ISBN: 978-3-95450-132-8 doi:10.18429/JACoW-IPAC2014-TUPRO002

FRINGE FIELDS MODELING FOR THE HIGH LUMINOSITY LHC LARGE APERTURE QUADRUPOLES *

B. Dalena, O. Gabouev, J. Payet, A. Chancé CEA/SACLAY, DSM/Irfu/SACM
F-91191 Gif-sur-Yvette, France
D. R. Brett, R. B. Appleby The University of Manchester and the Cockcroft Institute, UK
R. De Maria, M. Giovannozzi CERN, Geneva, Switzerland

Abstract

The HL-LHC Upgrade project relies on large aperture magnets (mainly the inner Triplet and the separation dipole D1). The beam is much more sensitive to non-linear perturbations in this region, such as those induced by the fringe fields of the low-beta quadrupoles. Different tracking models are compared in order to provide a numerical estimate of the impact of fringe fields for the actual design of the inner triplet quadrupoles. The implementation of the fringe fields in SixTrack, to be used for dynamic apertures studies, is also discussed.

INTRODUCTION

In order to increase the luminosity by a factor 10 a small β^* value is required at the two high luminosity Interaction Points (IPs). Additionally also an increase of beam brightness is required [1]. As a consequence the beam sizes increase in the inner triplet region. Moreover due to the crossing angle, the two beams enter off axis in the triplet, which implies that the beams are much more sensitive to non linear perturbations in this region. Analytical evaluations of detuning with amplitude and chromatic effects induced by the fringe fields of the inner triplet have shown that the effect is small but not negligible [2]. Therefore, the effect on long-term beam dynamics should be evaluated via symplectic tracking simulations. We present here an implementation of the method proposed by Venturini and Dragt [3], which uses 3D magnetic field data to compute the transfer map to be used in the tracking simulations. The application to the current design of the inner triplet quadrupole is presented, focusing on the source of errors in the method and on the comparison with the leading order analytical model and a symplectic 4th order integrator. Finally the possible implementation of this method in SixTrack [4–6] is briefly discussed.

MAGNETIC FIELD MAPS

The magnetic field data we use in this analysis have been computed by CERN magnet group [7]. The data correspond to the end region of a prototype model. They are provided on a 3D Cartesian grid of step 3 mm in the three dimensions and with z values which start at the nominal field value and ends where the field values are near zero (order of 10^{-5}).

* The research leading to these results has received funding from the European Commission under the FP7 project HiLumi LHC, GA no. 284404, co-funded by the DoE, USA and KEK, Japan.

This prototype design has two symmetric ends, although the actual magnets have asymmetric ends due to the current leads. They generate skew components on one end of the quadrupole and they increase the residual higher order multipoles in the two fringe field regions.

DESCRIPTION OF THE METHOD

In order to evaluate the non linear fringe field effect on the long term beam dynamics, a symplectic transfer map of the quadrupoles including the fringe region is required. The method we have implemented was first proposed by Venturini and Dragt [3]. It consists in three steps of calculation. First the field harmonics are calculated using the 3D magnetic field data computed by 3D FEM codes. Then, the vector potential of the quadrupole is computed using a Fourier and anti-Fourier transform of the field harmonics to calculate the generalized gradients. Finally, the transfer map of the z-dependent Hamiltonian of the quadrupole is evaluated through a second order symplectic integrator computed with Lie Algebra transformations. In the following, the main source of errors of the three steps of calculation and the main features of the method are discussed.

The major source of error in the harmonics analysis of 3D magnetic field data distributed on a Cartesian grid comes from the interpolation. We have compared two types of interpolator: a quadratic interpolator with weighted coefficients, which is usually implemented in PIC codes, and a cubic interpolator so called cubic Hermite spline or cubic Hermite interpolator [8]. The test field used to evaluate the errors is generated as a superposition of harmonics, analytically calculated. The value of the higher order harmonics are attenuated with respect to the lower ones, to simulate the different content of harmonics usually participating in a magnetic field. Each of the harmonics is shaped with a Gaussian form factor in z, to simulate the fringe field region. We have selected the cubic Hermite Spline Interpolator (HSI) because it gives a smaller error on the calculation of the lower order harmonics. Using this interpolator and the test field we have analyzed the resolution of the HSI as function of the grid steps in the three planes. The study shows that there is a smooth dependence with the grid step. The relative error in the reconstruction of the main quadrupole component with the HSI is always less than 10^{-5} and it increases up to 10^{-2} for the high order harmonics. A grid step of 3 mm in the three coordinates is a good compromise between computational speed and precision.

ISBN: 978-3-95450-132-8 doi:10.18429/JACoW-IPAC2014-TUPRO002

The three components of the quadrupole vector potential can be written as expansions of normal and of skew multipoles. Each of the multipole can be expanded in terms of x,y homogeneous polynomials and z-dependent coefficients $C_n^{[m]}(z)$, called generalized gradients. It can be demonstrated that these coefficients can be calculated using the multipolar expansion of the magnetic field [9]:

$$C_n^{[m]}(z) = f(n,m)\int_{-\infty}^{\infty} \frac{e^{\mathrm{ikz}}\, k^{n+m-1}}{I_n'(kR_{an.})}\tilde{B}_n(R_{an.},k)dk \qquad (1)$$

where, $f(n,m) = \frac{i^m}{2^n n! \sqrt{2\pi}}$, $I_n'(kR_{an.})$ is the derivative of the generalized Bessel functions, $\tilde{B}_n(R_{an.},k)$ is the Fourier transform of the harmonic coefficients and $R_{an.}$ is the radius used in the Harmonics Analysis calculation (50 mm in this case). The computation of this integral is non trivial when using discrete data, as in the case of the harmonics calculated from the magnetic field data. To solve the two Fourier integrals we use the method of Filon-Spline, which is an extension of the Filon method using a spline interpolation [10]. Special attention needs to be considered in the choice of the step in the frequency k and in the boundary conditions of the spline used to interpolate the function. Once these parameters have been fixed we obtain an analytical formula of which we know the Fourier transform, that approximates our data. We have applied the method to the harmonics calculated from the 3D magnetic field and studied the error in the reconstruction of the harmonics. Figure 1 shows the

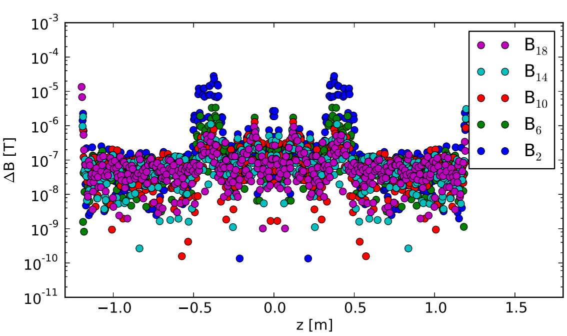

Figure 1: Difference between the calculated harmonics and the reconstructed harmonics for a grid step of 3 mm in the three dimensions, considering 16 derivatives in the generalized gradients computation.

difference between the harmonics calculated from the magnetic field data and the harmonics reconstructed from the generalized gradients, as:

$$B_n(r,z) = \sum_m (n+2m)\frac{(-1)^m n!}{4^m m!(n+m)!} r^{n+2m-1} C_n^{[2m]}(z) \qquad (2)$$

calculated summing up to 16 derivatives in the generalized gradients calculation. The absolute error on the reconstruction of the harmonics is 10^{-6} or less, except at the boundaries of the field map and in the region of the biggest slope of the field. At the boundary, the error in the reconstruction of the field reaches the same order of magnitude as the field itself (10^{-5}) this is known as Gibbs phenomena and can be cured without considering the first and last point in the tracking. The error in the region of biggest slope is related to the number of derivatives we use in the computation of the gradients, as shown in Fig. 2. In the following we will consider 16

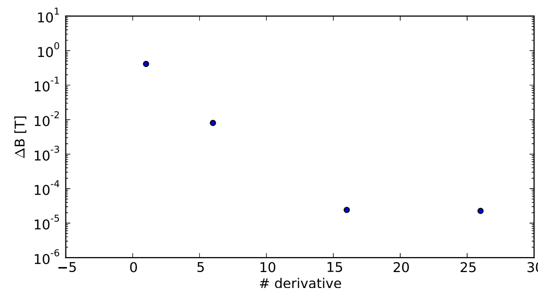

Figure 2: Maximum difference between the calculated harmonics and the reconstructed harmonics as a function of the number of derivatives, for grid steps of 3 mm in the three dimensions.

derivatives in the computation of the generalized gradients (more derivatives in fact would slow the tracking without improving its precision).

In order to describe the motion of the particles in a magnetic system the transfer map of the system is needed and in the case of multi-turn simulations this map needs to be symplectic. Using Lie Algebra formalism, this transfer map (which represents the equation of motion of the particle for the system under consideration) takes the following expression:

$$\mathcal{M}(\Delta\sigma) = \exp(-L : K :) \qquad (3)$$

where L and K are the length and the Hamiltonian of the system. In order to have the explicit dependence on z in the Hamiltonian we consider the 8 dimensions Hamiltonian given by Forest *et al.* in Ref. [11]. Since the expression of the Hamiltonian contains the terms $\left(p_{x,y} - a_{x,y}\right)^2$, the system is not exactly solvable, so we use a transfer map approximated to the second order:

$$\begin{aligned} \mathcal{M}(\Delta\sigma) = \exp(-\tfrac{\Delta\sigma}{2} : K_1 :) \exp(-\tfrac{\Delta\sigma}{2} : K_2 :) \\ \exp(-\tfrac{\Delta\sigma}{2} : K_3) \exp(-\Delta\sigma : K_4 :) \exp(-\tfrac{\Delta\sigma}{2} : K_3 :) \\ \exp(-\tfrac{\Delta\sigma}{2} : K_2 :) \exp(-\tfrac{\Delta\sigma}{2} : K_1 :) + O(\Delta\sigma^3) \\ = \mathcal{M}_2 + O(\Delta\sigma^3). \end{aligned} \qquad (4)$$

where $K = K_1 + K_2 + K_3 + K_4$, $K_1 = p_z - \delta$, $K_2 = a_z$, $K_3 = \frac{(p_x - a_x)^2}{2(1+\delta)}$, $K_4 = \frac{(p_y - a_y)^2}{2(1+\delta)}$, $: K : f = < K, f >$ is the Lie operator defined by the Poisson brackets [12], and we have used the generating function given in [11] to simplify the terms K_3 and K_4. They have been implemented in a Fortran90 code which reads a table of the computed vector potential, expressed as product of the generalized gradients and monomials in the x, y coordinates, for each step in z.

ISBN: 978-3-95450-132-8 doi:10.18429/JACoW-IPAC2014-TUPRO002

VALIDATION OF THE METHOD AND DISCUSSION

In order to validate the method we compare this symplectic integrator, that for brevity we will call Lie Tracking with a symplectic 4th order integrator and with leading order analytical kicks computed by Forest and Milutinovic in [13]. We track a single particle with different initial offset and zero angular deviation trough a single quadrupole and we look at the non-linear part of the final p_y coordinate (subtracting the linear contribution of the main quadrupole component) as a function of the initial offset. First we compared with a symplecting integrator, implemented for Linear Colliders interaction region studies [14]. It consists in interleaved drifts and kicks, accurately chosen to obtain a symplectic 4th order integrator [15], the kicks are given by the Lorenz force, computed interpolating the values of external 3D magnetic field data. We have used here the same HSI interpolator used previously for the harmonic analysis. Figure 3 shows

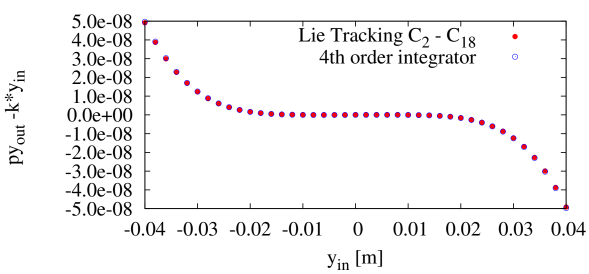

Figure 3: Non-linear part of p_y after tracking particles with different offsets in a single quadrupole. Lie Tracking is compared with a fourth order symplectic integrator.

the very good agreement of the two different tracking procedures, when using up to 18 harmonics to reconstruct the Vector Potential in the Lie Tracking. It proves the ability of the two tracking methods to describe effects at the nano scale. Figure 4 shows the non linear fringe field effect given

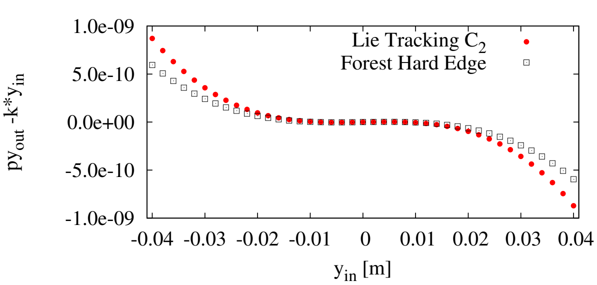

Figure 4: Non-linear part of p_y after tracking particles with different offsets in a single quadrupole. Lie Tracking of the main quadrupole component only is compared with a analytical leading order kicks.

by the Lie Tracking using the main quadrupole component of field only and by the leading order hard edge fringe field model given in [13]. There is a good agreement between the two models up to ±1.5 cm, but for larger initial offset values the Forest model underestimates the non linear fringe field effect of a factor 2 for this quadrupole design. This effect is due to the shape of the fringe field ends under study, which cannot be explained considering the lowest order derivative of the gradients only. In fact, to reduce the error of the field reconstruction in the region with biggest slope we have used up to 16 derivatives of the generalized gradients coefficients. By comparing Fig. 3 to Fig. 4 we see that the higher order components of the field in the fringe field increase the non-linear effect by one order of magnitude.

Therefore, to evaluate in a realistic way the non linear fringe field effects on the long term beam dynamics, a symplectic tracking needs to be included in a tracking code, like SixTrack. The force of the Lie Tracking we have implemented consists in the possibilities to control the field harmonics used in the simulations. Each field component can be switched on and off easily in the calculation of the generalized gradients. Moreover if the symplectic integrator is accurately written in the same coordinates as SixTrack the tracking in the central part of the quadrupole can be preserved as is, and only the fringe field region treated via the Lie Algebra tracking.

$$D(-L_d)I(L_{ff})Q^{-1}(L_q)Q(L_0)Q^{-1}(L_q)I(L_{ff})D(-L_d) \quad (5)$$

Equation 5 shows a possible scheme of integration of the Lie Tracking with an existing hard edge model of the quadrupole. In order to avoid over strength counting an equivalent anti quadrupole needs to be subtracted after the fringe tracking, which needs to be extended up to the z where the B_z component is vanishing again, before entering the central part. The limit of this method is that the computation of the generalized gradients, using the harmonics analysis of the field on a circle, is valid only inside the radius of analysis. As a consequence any DA calculation will be valid inside this region only.

CONCLUSION

We have implemented a method to compute a transfer map of a z-dependent Hamiltonian using 3D magnetic field data, to calculate the vector potential, and Lie Algebra transformation, to derive the symplectic integrator (transfer map). We have validated the method by comparing it with a 4th order symplectic integrator that uses the Lorenz force to evaluate the kick strength. By comparing the tracking we have implemented with analytical leading order fringe field model given by Forest-Milutinovic, we have found a discrepancy at large particle amplitudes due to the higher order derivatives needed to describe the fringe field shape of the magnet under study. Finally a possible implementation of the method in tracking code used for long term beam dynamics studies, like SixTrack, as been briefly discussed.

REFERENCES

[1] L. Rossi, ''LHC Upgrade Plans: Options and Strategy", in proceedings of IPAC11, p. 908.

ISBN: 978-3-95450-132-8 doi:10.18429/JACoW-IPAC2014-TUPRO002

[2] A.V. Bogomyagkov *et al.*, ''Analysis of the Non-linear Fringe Effects of Large Aperture Triplets for the HL LHC Project", in proceedings of IPAC13, p. 2615.

[3] M. Venturini, A. J. Dragt, Nucl. Inst. and Methods A 427 (1999) 387.

[4] F. Schmidt, CERN/SL/9456,2012.

[5] R. De Maria *et al.*, "Recent Developments and Future Plans for SixTrack", in proceedings of IPAC13, p. 948.

[6] SixTrack website, cern.ch/sixtrack-ng

[7] E. Todesco *et al.*, private comunication.

[8] E. Catmull and R. Rom, "A class of local interpolating splines", Computer Aided Geometric Design, R. E. Barnhill and R. F. Reisenfeld, Eds. Academic Press, New York, 1974, pp. 317–326.

[9] A. J. Dragt, www.physics.umd.edu/dsat

[10] B. Einarsson, "Numerical computation of Fourier integrals with cubic splines", 1968. C. Mitchell,"Calculation of realistic charged-particle transfer maps", PhD thesis, University of Maryland 2007.

[11] Y. Wu, E. Forest and D. S. Robin, Phys. Rev. E 68, 046502 (2003).

[12] A.J.Dragt *et al.*, Ann. Rev. Nucl. part. Sci. 1988, 38n 455-496.

[13] E. Forest *et al.*, Nucl. Inst. and Methods A 269 (1988) 474.

[14] Y. Levinsen *et al.*, Phys. Rev. ST Accel. Beams 17, 051002 (2014).

[15] E. Forest and R. D. Ruth, Physica D: Nonlinear Phenomena 43, 105 (1990).