# NEW BBA ALGORITHM FOR ELECTRON BEAM ORBIT STEERING IN LINEAR ACCELERATORS* 

A. Sargsyan \#, V. Sahakyan, G. Zanyan, CANDLE SRI, Yerevan, Armenia W. Decking, DESY, Hamburg, Germany

## Abstract

In linear accelerators or transfer lines beam-based alignment (BBA) techniques are important tools for beam orbit steering. In this paper BBA correction algorithm based on difference orbit multiple measurements is proposed. Numerical simulation results for European XFEL SASE1 and FLASH undulator section are presented, according to which the orbit alignment can be achieved within accuracy of about $2 \mu \mathrm{~m}$ and $5 \mu \mathrm{~m}$ respectively. The influence of quadrupole gradient errors is also discussed.

## INTRODUCTION

In linac driven SASE FEL or in future e+e- linear collider it is important to make fine electron beam orbit correction to have strong overlap between particle orbit and radiation cone (for SASE process), to prevent beam emittance dilution and meet design characteristics of the facility. In linear accelerators one of the strongest error sources of beam orbit distortion is quadrupole random misalignments. The most advanced approach for distorted beam orbit fine correction is the beam-based alignment (BBA) technique. In general, BBA techniques attempt to correct beam orbit by varying parameters of the beam or accelerator components (beam energy, quadrupole strength, acceleration gradient, etc.) and using BPM readings of trajectories before and after the parameter change.
In the present paper a modification of BBA algorithm developed at LCLS [1] is proposed. Unlike the LCLS BBA method, which uses several large, deliberate energy variations of the electron beam to detect quadrupole magnet and beam position monitor (BPM) transverse offsets simultaneously, the proposed method is based on difference orbit multiple measurements using single beam energy variation. Numerical simulation results for European XFEL SASE1 [2] and FLASH [3] undulator sections demonstrate that this algorithm provides alignment level of about $2 \mu \mathrm{~m}$ and $5 \mu \mathrm{~m}$ respectively. The impact of quadrupole gradient errors on correction results is also examined.

## ALGORITHM DESCRIPTION

Let's consider a section of linear accelerator with a focusing system represented by N FODO cells in which each quadrupole is accompanied with corresponding

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\# asargsyan@asls.candle.am

BPM. The proposed algorithm is based on measuring beam trajectories for two different energies (design and test), which is organized by changing the gradient of accelerator modules upstream. For energies $\mathrm{E}_{1}$ (design) and $\mathrm{E}_{2}$ (test) trajectory readings of BPMs can be presented in the following matrix form [4]

$$
\begin{align*}
& \mathrm{M}_{1}=\mathrm{R}_{\mathrm{LRM}}^{(1)} \mathrm{X}_{\text {launch }}^{(1)}+\mathrm{R}_{\mathrm{ORM}}^{(1)} \Delta \mathrm{Q}+\Sigma_{1}-\mathrm{B},  \tag{1}\\
& \mathrm{M}_{2}=\mathrm{R}_{\mathrm{LRM}}^{(2)} \mathrm{X}_{\text {launch }}^{(2)}+\mathrm{R}_{\mathrm{ORM}}^{(2)} \Delta \mathrm{Q}+\Sigma_{2}-\mathrm{B},
\end{align*}
$$

where $M$ is the column of $2 N$ BPM readings, $R_{\text {LRM }}$ is the response matrix of launch conditions from the entrance of the section to each BPM, $\mathrm{R}_{\mathrm{ORM}}$ is the orbit response matrix which maps quadrupole offsets $\Delta \mathrm{Q}$ to the BPM readings downstream, $\mathrm{X}_{\text {launch }}$ represents launch conditions (initial position and slope), $\Sigma$ and B-BPM resolution errors and misalignments respectively.

The proposed correction scheme consists of the following steps:

- In two operation modes (with design and test energies) one takes BPM readings for many bunches and calculates the average of taken data for each energy case. From (1) follows

$$
\begin{align*}
& \left\langle\mathrm{M}_{1}\right\rangle=\mathrm{R}_{\mathrm{LRM}}^{(1)}\left\langle\mathrm{X}_{\text {launch }}^{(1)}\right\rangle+\mathrm{R}_{\mathrm{ORM}}^{(1)} \Delta \mathrm{Q}+\left\langle\Sigma_{1}\right\rangle-\mathrm{B},  \tag{2}\\
& \left\langle\mathrm{M}_{2}\right\rangle=\mathrm{R}_{\mathrm{LRM}}^{(2)}\left\langle\mathrm{X}_{\text {launch }}^{(2)}\right\rangle+\mathrm{R}_{\mathrm{ORM}}^{(2)} \Delta \mathrm{Q}+\left\langle\Sigma_{2}\right\rangle-\mathrm{B} .
\end{align*}
$$

- Then one calculates systematic part of average launch conditions using readings of two BPMs located upstream the section of consideration and excludes from further calculations.
- Furthermore, one derives the difference $\langle\Delta \mathrm{M}\rangle_{\text {calc }}=\left\langle\mathrm{M}_{2}\right\rangle-\left\langle\mathrm{M}_{1}\right\rangle$ for which Eq. (2) yields

$$
\begin{equation*}
\langle\Delta \mathrm{M}\rangle_{\mathrm{calc}}=\Delta \mathrm{M}_{\mathrm{real}}+\Delta_{\mathrm{err}} \tag{3}
\end{equation*}
$$

where

$$
\begin{align*}
& \Delta \mathrm{M}_{\text {real }}=\left(\mathrm{R}_{\mathrm{ORM}}^{(2)}-\mathrm{R}_{\mathrm{ORM}}^{(1)}\right) \Delta \mathrm{Q} \equiv \Delta \mathrm{R}_{\mathrm{ORM}} \Delta \mathrm{Q}, \\
& \Delta_{\text {err }}=\mathrm{R}_{\mathrm{LRM}}^{(2)}\left\langle\mathrm{X}_{\text {rand }}^{(2)}\right\rangle-\mathrm{R}_{\mathrm{LRM}}^{(1)}\left\langle\mathrm{X}_{\text {rand }}^{(1)}\right\rangle+\langle\Delta \Sigma\rangle . \tag{4}
\end{align*}
$$

- Finally, by using singular value decomposition (SVD) method one obtains quadrupole misalignments. According Eq. (3)

$$
\begin{equation*}
\Delta \mathrm{Q}_{\mathrm{calc}}=\Delta \mathrm{R}_{\mathrm{ORM}}^{+}\langle\Delta \mathrm{M}\rangle_{\mathrm{calc}}=\Delta \mathrm{Q}+\Delta \mathrm{Q}_{\mathrm{err}} \tag{5}
\end{equation*}
$$

where $\Delta \mathrm{Q}_{\text {err }}=\Delta \mathrm{R}_{\text {ORM }}^{+} \Delta_{\text {err }}, \Delta \mathrm{R}_{\text {ORM }}^{+}$is the inverse (or pseudo-inverse) of $\Delta \mathrm{R}_{\mathrm{ORM}}$.
As can be seen from Eq. (4) - (6), in order to improve the precision of calculations one should provide the condition $\Delta \mathrm{M}_{\text {real }} \gg \Delta_{\text {err }}$, which can be achieved by considering big energy difference for two operation modes and/or large number of taken data for averaging.

## NUMERICAL SIMULATION RESULTS

The described algorithm has been numerically tested for European XFEL SASE1 and FLASH undulator sections. Lattice parameters and errors used in ELEGANT [5] simulations are presented in Table 1.
Table 1: Lattice Parameters and Errors used in Simulations

| Description | SASE1 | FLASH |
| :--- | :---: | :---: |
| Number of quadrupoles | 34 | 6 |
| FODO cell length $[\mathrm{m}]$ | 12.2 | 10 |
| Quadrupole strength $\left[\mathrm{m}^{-2}\right.$ ] | $\pm 0.64$ | $\pm 2$ |
| Quadrupole length $[\mathrm{m}]$ | 0.1 | 0.13 |
| Quad. rms offset $[\mu \mathrm{m}]$ | 100 | 300 |
| BPM rms resolution $[\mu \mathrm{m}]$ | 1 | 20 |
| Orbit rms initial offset $[\mu \mathrm{m}]$ | 40 | 100 |
| Orbit rms initial slope $[\mu \mathrm{rad}]$ | 1 | 10 |

The correction procedure and data analysis were performed with MATLAB. In Fig. 1 the electron beam orbit is shown before and after correction in SASE1 for one particular seed of quadrupole offsets. $20 \%$ relative energy difference between operation modes and 3000 launch conditions are considered in correction algorithm.


Figure 1: Beam orbit before and after correction.
Figure 2 shows rms corrected orbit and rms deviation of corrected orbit from linear fit for SASE1. Ten seeds of quadrupole offsets and ten seeds of 3000 launch conditions per quadrupole seed were considered.

The results are summarized in Table 2. For FLASH undulator section $50 \%$ relative energy difference and 10000 launch conditions were considered.


Figure 2: RMS corrected orbit and rms deviation of corrected orbit from linear fit for SASE1.

Table 2: Simulation Results

| Description | SASE1 | FLASH |
| :--- | :---: | :---: |
| rms corrected orbit $[\mu \mathrm{m}]$ | $\leq 110$ | $\leq 33$ |
| rms dev. from linear fit $[\mu \mathrm{m}]$ | $\leq 3$ | $\leq 5$ |

To improve the trajectory after correction for SASE1 the algorithm was also applied considering $40 \%$ relative energy difference and 10000 launch conditions. In this case the rms orbit was below $35 \mu \mathrm{~m}$ and rms deviation of corrected orbit from linear fit was below $2 \mu \mathrm{~m}$.
Note that similar results have been obtained by numerical implementation of LCLS method for SASE1 [4] and FLASH undulator section [6], where several energy large variations of the electron beam have been considered and a number of iterations were applied. The proposed algorithm uses a single energy large variation for difference orbit multiple measurements and needs one iteration.

One should note also that the proposed algorithm allows estimating BMP offsets. As it can be seen from Fig. 1 the trajectory becomes almost linear after correction. Using this fact one can estimate BPM offsets by taking BPM readings after correction, subtracting launch conditions impact and calculating deviations from linear fit.

## QUADRUPOLE GRADIENT ERRORS INFLUENCE

To estimate the effect of quadrupole gradient errors which yield to errors of response matrix, at first the procedure was repeated for SASE1 considering $1 \%$ and $3 \%$ quadrupole gradient errors, i.e. the response matrix of ideal lattice was used for orbit correction in lattices with corresponding quadrupole gradient errors. Figure 3 shows rms orbits after correction and rms deviations of corrected orbits from linear fits. In simulations ten seeds of gradient errors were considered.

As it can be seen from Fig. 3 the response matrix of ideal lattice is not suitable for precise orbit correction in lattice with quadrupole gradient errors.
a)

b)

Figure 3: RMS corrected orbits a) and rms deviations of corrected orbits from linear fits b) for SASE1 obtained by using response matrices of ideal lattice.

Further, instead of using response matrix of ideal lattice in correction procedure we used the response matrix which was formed (for the misaligned lattice with quadrupole gradient errors) by applying the following procedure:

1. For particular seed of quadrupole offsets the average orbit over 3000 seeds of initial coordinates and BPM resolution errors (with rms values from Table 1) was calculated to have "experimental reference orbit".
2. Unit kick was added at each quadrupole position separately and the average orbits over 3000 seeds of initial coordinates and BPM resolution errors were calculated.
3. By subtracting "experimental reference orbit" from average orbits described in step 2 the so called measured response matrix was formed.
It is clear that BPM misalignments have no contribution in measured response matrix because of the subtraction in step 3.
Figure 4 shows rms corrected orbits and rms deviations of corrected orbits from linear fits obtained by using measured response matrices for lattices with $1 \%$ and $3 \%$ quadrupole gradient errors.
As it can be seen from Figs. 2 and 4 the usage of measured response matrices in correction procedure makes the influence of quadrupole gradient errors insignificant.


Figure 4: RMS corrected orbits a) and rms deviations of corrected orbits from linear fits b) for SASE1 obtained by using measured response matrices.

## SUMMARY

New BBA procedure based on difference orbit multiple measurements is proposed in the paper. The algorithm was numerically tested for European XFEL SASE1 and FLASH undulator section. The results demonstrate that using this method about $2 \mu \mathrm{~m}$ final orbit with respect to a straight line can be achieved for SASE1 and about $5 \mu \mathrm{~m}$ for FLASH undulator section. It is also shown that the application of so called measured response matrices makes the algorithm implementation results insensible to quadrupole gradient errors.

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