# BENCHMARKING STUDIES OF INTRA BEAM SCATTERING FOR HL-LHC 

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## Abstract

The effects of Intra Beam Scattering (IBS) in the High Luminosity upgrade of the LHC (HL-LHC) will be stronger compared to effects in the present LHC because of the high intensity of the proton bunches and the new proposed optics. We present benchmarking studies carried out for the present LHC at injection and collision energies as well as HL-LHC at collision energy with the Achromatic Telescopic Squeezing optics. The results of IBS growth-rate calculations using the full BjorkenMtingwa formulae [1] are compared with simplified formulae [2], Bane's high energy approximation [3], and the Completely Integrated Modified Piwinski (CIMP) approximation [4,5]. The results of calculations based on these methods carried out in Mathematica are compared with results from the codes MAD-X [6] and ZAP [7].

## INTRODUCTION

A multiple small angle scattering of charged particles within highly dense bunches, known as Intra Beam Scattering (IBS) leads to the growth in beam emittances, which can degrade the luminosity lifetimes in hadron colliders. The upgrade of LHC to High Luminosity (HLLHC) involves increasing the bunch intensity combined with small emittances and highly focussed beams at the Interaction Points (IP) through Achromatic Telescopic Squeezing (ATS) optics [8].

Full IBS theory is described in number of publications and different formalisms are used in different software codes to compute the longitudinal and transverse growth rates. In this paper, we present the benchmarking of results based on [2] carried out in Mathematica compared with results from the codes MAD-X and ZAP. ZAP uses Bjorken-Mtingwa formalism without vertical dispersion [1]. As described in [9], MAD-X uses extended formulae starting from Bjorken-Mtingwa and following the approach of Conte and Martini [10], but includes the effect of vertical dispersion.

The goal of these studies is to quantify the IBS growth rates that should be expected in HL-LHC under different conditions, validating the results by comparing different codes and formalisms, and placing the results in context by making comparisons with the LHC. The effects of IBS growth rates on evolution of different beam parameters such as emittance, bunch length, intensity and luminosity during a fill as addressed in [11] using Collider Time Evolution (CTE) program [12] are not treated here.

## IBS GROWTH RATES

The IBS growth rates are calculated for the present LHC (lattice "A", LHCV6.503) and HL-LHC (lattice "B", HLLHCV1.0). Relevant beam parameters are given in Table 1. For LHC, a single "nominal" parameter set is considered; for HL-LHC, two different parameter sets are considered based on bunch separations of 25 ns or 50 ns .

Table 1: Lattice and Beam Parameters used for IBS Growth Rates

| Case Number | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Lattice | A | A | A | B | B | B | B | B | B |
| Optics: Injection or Collision | Inj. | Coll. | Coll. | Inj. | Inj. | Coll. <br> Round | Coll. <br> Round | Coll. <br> Flat | Coll. <br> Flat |
| Parameter Set | Nom. | Nom. | Nom. | 25ns | 50ns | 25ns | 50ns | 25ns | 50ns |
| Beam Energy (GeV) | 450 | 3500 | 7000 | 450 | 450 | 7000 | 7000 | 7000 | 7000 |
| Particles per bunch $\left(\times 10^{11}\right)$ | 1.15 | 1.15 | 1.15 | 2.2 | 3.5 | 2.2 | 3.5 | 2.2 | 3.5 |
| Relativistic gamma | 479.6 | 3730 | 7461 | 479.6 | 479.6 | 7461 | 7461 | 7461 | 7461 |
| Norm. emittance (m.rad) $\left(\times 10^{-6}\right)$ | 3.75 | 3.75 | 3.75 | 2.5 | 3.0 | 2.5 | 3.0 | 2.5 | 3.0 |
| Bunch length (m) | 0.102 | 0.079 | 0.079 | 0.102 | 0.102 | 0.079 | 0.079 | 0.079 | 0.079 |
| Relative Energy spread ( $\times 10^{-4}$ ) | 3.78 | 2.2 | 1.1 | 3.78 | 3.78 | 1.1 | 1.1 | 1.1 | 1.1 |
| Longitudinal emittance (eVs) | 0.73 | 2.55 | 2.55 | 0.73 | 0.73 | 2.55 | 2.55 | 2.55 | 2.55 |

IBS growth rates calculated using different software codes/formalisms with crossing angle off are given in Table 2 (a); results with crossing angle on are given in Table 2 (b). Flat beam HL-LHC lattices (cases 8, 9) are available only with crossing angle off. In crossing angle off cases, there are some accuracy issues with the integral
in vertical plane using Bane's approximation. ZAP does not include vertical dispersion in the IBS growth rates and therefore the vertical growth rates with crossing angle on calculated using ZAP are the same as with crossing angle off. As can be seen in Tables 2 (a) and (b), the results of different formalisms and codes are in good agreement.

Table 2 (a): IBS Growth Rates (with crossing angle off)

| Case Number | 1 | 4 | 5 | 6 | 7 | 8 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| IBS Growth Rates $\times 10^{-5}$ |  |  |  |  |  |  |  |
| MAD-X |  |  |  |  |  |  |  |
| Longitudinal (1/sec) | 1.62 | 4.99 | 6.38 | 1.32 | 1.68 | 1.29 | 1.64 |
| Horizontal (1/sec) | 0.661 | 3.34 | 3.58 | 1.50 | 1.61 | 1.63 | 1.75 |
| Vertical (1/sec) | -0.00483 | -0.0224 | -0.024 | -0.0000434 | -0.0000461 | -0.0000402 | -0.0000428 |
| Full Bjorken-Mtingwa |  |  |  |  |  |  |  |
| Longitudinal (1/sec) | 1.65 | 5.06 | 6.47 | 1.38 | 1.75 | 1.35 | 1.72 |
| Horizontal (1/sec) | 0.685 | 3.44 | 3.68 | 1.60 | 1.71 | 1.75 | 1.88 |
| Vertical (1/sec) | -0.00468 | -0.0216 | -0.0232 | -0.0000418 | -0.0000444 | -0.0000387 | -0.0000411 |
| BANE |  |  |  |  |  |  |  |
| Longitudinal (1/sec) | 1.95 | 6.10 | 7.72 | 1.35 | 1.72 | 1.32 | 1.66 |
| Horizontal (1/sec) | 0.804 | 4.11 | 4.36 | 1.55 | 1.66 | 1.65 | 1.78 |
| Vertical (1/sec) | - | - | - | - | - | - | - |
| CIMP |  |  |  |  |  |  |  |
| Longitudinal (1/sec) | 1.95 | 6.13 | 7.75 | 1.35 | 1.73 | 1.32 | 1.69 |
| Horizontal (1/sec) | 0.802 | 4.10 | 4.35 | 1.55 | 1.67 | 1.65 | 1.78 |
| Vertical (1/sec) | -0.00598 | -0.0289 | -0.0304 | -0.0000465 | -0.0000490 | -0.0000437 | $-0.0000461$ |
| ZAP |  |  |  |  |  |  |  |
| Longitudinal (1/sec) | 1.58 | 4.66 | 8.01 | 1.30 | 1.65 | 1.26 | 1.61 |
| Horizontal (1/sec) | 0.644 | 3.12 | 4.49 | 1.47 | 1.58 | 1.60 | 1.71 |
| Vertical (1/sec) | -0.00462 | -0.0201 | -0.0296 | -0.0000419 | -0.0000445 | -0.0000388 | $-0.0000413$ |

Table 2 (b): IBS Growth Rates (with crossing angle on)

| Case Number | 2 | 3 | 6 | 7 |
| :---: | :---: | :---: | :---: | :---: |
| IBS Growth Rates $\times 10^{-5}$ |  |  |  |  |
| MAD-X |  |  |  |  |
| Longitudinal (1/sec) | 0.280 | 0.467 | 1.32 | 1.68 |
| Horizontal (1/sec) | 0.300 | 0.256 | 1.51 | 1.62 |
| Vertical (1/sec) | 0.00569 | 0.00475 | 0.0258 | 0.0275 |
| Full Bjorken-Mtingwa |  |  |  |  |
| Longitudinal (1/sec) | 0.285 | 0.478 | 1.37 | 1.75 |
| Horizontal (1/sec) | 0.310 | 0.266 | 1.60 | 1.72 |
| Vertical (1/sec) | 0.00542 | 0.00456 | 0.0261 | 0.0279 |
| BANE |  |  |  |  |
| Longitudinal (1/sec) | 0.289 | 0.478 | 1.34 | 1.71 |
| Horizontal (1/sec) | 0.313 | 0.266 | 1.54 | 1.66 |
| Vertical (1/sec) | 0.00554 | 0.00457 | 0.0247 | 0.0265 |
| CIMP |  |  |  |  |
| Longitudinal (1/sec) | 0.291 | 0.480 | 1.35 | 1.72 |
| Horizontal (1/sec) | 0.315 | 0.267 | 1.55 | 1.67 |
| Vertical (1/sec) | 0.00552 | 0.00458 | 0.0248 | 0.0266 |
| ZAP |  |  |  |  |
| Longitudinal (1/sec) | 0.274 | 0.459 | 1.30 | 1.65 |
| Horizontal (1/sec) | 0.293 | 0.251 | 1.48 | 1.59 |
| Vertical (1/sec) | -0.0000461 | -0.0000086 | -0.0000421 | -0.0000447 |



Figure 1: Left - Case 3 (LHC Collision), Right - Case 7 (HL-LHC Collision); Top - Longitudinal Local (Green) and accumulated (Dark blue) IBS growth rates; Middle - Horizontal Local (blue) and accumulated (Brown) IBS growth rates, Bottom - Vertical Local (red) and accumulated (dark blue) IBS growth rates.

The slight differences at injection energies in Bane's approximation and CIMP are due to the high energy approximations used in these formalisms. The crossing angle has little effect on the longitudinal and horizontal growth rates but it changes the sign of the vertical growth rates and increases the absolute value. Figure 1 shows local and accumulated IBS growth rates for cases 3 and 7 with crossing angle on, for the longitudinal, horizontal and vertical planes respectively. The increase in local contributions to growth rates from ATS optics and HLLHC beam parameters are seen on plots on the right. There is no significant effect on IBS growth rates for round or flat beams with ATS optics and even though with crossing angle on, the vertical growth rates change sign with increased value, the vertical growth rates are still very low.

The IBS growth rates previously presented in references $[11,13]$ are in good agreement at collision energy (Cases 8 and 9 in Table 2a). The slight differences at collision energy and marginal differences at injection energy are due to revised beam parameters and the latest HL-LHC lattices used here. The differences in growth rates at injection energy will affect the evolution of beam parameters during a fill, which are not included in this paper and need to be updated using CTE program.

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