

ELECTROMAGNETIC SIMULATIONS FOR NON-ULTRARELATIVISTIC BEAMS AND APPLICATION TO THE CERN LOW ENERGY MACHINES

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Abstract

In the framework of the PS-Booster upgrade project an accurate impedance model is needed in order to determine the effect on the beam stability and assess the impact of the new devices to be installed in the machine. CST 3-D EM simulations are widely used to estimate the impedance contribution of the different devices along the CERN accelerator complex. Unlike the highly relativistic case, in which the reliability of the EM solver has been proved in many specific cases by comparing simulations with analytical results, the nonrelativistic case has been so far not yet benchmarked. In order to use systematically CST 3-D EM simulations for the PS-Booster, or even lower energy machines like the antiproton decelerator ELENA, a validation campaign has been carried out. The main complication to single out the beam coupling impedance, as resulting from the interaction of the beam with the surroundings, consisted of removing reliably the strong contribution of the direct space charge of the source bunch, which is included in the EM calculation. The simulation results were then benchmarked with the analytical results for the case of a PEC cylindrical tube and of a ferrite loaded kicker.

INTRODUCTION

The beam coupling impedance defines the electromagnetic (EM) interaction of the particle beam with the external surroundings. According to its definition [1] the beam coupling impedance of a device of length L can be written directly in frequency domain as follows

$$Z_{\parallel} = -\frac{1}{q_0} \int_0^L E_s e^{jks} ds \quad (1)$$

$$Z_{x,y} = \frac{j}{q_0} \int_0^L [E_{x,y} - \beta Z_0 H_{y,x}] e^{jks} ds$$

where $(E_{x,y,s}, H_{x,y,s})$ are respectively the electric and magnetic induced fields in frequency domain, $k = \frac{\omega}{v}$ is the wave number, $v = \beta c$ is the particle velocity and Z_0 is the free space impedance ($120\pi\Omega$). EM codes like CST Particle Studio [2] cannot single out the induced fields, therefore the beam coupling impedance is obtained as calculated from the total fields (sum of induced and self-field of the source):

$$Z^{tot}(\beta) = Z(\beta) + Z^{SC}(\beta) \quad (2)$$

where $Z^{SC}(\beta)$ is the so called direct space charge impedance. The direct space charge impedance is insignificant for accelerators with high relativistic beta (in the ultrarelativistic limit ($\beta = 1$) it results $Z^{SC} = 0$), and the smaller the beta the larger the direct space charge impedance.

CST PARTICLE STUDIO SIMULATIONS

In the non ultrarelativistic case ($\beta < 1$) the main complication of CST 3D EM simulations to single out the beam coupling impedance, as resulting from the interaction of the beam with the surroundings, consisted of removing reliably the strong contribution of the direct space charge of the source bunch.

Analytical cancellation of $Z^{SC}(\beta)$

The direct space charge impedance does not depend on the surroundings and can be calculated analytically [3, 4] according to the definition of the beam coupling impedance (see Eq. (1)) from the EM self-fields of the particle source [5, 6]. Therefore, according to Eq. (2), the beam coupling impedance $Z(\beta)$ is obtained subtracting to the CST simulation results the direct space charge impedance analytically calculated. However, as it will be described in the next section, when $Z^{SC}(\beta) \gg Z(\beta)$ the required accuracy on the simulation results becomes prohibitive to be reached.

Numerical cancellation of $Z^{SC}(\beta)$

Due to the limitations of the analytical cancellation of $Z^{SC}(\beta)$, it would be very important to establish a solid technique to remove the direct space charge impedance directly from simulations. CST allows to remove all the elements of the device under test (DUT) without changing the mesh structure. Therefore, the beam coupling impedance of the simulation box (i.e. a rectangular vacuum chamber) can be obtained by using exactly the same discretization of the domain of calculus of the DUT. Writing the impedance of this simulation as follows:

$$Z_1^{tot}(\beta) = Z_1(\beta) + Z^{SC}(\beta) \quad (3)$$

and by subtracting this impedance $Z_1^{tot}(\beta)$ to the impedance of the DUT $Z^{tot}(\beta)$ (see Eq.(2)) we obtain:

$$Z^{sim}(\beta) = Z^{tot}(\beta) - Z_1^{tot}(\beta) = Z(\beta) - Z_1(\beta) \quad (4)$$

The beam coupling impedance $Z_1(\beta)$ of the rectangular vacuum chamber with perfect electric conductive walls (also called indirect space charge impedance [4]) can be calculated analytically [7]. Moreover, by definition it results $Z_1(\beta) \leq Z(\beta)$ ensuring a good accuracy for the extrapolation of $Z(\beta)$ obtained by summing $Z^{sim}(\beta)$ and the analytical calculation of $Z_1(\beta)$.

EXAMPLE OF APPLICATION

The proposed methods have been applied to the case of a cylindrical vacuum chamber with walls made of perfect

electric conductor (PEC) and to the case of two parallel plates of ferrite surrounded by PEC (i.e. the Tsutsui model [8]). For both structures the existing analytical solutions [3, 9, 10] have been used to validate the simulations.

Cylindrical vacuum chamber with PEC walls

The beam coupling impedance $Z(\beta)$ of a cylindrical vacuum chamber with PEC walls would be exactly zero in the ultrarelativistic limit ($\beta = 1$). The longitudinal direct space charge impedance is singular when source and test particle are placed at the same transverse position (x, y). However, also in this case, the 3D simulation give a finite value of the impedance because of the numerical transverse effective size of the source. As reference Figs. 1 and 2 show the analytical calculation of the longitudinal and transverse impedance as function of the source offset (transverse displacement of the source with respect to the center of the vacuum chamber (position of the test particle)) at a given particle velocity and frequency for the cylindrical vacuum chamber with PEC walls of radius $r = 31.5 \text{ mm}$ and length $L = 0.2 \text{ m}$. In the example of Fig. 2, the extrapolation of the transverse beam coupling impedance is expected to be very critical since the direct transverse space charge impedance $Z^{SC}(\beta)$ is up to 5 orders of magnitude larger than the beam coupling impedance $Z(\beta)$. The required accuracy in percentage on the calculation of the total impedance $Z^{tot}(\beta)$ to keep the error on the extrapolated beam coupling impedance $Z(\beta)$ below the 10% ranges between $10^{-4}\%$ (source offset=0.1 mm) and 3.5% (source offset=20 mm). The required accuracy is a decreasing function with the source offset (the direct space charge impedance is a decreasing function with the source offset while the impedance $Z(\beta)$ is constant with it). Therefore, using large source offset would dramatically reduce the required accuracy of the simulation. In general this trick could be applied to any kind of device. However, it is important to consider that the linear expansion of the beam coupling impedance is questionable as the source offset becomes comparable with the aperture.

The extrapolation of the longitudinal beam coupling impedance is expected to be much less critical ($Z(\beta)$ and $Z^{SC}(\beta)$ are of the same order of magnitude). Figures 3 and 4 show a comparison between the analytical calculation and the CST simulation of the beam coupling impedance as obtained using the analytical cancellation method of the direct space charge impedance respectively for the longitudinal and transverse beam coupling impedance. As expected, the longitudinal impedance is very close to the analytical one and the accuracy in the extrapolation of the transverse impedance is offset dependent (as expected large offsets lead to higher accuracy).

Parallel plates of ferrite surrounded by PEC

The beam coupling impedance $Z(\beta)$ of two parallel plates of ferrite surrounded by PEC has been calculated by H. Tsutsui in the ultra-relativistic case [8] and extended to the nonrelativistic regime in [9, 10]. The impedance of these structures have been simulated by using the numerical cancellation

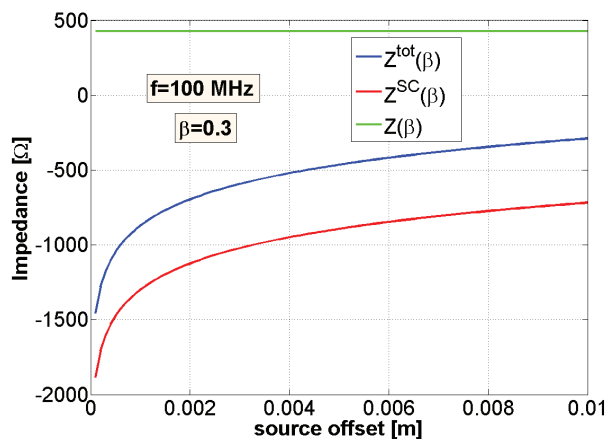


Figure 1: Analytical calculation of the imaginary part of the longitudinal impedance for the cylindrical vacuum chamber with PEC walls ($r = 31.5 \text{ mm}$, $L = 0.2 \text{ m}$).

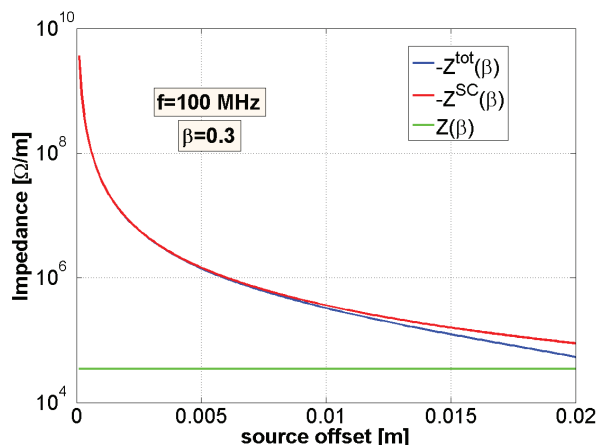


Figure 2: Analytical calculation of the imaginary part of the transverse impedance for the cylindrical vacuum chamber with PEC walls ($r = 31.5 \text{ mm}$, $L = 0.2 \text{ m}$).

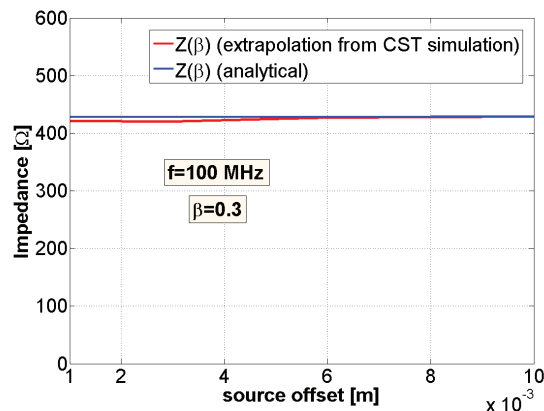


Figure 3: Comparison between the simulated imaginary part of the longitudinal impedance (red) and the analytical calculation (blue).

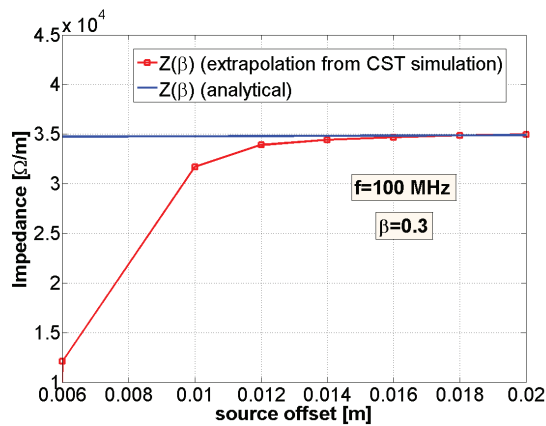


Figure 4: Comparison between the simulated imaginary part of the transverse impedance (red) and the analytical calculation (blue).

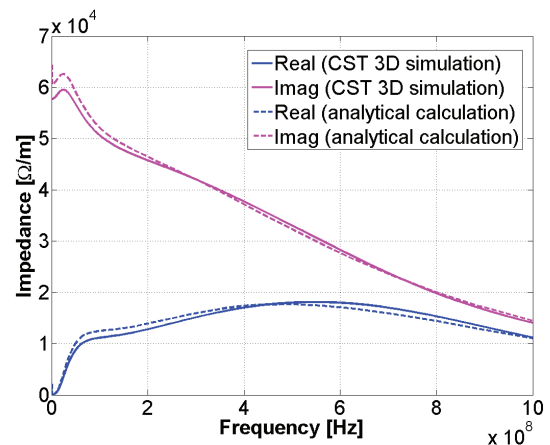


Figure 6: Comparison between the simulated vertical detuning impedance (full lines) and the analytical calculation (dashed lines) for the Tsutsui model of the extraction kicker of the PSB at 160 MeV kinetic energy ($\beta = 0.519$).

method of the direct space charge. As example, Figures 5 and 6 show a comparison between the CST simulations and the analytical calculations for the vertical driving and detuning impedances. The beam coupling impedances obtained from the simulation results by using the numerical cancellation method exhibit a good agreement with the analytical results.

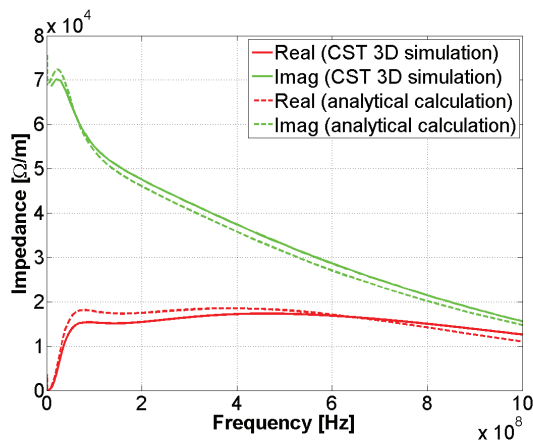


Figure 5: Comparison between the simulated vertical driving impedance (full lines) and the analytical calculation (dashed lines) for the Tsutsui model of the extraction kicker of the PSB at 160 MeV kinetic energy ($\beta = 0.519$).

CONCLUSIONS

The problem of the beam coupling impedance simulation in the nonrelativistic regime has been discussed. Two different methods have been proposed to single out the beam coupling impedance from the simulation results. The methods have been successfully benchmarked with analytical results.

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