

# ELECTROMAGNETIC MODELING OF OPEN CELL CONDUCTIVE FOAMS FOR HIGH SYNCHROTRON RADIATION RINGS

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## Abstract

Open cell conductive foams (OCMF) have been recently suggested as a possible alternative to perforated metal patches for efficiently handling gas desorption from the beam pipe wall in high synchrotron radiation machines, in view of its superior performance in terms of residual gas concentration and beam shielding. Here we discuss the electromagnetic properties (characteristic impedance and propagation constant) of OCMFs and how they affect the beam coupling impedance.

## OPEN CELL METAL FOAMS

Open cell metal foams (OCMF) are highly gas-permeable reticular materials, consisting of a 3D web of thin conducting ligaments (Fig. 1), whose typical structure is shown in Figure 1. OCMF have remarkable struc-

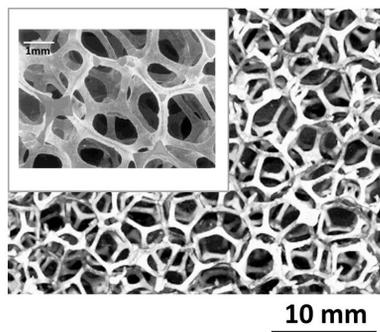


Figure 1: A typical OCMF at two different viewing scales.

tural properties (low density and weight, high (tensile and shear)-strength to weight ratio, nearly isotropic load response, low coefficient of thermal expansion), which qualified them among the most interesting new materials for aerospace applications [1]. The key morphological parameters of OCMF are the "pore" size, and the porosity (volume fraction of pores). Pore sizes in the range from  $10^{-6}$  to  $10^{-3}$  m and porosities in the range 0.7 - 0.99 are typical. These parameters determine the gas-permeability of the material, and, together with the electrical properties of the metal matrix, its electrical characteristics. OCMF may replace perforated metal patches in high synchrotron radiation accelerators [2], in view of their potentially superior performance in terms of outgassing and beam shielding [3].

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## Electrical Properties of Conducting Foams

Electromagnetic modeling of OCMFs has been studied by several Authors during the last decade. A numerical approach based on Weiland finite integration technique (FIT, [4]) has been proposed by Zhang et al. [5] to compute the (frequency, thickness and angle of incidence dependent) reflection coefficient of *SiC* foam. In the quasi-static limit  $\lambda \rightarrow 0$ , the conductivity of OCMFs can be computed using effective medium theory (EMT), for which several formulations exist giving comparable results (see, [6],[7] for a review), e.g.,

$$\sigma_{eff} = \sigma_0(1-p)^\nu, \quad (1)$$

where  $\sigma_0$  is the bulk metal conductivity,  $p$  the porosity, and  $\nu$  a morphology-dependent factor.

Measurements of the microwave electromagnetic shielding efficiency of OCMF panels [8] indicate that a simple Drude model

$$\sigma(\omega) = \frac{\omega_p^2 \epsilon_0}{1\omega + \nu} \quad (2)$$

provides a good description of the frequency-dependent conductance of metallic foams. Typically, the relevant plasma and collision frequencies,  $\omega_p$  and  $\nu$  are of the order of a few tens of GHz and a few tens of KHz, respectively, much smaller than their solid metal counterparts (typically in the PHz and GHz range, respectively [9]).

## OCMF Impedance and Skin Depth

Throughout a typical beam current frequency spectrum, OCMF and bulk metals behave differently<sup>1</sup>. In bulk metals,  $\omega \ll \nu \ll \omega_p$  so that the characteristic impedance  $Z_m$  and (complex) propagation constant  $\tilde{k}_m$  can be written

$$Z_m \sim \frac{1+i}{\sqrt{2}} \left( \frac{\omega\nu}{\omega_p^2} \right)^{1/2} Z_0, \quad \tilde{k}_m \sim \frac{1-i}{\sqrt{2}} k_0(\omega_p) \sqrt{\frac{\omega}{\nu}} \quad (3)$$

with  $Y_0 = (\epsilon_0/\mu_0)^{1/2} = 1/Z_0$  the vacuum admittance, and  $k_0(\omega) = \omega/c$ . In metallic conductors, both  $Z_m$  and  $\tilde{k}_m$  are  $\propto \omega^{1/2}$ .

In OCMFs  $\nu \ll \omega \ll \omega_p$  throughout the beam current spectrum, so that the material exhibits a *plasmonic* behavior. The OCMF wall (characteristic) impedance  $Z_f$  and (complex) propagation constant  $\tilde{k}_f$  are thus given (to lowest order in the small quantities  $\nu/\omega$  and  $\omega/\omega_p$ ) by

$$Z_f \sim Z_0 \left( \frac{\nu}{2\omega_p} + i \frac{\omega}{\omega_p} \right), \quad \tilde{k}_f \sim k_0(\omega_p) \left( \frac{\nu}{2\omega} - i \right) \quad (4)$$

<sup>1</sup>In the case of LHC, the beam current power spectrum consists of lines at integer multiples of  $f_0 = c/\delta_b$ ,  $\delta_b$  being the bunch spacing, with an envelope approximately  $\propto \cos^2$ . The -20 dB bandwidth is  $\sim 1$  GHz, roughly  $10^5$  times the circulation frequency  $\omega_R$ , and  $10^{-1}$  times the cut-off frequency of the lowest beam pipe waveguide mode [10].

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Hence, the OCMF characteristic resistance  $R_f = \text{Re}[Z_f]$  and skin depth  $\delta_f$  are both frequency independent, and, e.g., for the case of high-grade ( $\rho \approx 5.510^{-10}$  ohm cm<sup>-1</sup> at 20K) Copper foam with  $p = 0.9$ , both fairly small:

$$R_f \sim \frac{Z_0}{2} \frac{\nu}{\omega_p} \approx 1.4 \cdot 10^{-5} \text{ ohm}, \quad \delta_f \sim \frac{c}{\omega_p} \approx 6 \cdot 10^{-4} \text{ m}. \quad (5)$$

The OCMF characteristic reactance

$$X_f = \text{Im}[Z_f] \sim \nu Z_0 \frac{\omega}{\omega_p} \quad (6)$$

on the other hand, is large compared to that of bulk metal, and grows linearly with  $\omega$ . For the case, e.g., of high-grade Copper foam with  $p = 0.9$ ,  $X_f \approx 0.5$  ohm at 10<sup>4</sup>Hz.

## BEAM COUPLING IMPEDANCES

The following relationship exists (to 1st order in the wall impedance) between the longitudinal impedance per unit length  $\bar{Z}_{\parallel}$  of a patched-wall beam liner and the (known) longitudinal impedance per unit length  $\bar{Z}_{\parallel}^{(0)}$  of the same pipe with a perfectly conducting wall [11]

$$\bar{Z}_{\parallel} = \bar{Z}_{\parallel}^{(0)} + \frac{\epsilon_0 Y_0}{c \Lambda^2} \oint_{\partial S} Z_w(s) |E_{0n}(s)|^2 ds, \quad (7)$$

where  $Z_w$  is the (local) Leontóvich impedance [12] of the patched wall,  $\partial S$  is the pipe cross-section contour,  $\Lambda$  the beam linear charge density, and  $E_n^{(0)}$  the (known) field component normal to the pipe wall in the perfectly conducting pipe. A similar formula, not included for brevity, exists for the (dyadic) transverse impedance [11] as well.

Beam coupling impedances embody a synthetic description of beam stability. The absolute value and imaginary part of  $\bar{Z}_{\parallel}$  are, e.g., inversely proportional to the threshold currents for (single-bunch) microwave instability and Landau damping suppression [13]). The real part of  $\bar{Z}_{\parallel}$  determines the parasitic loss (energy lost by the beam per unit pipe length), via [13]

$$\Delta \mathcal{E} = \frac{1}{2\pi} \int_{-\infty}^{+\infty} |I(\omega)|^2 \Re[\bar{Z}_{\parallel}(\omega)] d\omega, \quad (8)$$

where  $I(\omega)$  and  $Z^{\parallel}(\omega)$  is the beam-current spectrum.

According to (7), perforated/slotted or OCMF patches, placed where the normal field component  $E_n^{(0)}$  is minimum, have minimum impact on the longitudinal impedance.

The circular beam liner with radius  $s$ , where  $E_n^{(0)}$  is uniform along  $\partial S$ , represents a worst case, where [13] the longitudinal and transverse impedances per unit length

$$\bar{Z}_{\parallel} = \frac{\langle Z_{wall} \rangle}{2\pi a}, \quad \bar{Z}_{\perp} = \frac{\langle Z_{wall} \rangle}{2\pi k_0 a^3} \quad (9)$$

are simply proportional to the average wall impedance, assumed piecewise constant

$$\langle Z_{wall} \rangle = \sum_i \xi_i Z_{wall}^{(i)} \quad (10)$$

$\xi_i$  being the surface fraction covered by patches with wall impedance  $Z_{wall}^{(i)}$ .

## OCMF Impedance Budget

Equations (7) to (10) allow to evaluate the impedance budget of an OCMF patched beam pipe. A straightforward solution, which is not included for brevity, of the electromagnetic boundary value problem for a (relativistic, vanishingly thin) axial beam in a circular conducting liner, with radius  $a$  and thickness  $\Delta$  surrounded by a co-axial (infinitely thick) conducting circular tube (the cold bore) with radius  $b > a + \Delta$ , shows that if  $\Delta > \delta_f$  in eq. (5) across the whole beam current spectrum, the Leontóvich impedance of the cold bore backed liner wall OCMF patch is fairly well approximated by the intrinsic impedance of the OCMF.

The contribution of the surface roughness of the foam to the OCMF wall impedance can be estimated as [14]

$$Z_f^{(rough)} \approx \nu \sqrt{\frac{\pi}{32}} Z_0 \left( \frac{h}{L} \right) \frac{\omega}{c} h, \quad (11)$$

$h$  and  $L$  being the r.m.s. height and correlation length of the surface roughness, respectively.

The numerical values of the above impedance components, normalized to the mode number (i.e., multiplied by the  $(\omega_R/\omega)$  factor), have been collected in Table 1, where the solid metal values are also shown for comparison. Here we assume high-grade Copper ( $\rho \approx 5.510^{-10}$  ohm cm<sup>-1</sup> at 20K) for the solid and foamed material. For this latter we assume a pore diameter  $\sim 1$  mm, and a ligament size  $\sim 0.1$  mm, yielding  $\omega_p \approx 7.93 \cdot 10^{10}$  rad sec<sup>-1</sup> and  $\nu \approx 3.66 \cdot 10^4$  Hz in the Drude model of Section , consistent with typical measured values of the static conductance, and a r.m.s. roughness scale  $h \approx .125$  mm with correlation length  $L \approx 0.25$  mm.

Table 1:  $Z_{wall}$  of Solid and OCMF High-Grade Cu

[ohm]	Solid	OCMF
$(\omega_R/\omega)R_{wall}$	$4.910^{-6} \sqrt{\omega_R/\omega}$	$1.410^{-5} (\omega_R/\omega)$
$(\omega_R/\omega)X_{wall}$	$4.910^{-6} \sqrt{\omega_R/\omega}$	$5.510^{-5}$

As seen from Table 1, the Copper wall resistance is larger than that of the OCMF up to  $\omega = 10^5 \omega_R$ . On the other hand, as anticipated in Section , the OCMF wall reactance is relatively large .

It exceeds that of a perfectly conducting slotted by roughly one order of magnitude, at a 10% escape probability level). as well as that of solid Copper. Indeed, while the typical escape probability of an OCMF wall is roughly one

order of magnitude larger than the limiting one of a slotted wall the wall electrical reactance is roughly one order of magnitude larger [3].

However, given that to obtain the same pumping capacity, the patched beam-liner surface in the OCMF-patch case is only a small fraction of that for the slotted-patch case, it is easy to place the OCMF patches where the (unperturbed) field in eq. (7) is minimum, so as to minimize their impact on the beam coupling impedance.

## CONCLUSIONS

The OCMF characteristic impedance and skin depth are nicely low, and almost frequency independent. The (inductive) wall reactance, on the other hand, is relatively large. This drawback could be mitigated by clever placement of the OCMF patches, and/or appropriate reactive loading of the (solid portion of) the pipe wall. Measurement of the complex surface impedance of metal foams are now underway [15].

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