# SOLENOID SIBERIAN SNAKE WITHOUT COMPENSATION OF BETATRON OSCILLATION COUPLING IN NUCLOTRON@JINR 

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## Abstract

The influence of solenoids on spin is very efficient, but beam focusing is determined mainly by structural quadrupoles. The condition of stable orbital motion of particles does not require compensation of the betatron oscillation coupling. To reduce the influence of the Snake on orbital motion it is desirable to exclude compensating quads completely. The design of solenoid Siberian snake for the Nuclotron lattice is presented. The orbital functions of the lattice were calculated and the results are discussed.

## INTRODUCTION

A possibility to use Nuclotron as a polarized protons injector for NICA collider up to momentum of $6 \mathrm{GeV} / c$ was formulated in 2012 [1]. A solenoid Siberian snake was proposed to preserve the polarization during the acceleration cycle. Nevertheless, solenoid magnetic fields lead to the betatron oscillation coupling. The coupling is usually compensated by means of additional quads [2-4]. The design of the snake without the quads would give additional space for the solenoids and reduce their required longitudinal field respectively. Any solenoid insertion into synchrotron lattice changes the betatron oscillations behavior, nevertheless two independent oscillation modes with betatron tunes $v_{1}$ and $v_{2}$ will occur. The main difference in comparison with uncoupling case is rotation of the oscillation mode planes. Coupling angle - the angle between orbit plane and oscillation mode plane - is a periodical function of azimuth determined by the total ring lattice. The regions of beam stability in FODO structure of Nuclotron with solenoid Siberian snake is analyzed in this paper.

## NUCLOTRON WITH SOLENOID SIBERIAN SNAKE

Nuclotron is a conventional strong-focusing 8 superperiods synchrotron with the magnetic rigidity up to $B \rho=45 \mathrm{~T} \cdot \mathrm{~m}$. The magnetic system consists of superconducting elements with guiding magnetic field ramp of about $1 \mathrm{~T} / \mathrm{s}$. The $\beta$-functions of a superperiod for the tunes . $v_{x}=6.8$ and $v_{y}=6.85$, are shown in Fig. 1. The tunes are determined by two families of focusing and defocusing quads $F$ and $D$.


Figure 1: $\beta$-functions of Nuclotron superperiod.
There are two free spaces of 3.505 m long each separated by structural $D$ quad in the second superperiod aimed at the Snake insertion. Structure of the straight section with the solenoids is presented in Fig. 2, where $K_{F}$, $K_{D}$ are focusing and defocusing quads gradients $\left(K=\left(\partial B_{y} / \partial x\right) / B \rho\right)$ and $K_{S}=B_{s} / B \rho$ is the solenoid field in the units of magnetic rigidity.


Figure 2: Layout of the straight section with the solenoid Siberian snake without compensating quads.

In the case of a full Siberian snake, which rotates spin around longitudinal direction by an angle of $\Psi=\pi$ radian at the momentum of $6 \mathrm{GeV} / c$, one has to provide longitudinal field integral of $B L=22.5 \mathrm{~T} \cdot \mathrm{~m}$, i.e. the solenoid field should reach of 3.6 T . The field value of 1.8 T is sufficient for the half snake respectively.

## STABLITY REGIONS IN THE STRUCTURE WITH SOLENOIDS

Stability of betatron motion in accelerator is characterized by four eigenvalues $\lambda_{1}, \lambda_{2}, \lambda_{3}, \lambda_{4}$ of the transfer matrix for passage through one revolution. For stable betatron oscillations all four eigenvalues must lie on the unit circle, forming two complex conjugate pairs [5]:

$$
\left|\lambda_{1}\right|=1,\left|\lambda_{2}\right|=1, \quad \lambda_{3}=\lambda_{1}^{*}, \quad \lambda_{4}=\lambda_{2}^{*} .
$$

Betatron tunes $v_{1}$ and $v_{2}$ are defined by eigenvalues:

$$
\lambda_{1}=\exp \left(2 \pi i v_{1}\right), \quad \lambda_{2}=\exp \left(2 \pi i v_{2}\right)
$$



Figure 3: Diagrams of beam stability as a function of structural quad strengths in Nuclotron without ( $\Psi=0$ ) and with ( $\Psi=\pi$ ) full solenoid Siberian snake.


Figure 4: The dependence of stability regions deformations on spin angle $\Psi$ of solenoid Siberian snake.

Diagrams of beam stability for the cases without ( $\Psi=0$ ) and with ( $\Psi=\pi$ ) full solenoid snake are shown in Fig. 3. The $\Psi$ is a spin rotation angle around longitudinal direction. The diagrams show the regions of $\cos 2 \pi \nu_{1}$ and $\cos 2 \pi \nu_{2}$ values for different normalized quads gradients $K_{F}$ and $K_{D}$. The diagram consists of different areas, namely: regions of stability (not colored) and regions of unstable motion (colored in black, grey and red). On the boundary of regions marked with black the value of the cosines are «+1», i.e. condition of the integer resonances $v_{1,2}=k$ is fulfilled. The boundary of regions marked with grey is correspond to the cosines value of «-1», i.e. to half-integer resonances $v_{1,2}=k+1 / 2$. The boundary of red regions is correspond to $\cos 2 \pi \nu_{1}=\cos 2 \pi \nu_{2}$, i.e. to condition of coupling resonances $v_{1}=k \pm v_{2}$. Deformation of the stability regions due to spin angle $\Psi$ is shown in more detailed in Fig. 4. The constant cosines lines (geodetic levels) calculated with step of 0.2 are shown inside the stability regions with thin black and grey lines. The deformation of stability regions is different. For instance, the topology of the regions «1» and «4» doesn't change in fact, whereas the
regions «2» and «3» join to each other and no coupling resonance is occurred when the snake is switch on. The density of geodetic levels is reduced with the increasing of $\Psi$ and the behavior of their crossing changes also. The areas, which geodetic levels for different tunes are parallel to each other are formed. Beam stability within these areas is better in respect to variation of the quad gradients. Grey triangles in Fig. $4\left(K_{F}=0.758, K_{D}=0.770\right)$ corresponds to the tunes $v_{1}=6.8$ and $v_{2}=6.85$. In this case the eigenvalues lie at the unit circle closely to each other in the same quadrants (see Fig. 4, from the right). The betatron motion becomes unstable due to increasing normalized solenoid field $K_{S}$. The choice of another point in the diagram may provide beam stability during the increasing of solenoid field up to 3.6 T (full snake). An example of such point is $K_{F}=0.760, K_{D}=0.755$ (marked with black square in Fig. 4). The cosines values are $\cos 2 \pi \nu_{1}=1 / \sqrt{ } 2, \cos 2 \pi \nu_{2}=-1 / \sqrt{ } 2$ for $\Psi=0$. In this case the eigenvalues are at maximum distance from each other at the unit circle. Thus the deviations of betatron motion parameters are the least sensitive to quad gradients variations. The dependences of the cosines on

Figure 5: (a) The dependence of $\cos 2 \pi \nu_{1}$ and $\cos 2 \pi \nu_{2}$ on spin angle $\Psi$ in the snake ( $K_{F}=0.760, K_{D}=0.755$ ). (b) Quads corrections $\delta K_{F}$ and $\delta K_{D}$ for compensation of the tunes shifts (for $\Psi=0: K_{F}=0.760, K_{D}=0.755$ ).


change of structural quads gradients (see Fig. 5b).
(b)

slope of the tunes can be easily compensated by a small

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