# nuSTORM HORN OPTIMIZATION STUDY\*

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#### Abstract

title of the work, publisher, and DOI. The efficiency of using magnetic horns as a pion collection device has been recognized by several neutrino projects. In the study, we began with a NuMI-like horn, which was applied to collect the secondary pions from bombarding the target with 120 GeV/c protons in the nuS-TORM proposal. The necessity of optimizing the horn for a non-conventional neutrino beamline like the nuSTORM attribution pion beamline was then acknowledged. This paper presents a detailed description of the optimization objectives, the Multi-objective Genetic Algorithm developed for this spenaintain cific purpose, and the results of the optimization. With the full G4beamline simulation results, the success of the optimization provides an increase of 16% in the useful muons must in the ring. This methodology can be applied to any neu-

## **INTRODUCTION**

in the fing. This includoiog trino beamline configuration. **INTROD** Magnetic horns have been to projects as a pion collection of strong pulsed electric current Magnetic horns have been widely used in neutrino projects as a pion collection device. The horns are fed by ġ; strong pulsed electric currents of several hundred kA and can provide fast powerful focusing for the secondary particles produced by bombarding a target placed before or inside the horn. With a 230 kA horn current and a 10 mm 201 neck radius, the peak magnetic field generated in the horn O can reach 4.6 Tesla.

licence ( nuSTORM ( a "non-conventional" short baseline neutrino program) uses a horn configuration similar to the 3.01 NuMI horn, with a slightly changed horn length and a tar- $\succeq$  get position in the horn. The horn shown in Figure 1 is a de-Scription of the configuration. The parabolic shape on both gends of the horn was designed to provide particles with a 5 monochromatic focus.

One of the physics motivations of nuSTORM is to provide neutrino beams with well known flavor and flux at the near and far detectors. To obtain high intensity neutrino beams, the momentum acceptance for the pion beam and the resulting muon beam must be very large. The nsed monochromatic horn in Figure 1 may not provide the maxg imum number of muons, and a broad variety of horn pa- $\frac{1}{2}$  paper, the full optimization process is demonstrated by de-scribing first the optimization goals of

The horn revolution 200 Before Opt. 180 160 230000 140 120 100 80 500 1000 3000 1500 2000 2500 Z (mm

Figure 1: Schematic drawing of the horn used in the nuS-TORM proposal, before optimization.

## PION BEAMLINE AND OPTIMIZATION **GOALS**

The nuSTORM FODO decay ring was designed to achieve a transverse phase space acceptance of 2 mm and a momentum acceptance of  $3.8\pm10\%$  GeV/c. The neutrino flux can be enlarged by increasing the number of muons in both the phase space and momentum acceptance. A pion beamline, with the reference pion momentum  $P_0=5$  GeV/c, is used to transport the pions after the horn collection into the decay ring. The pions are injected by the Beam Combination Section (BCS) and decay to muons while traversing the first decay straight. The muons are then captured in the ring acceptance. A BCS mirror is used at the other end to extract the remaining pions and high energy muons. The layout of the pion beamline is shown in Figure 2.



Figure 2: The layout of nuSTORM pion beamline.

It is straightforward to note that both the pion beamline and the horn can be optimized to produce more muons

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in the phase space and momentum acceptance at the BCS mirror. This could be the single goal of the optimization. However, because different horn designs give different pion phase space distributions at the downstream end of the horn, the pion beamline optics needs to be re-designed to match the optics parameters. Following that, the design must be converted to an input file in G4Beamline to do the full MC tracking. The whole process would require an intolerably long computing time.

Alternatively, the paper gives a method of deriving the two numbers at the downstream end of the horn in order to avoid the MC tracking. the goal can be reached by simply maximizing the number of muons in the phase space acceptance and the momentum acceptance separately and simultaneously.

#### Muons in the Momentum Acceptance

To derive the number of muons in the  $3.8\pm10\%$  GeV/c acceptance at the end of the first decay straight, denoted as  $N_{\mu,end}$ , both the pion decay kinematics and the pion beamline transmission efficiency need to be considered.

Consider a pion beam after the horn with  $N_0$  pions in the range of  $P_0(1 \pm m)$  GeV/c where  $m = \Delta P/P_0$ . It is observed in MARS[1] that, for medium or high Z targets and 120 GeV Proton On Target (POT), the momentum probability density function of the  $\pi$ + produced at the target can be written as  $f_{p\pi}(p_{\pi}) = ap_{\pi} + b$ , where the negative coefficient *a* means the productivity decreases with pion momentum  $p_{\pi}$ . The maximum number of muons within  $3.8\pm10\%$ GeV/c produced from this pion beam without any beam loss is

$$N_{\mu,max}(m) = 2.34 \left[ 2mP_0 a + \ln\left(\frac{1+m}{1-m}\right) b \right] \times 760 \times N_0$$
(1)

Combining the effects of the decay kinematics and the transmission efficiency,  $N_{\mu,end}$  can thus be written as  $N_{\mu,end}(m) = N_{\mu,max}(m)P_{mom}(m)P_{\Phi}$ , where  $P_{mom}$  is the momentum transmission efficiency of the pion beam with momentum spread m, and  $P_{\Phi}$  is the phase space transmission efficiency of the 2 mm rad assumed to be independent of m.  $P_{\Phi}$  is taken as 1 to simplify the problem.  $P_{mom}(m)$  can be derived by tracking pion beams with different m.

 $N_{\mu,end}(m)$  as a function of m is plotted in Figure 3. The formula was compared with two MC tracking results in G4Beamline, which are two pion beamlines designed for pion beams after the horn collection of two graphite and Inconel targets. For different horn configurations, the three parameters  $N_0$ , a and b are different, where  $N_0$  controls the height and a, b modify the shape of the trend. However, the rule of thumb from this derivation indicates that pions up to  $5 \pm 18\%$  GeV/c are useful in generating muons in the correct momentum band. Therefore, using the 18% momentum cufoff, the  $N_{\mu,end}$  to be compared for each horn configuration is

$$N_{\mu,\text{end}}(0.18) \sim 8.82 \times 10^2 N_0 \left[ 1.8 \times 10^3 a + 0.36b \right]$$
(2)

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Figure 3: Comparison of  $N_{\mu,\text{end}}(m)$  (formula) and  $\mathbf{N}_{\mu,\text{end}}(m)$  (G4Beamline simulation on Inconel: red; on Graphite: green). The numbers are scaled to make the comparison.

This avoids the full tracking for each of the horn configurations.

### Muons in the Phase Space Acceptance

In order to further avoid complete Monte Carlo tracking in the optimization, the number of pions in the 2 mm·rad after the horn collection, denoted as  $N_{\pi}$ , is used to approximate the number of muons in the same acceptance. This is based on the previous assumption that the transmission efficiency of the particles within the 2 mm·rad Gaussian acceptance ellipse of any shape is the same. Only ellipse that give Twiss parameters which can be matched using a conventional quadrupole channel without exceeding the limit of  $\beta_{\text{max}} = 16$  m are valid.

A fitting algorithm that is useful specifically for fitting beam phase space distribution with a bivariate Gaussian was proposed in [2]. The algorithm is applied to the pion beam after each horn design to obtain the initial Twiss parameters and count  $N_{\pi}$  in the corresponding acceptance ellipse.

#### **OPTIMIZATION TOOL**

The Multi-Objective Genetic Algorithm (MOGA) has been widely recognized for its efficiency in searching for extrema of various types of problems, especially when there are more than one optimization objective (goal) and the analytical form of extremum conditions is very hard to find. In this optimization study, a Python based MOGA code was used as the optimization tool. It was also realized that the idea of crossover and mutation operators in MOGA and the independence of each horn design make this problem open to MPI coding. Therefore, the mpi4py [3] module in the Python-based code allows the code to be

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run on NERSC in a multi-core environment. The MOGA process is described in Figure 4.



Figure 4: Illustration of the MOGA process to do the horn optimization

### **OPTIMIZATION RESULT**

Three Inconel targets, which are 2.5, 3, and 4 interaction lengths long, were checked in the optimization.

## 38 cm (2.5 Interaction Lengths) Inconel Target

of The algorithm took 66 generations to stop at the one in Figure 5. The red cross on the front part of the horn results from noting that backward-propagating pions cannot con-tribute to the forward beam. Truncating the horn at the beginning of the target will provide the same captured beam with reduced horn requirements. The optimization effect



Figure 5: The optimized horn for the 38 cm long Inconel target, stopped at the 66th generation

was confirmed using the full G4Beamline tracking with a re-match of the optics. The number of muons in both 2

Content TUPRI005 mm·rad and  $3.8\pm10\%$  GeV/c was improved by 8% from the starting horn.

## 46 cm or 62 cm (3 or 4 Interaction Lengths) Inconel Target

It was also shown in the optimization results that the effect of elongating the target was roughly the same for 3 and 4 interaction lengths of Inconel. The optimized horn configuration for the 46 cm Inconel target is shown in Figure 6. The number of muons in both 2 mm·rad and  $3.8\pm10\%$ 



Figure 6: The optimized horn for the 46 cm long Inconel target, stopped at the 61st generation

GeV/c was improved by 16% compared with that from the horn and target configuration in Figure 1.

#### **SUMMARY**

In this study, the horn optimization enables for an 8% increase in the stored muons, and therefore the useful muon decays with a 38 cm Inconel target. An increase of 16% on the same number can be obtained if the 46 cm Inconel target is used.

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