smaller errors after correction (see Table 5).

Quadrupole axial rotations give the most significant discrepancy in the fixed lattice for the project BPMs locations. It does not cause unacceptable errors of betas, dispersions and tunes, however it may affect nonlinear dynamics in unexpected way.

Table 5: Results of The Linear Lattice Correction Modeling for Two Configurations of the BPMs

| Parameter | Error | Fixed <br> project | Fixed <br> relocated |
| :--- | :---: | :---: | :---: |
| $N_{\text {sing.val. }}$ | - | 177 | 186 |
| $\chi^{2}$ | 1.0 E 9 | 2580 | 2510 |
| $\langle \| \Delta v_{x}\| \rangle$ | 0.0158 | $8.610^{-5}$ | $4.510^{-5}$ |
| $\langle \| \Delta v_{y}\| \rangle$ | 0.0134 | $1.610^{-4}$ | $4.910^{-5}$ |
| $\left\langle\beta_{x}^{2}\right\rangle^{0.5}, \%$ | 31.7 | 0.16 | 0.10 |
| $\left\langle\beta_{y}^{2}\right\rangle^{0.5}, \%$ | 18.2 | 0.74 | 0.27 |
| $\left\langle D_{x}^{2}\right\rangle^{0.5}, \mathrm{~cm}$ | 22.3 | 0.03 | 0.019 |
| $\left\langle D_{y}^{2}\right\rangle^{0.5}, \mathrm{~cm}$ | 6.0 | 0.03 | 0.014 |
| $\langle \| \beta_{x, \text { insertion }}\| \rangle, \%$ | 27.1 | 0.098 | 0.056 |
| $\left\langle\beta_{y, \text { insertion }} \mid\right\rangle, \%$ | 13.4 | 0.166 | 0.096 |
| Quad rot., deg. | 0.1 | 0.049 | 0.019 |
| Quad $\Delta G / G_{0}, \%$ | 0.97 | 0.122 | 0.059 |

## CONCLUSION

The present study reveals possible problems with linear optics correction in the project lattice configuration that can be fixed with relocation of BPMs. The closest iron-to-iron distance between the quadrupoles is 13 cm , which is enough for the placement of an electrostatic pickup. To further support the proposed improvement more detailed tests must be done with expanded sources of errors, such as gradient fields in the main dipoles, longitudinal displacements of all elements, etc.

## REFERENCES

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