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STUDY COOLING PERFORMANCE IN A HELICAL COOLING CHANNEL FOR MUON COLLIDERS*

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Abstract

Six-dimensional (6D) muon helical ionization cooling channel (HCC) has been proposed for muon colliders. The linear HCC theory accurately represents the helical beam dynamics and the cooling performance. It is extended to the generic cooling theory to involve the non-linear beam dynamics including with a collective effect. The modified theory is tested by comparing with numerical simulation with various conditions.

INTRODUCTION

The HCC has been proposed to make fast 6D muon ionization cooling for muon colliders [1]. The HCC magnet consists of a helical dipole and a solenoid components to generate a continuous dispersion in the Larmor motion. A helical field gradient is applied to stabilize the helical beam phase space. Since there is no betatron resonance in the continuum field the HCC realizes a large momentum acceptance. Hydrogen gas-filled RF cavities are incorporated continuously into the HCC magnet, in which muons lost their kinetic energy via the ionization process with dense hydrogen gas and simultaneously regain the lost energy from the RF field. It realizes a compact 6D ionization cooling channel.

The linear HCC theory predicts accurately the helical beam dynamics and the cooling performance. The theory has been verified in various conditions by comparing with the numerical simulation [2]. It is extended to the generic cooling theory for a following purpose. From the recent beam-induced gas plasma simulation study, a beam-induced gas-plasma in the gas-filled RF cavity could be condensed in the beam path by the wake field. As a result, the dense gas plasma is formed in the cavity and could influence the beam dynamics. Besides, since the energy transfer in the HCC is above transition the space charge makes longitudinal focusing. These collective effects should be involved to evaluate the cooling performance.

EXTEND HCC THEORY WITH GENERIC COOLING FORMULAE

Transverse Motion and Beta Function

The transverse beam motion in the HCC is solved with the linear approximation method [1]. The beta tune and related equations are given,

$$Q^2 = Q_\pm^2 = R \pm \sqrt{R^2 - G}, \tag{1}$$

$$R = \frac{1}{2}\left(1 + \frac{q^2}{1+\kappa^2}\right). \tag{2}$$

$$G = \left(\frac{2q+\kappa^2}{1+\kappa^2} - \hat{D}^{-1}\right)\hat{D}^{-1}, \tag{3}$$

$$q = \sqrt{\frac{1+\kappa^2 - 1/2\kappa^2\hat{D}}{1+1/2\kappa^2\hat{D}/(1+\kappa^2)}}, \tag{4}$$

where κ is the helical pitch ($= p_\perp/p_z$). $Q_\pm$ are the eigen values in the helical coordinate system. Thus, the general solution of helical beam motion is the sum of these eigen modes. The beam stability condition, $0 < G < R^2$ is given from eq. (1). q is a reference to represent the ratio between solenoid and helical dipole field strengths. Eq. (4) is derived from the condition in which the cooling decrements in the two eigen modes are equal ($\Lambda_+ = \Lambda_-$ in ref. [3]). The transverse beta functions are given,

$$\beta_\pm = \frac{1}{kQ_\pm} = \frac{\lambda}{2\pi Q_\pm}. \tag{5}$$

We define the transverse beta function for the generic cooling theory,

$$\beta_T = \sqrt{\beta_+\beta_-}. \tag{6}$$

The ratio between solenoid (B_z) and helical dipole (b) field is given,

$$\frac{b}{B_z} = \frac{\kappa}{1+\kappa^2}\left(\frac{q}{q+1}\right). \tag{7}$$

Since $\kappa = 1$ and q is typically ~0.8 the field ratio is 0.22. Thus, the HCC is a solenoid dominant channel. The β_T is only ~ 20% shorter than the beta function in a uniform solenoid field ($\beta^\star = 2p/eB_z$).

Longitudinal Motion and Beta Function

The admittance of HCC is given,

$$(I_s)_{adm} = \frac{2}{\pi\omega}\sqrt{\frac{\gamma'_{max}}{\eta\omega}}. \tag{8}$$

The momentum slip factor is given,

$$\eta = \frac{d}{d\gamma}\frac{\sqrt{1+\kappa^2}}{\beta} = \frac{\sqrt{1+\kappa^2}}{\gamma\beta^3}\left(\frac{\kappa^2}{1+\kappa^2}\hat{D} - \frac{1}{\gamma^2}\right). \tag{9}$$

The longitudinal beta function is defined,

$$\beta_L = \frac{1}{\omega Q_L} = \sqrt{\frac{m_\mu c}{\eta\omega eV'}\frac{1+\sin(\phi_s)}{1-\sin(\phi_s)}}, \tag{10}$$

where ω is the RF resonant frequency, V' is the peak RF gradient, and ϕ_s is the synchrotron phase. It is worth to note that eqs. (1) ~ (10) show that the all HCC parameters are determined from the dispersion factor, $\hat{D}$ (= aD where a is the radius of reference orbit and D is the dispersion).

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Emittance

The emittance evolution is given as a function of the channel length (s),

$$\frac{d\varepsilon_n}{ds} = \varepsilon\frac{d\beta\gamma}{ds}+\beta\gamma\frac{d\varepsilon}{ds} = \varepsilon_n\frac{1}{\beta^2 E}\frac{dE}{ds}+\frac{\beta\gamma}{2}\left(\beta_r\sigma_r^2\right), \quad (11)$$

where β_r is the beta function in r-th phase space, E is the total energy of incident particle, and σ_r is the statistic fraction due to the stochastic process. Cooling partition functions for transverse and longitudinal, g_T and g_L, respectively are introduced [4]. Emittance evolutions in transverse and longitudinal phase spaces are given,

$$\frac{d\varepsilon_T}{ds} = -\frac{g_T}{\beta^2 E}\frac{dE}{ds}\varepsilon_T+\frac{\beta\gamma}{2}\beta_T\frac{d\langle\theta_{rms}^2\rangle}{ds}, \quad (12)$$

$$\frac{d\varepsilon_L}{ds} = -\frac{g_L}{\beta^2 E}\frac{dE}{ds}\varepsilon_L+\frac{\beta\gamma}{2}\beta_L\frac{d\langle(\delta p/p)_{rms}^2\rangle}{ds}, \quad (13)$$

where

$$g_L \rightarrow g_{L,0}+\delta g_L, \quad (14)$$

$$g_T \rightarrow 1-\frac{\delta g_L}{2}, \quad (15)$$

$$g_{L,0} = -\frac{2}{\gamma^2}+2\frac{\left(1-\beta^2/\gamma^2\right)}{\ln\left(2m_e c^2\beta^2\gamma^2/I(Z)\right)}. \quad (16)$$

Neuffer describes that δg_L in the HCC is given [4],

$$\delta g_L = \frac{\kappa^2}{1+\kappa^2}\hat{D}. \quad (17)$$

Use simplified formulae to represent eqs. (12) and (13),

$$\frac{d\varepsilon_{T(L)}}{ds} = -\Lambda_{T(L)}\varepsilon_{T(L)}+\varepsilon_{T,eq(L,eq)}, \quad (18)$$

$$\Lambda_T = \frac{g_T}{\beta^2 E}\frac{dE}{ds}, \quad (19)$$

$$\Lambda_T = \frac{g_T}{\beta^2 E}\frac{dE}{ds}. \quad (20)$$

The equilibrium emittance is obtained from eqs. (12) and (13) with $d\varepsilon_r/ds=0$,

$$\varepsilon_{T,eq} = \frac{\beta_T(13.6\ \ MeV)^2}{2m_\mu\beta g_T X_0\langle dE/ds\rangle}, \quad (21)$$

$$\varepsilon_{L,eq} = \frac{m_e c^2\gamma^2\beta(1-\beta^2/2)\beta_L}{2m_\mu g_T\left(\ln\left(\frac{2m_e c^2\beta^2\gamma^2}{I(Z)}\right)-\beta^2\right)}. \quad (22)$$

It is worth to note that the cooling decrements eqs. (19) and (20) are exactly same as the cooling decrements derived in the HCC theory [3]. Eq. (18) can be solved,

$$\varepsilon_{T(L)}(s) = (\varepsilon_{T(L),0}-\varepsilon_{T(L),eq})e^{-\Lambda_{T(L)}s}+\varepsilon_{T(L),eq}. \quad (23)$$

The equilibrium emittances in eqs. (21) and (22) do not take into account the mixing of transverse and longitudinal phase space due to the stochastic effect. The HCC theory involves the mixing term and its amplitude is ~30 % of the estimated equilibrium emittance in eqs. (21) and (22).

RF WINDOW THICKNESS DEPENDENCE

Transverse and longitudinal beta functions in the HCC are much longer than the RF cavity length. In this case, the RF window thickness dependence can be simply evaluated by using eq. (21) with the averaged radiation length and the effective energy loss rate. The average radiation length is

$$X_0 \rightarrow \bar{X_0} = \frac{X_{0,GH_2}X_{0,Be}}{X_{0,GH_2}+X_{0,Be}}, \quad (24)$$

$$\left\langle\frac{dE}{ds}\right\rangle \rightarrow \left\langle\frac{dE}{ds}\right\rangle_{total} = \left\langle\frac{dE}{ds}\right\rangle_{GH_2}+\left\langle\frac{dE}{ds}\right\rangle_{Be}. \quad (25)$$

where X_{0,GH_2} and $X_{0,Be}$ involve abundance of the material. Figures 1 and 2 show the estimated radiation length and the total energy loss as a function of *Be* RF window thickness, in which the number of RF windows is 20 per meter and the pressure of gaseous hydrogen is 160 *atm* at room temperature.

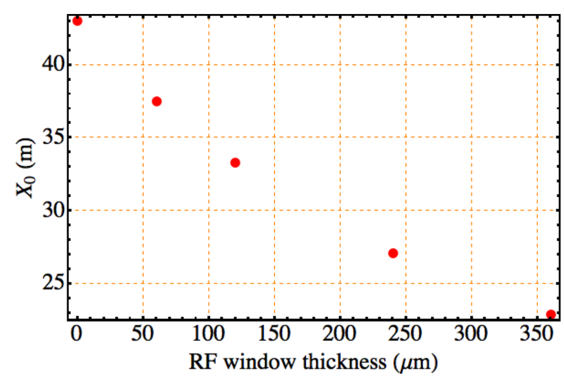

Figure 1: Estimated radiation length as a function of RF window thickness.

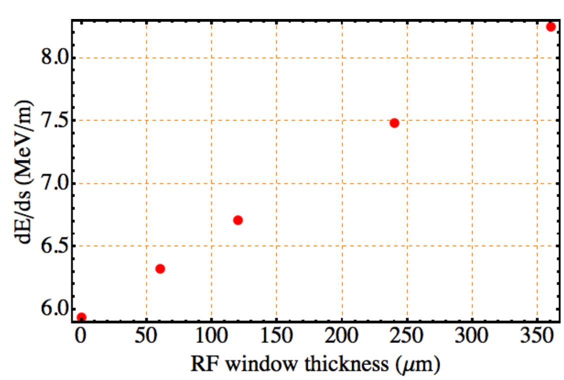

Figure 2: Estimated energy loss rate as a function of RF window thickness.

Figures 3 and 4 show the estimated transverse and longitudinal equilibrium emittances, respectively in G4BL [5] (red point) and analytic calculations from eqs. (21) and (22) and the HCC theory [3]. The reference momentum is 200 MeV/c, the helical period is 0.5 m, the helical pitch is 1.0, the gas pressure is 160 *atm* at room temperature, the RF frequency is 650 MHz, the peak RF gradient is 20 MV/m, and

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the number of RF cavities is 10 per one helical period. The RF phase is tuned with respect to the RF window thickness to compensate the energy loss rate. Blue line is the analytic estimation by using the generic cooling formulae (eqs. (21) and (22)) while magenta one is the analytic estimation from the HCC theory. The discrepancy between the analytic and numerical results is less than 20 %.

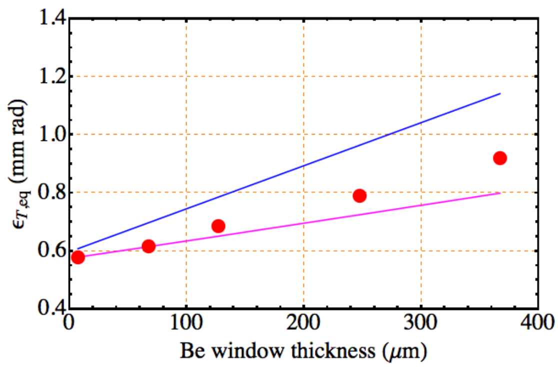

Figure 3: Estimated transverse equilibrium emittance.

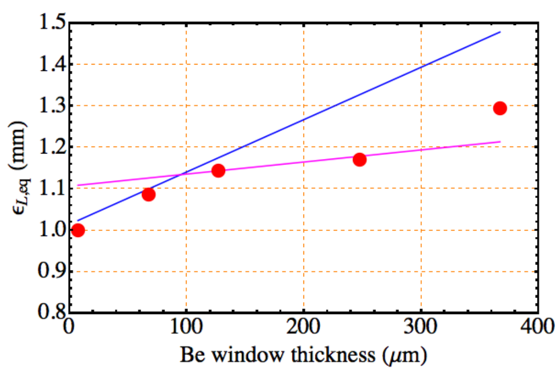

Figure 4: Estimated longitudinal equilibrium emittance.

EMITTANCE EVOLUTION

The emittance evolution is evaluated in the same condition as the above with fixed window thickness 60 μm. The estimated cooling decrements are under the assumption of equal cooling decrements in this analysis [3],

$$\Lambda_T = \Lambda_L = \frac{(2+g_{L,0})/3}{\beta^2 E}\left\langle \frac{dE}{ds}\right\rangle_{total} = 0.0307 \ m^{-1}. \quad (26)$$

The estimated equilibrium transverse and longitudinal emittances in the HCC (generic) theory are 0.615 (0.697) $mmrad$ and 1.12 (1.10) mm, respectively. Substituting Λ_T, Λ_L, $\varepsilon_{T,eq}$, and $\varepsilon_{L,eq}$ into the emittance evolution function eq. (18), it show in Figures 5 and 6. The HCC theory (red line) are agreed well with the numerical result (red point).

NEXT STEP

The extended HCC theory is consistent with the numerical simulation. The wake field amplitude becomes comparable with the RF gradient when the beam intensity is > 10^{12} muons/bunch in ~ mm^3 volume. The collective effect with such intense muon beam will be evaluated in the theory by adding the collective term in the beta tunes. Simulation effort just begun [6].

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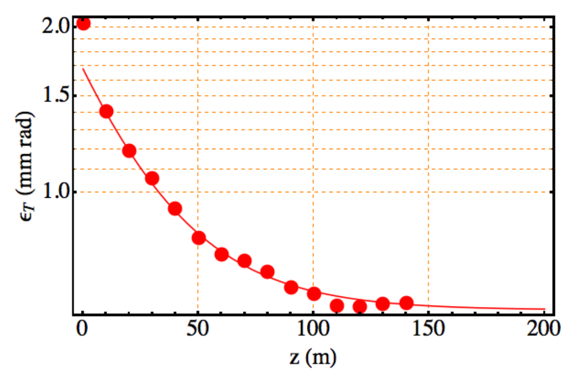

Figure 5: Estimated transverse emittance evolution. Red line is the predicted emittance evolution curve, $\varepsilon_T(s) = (1.68 - 0.62)e^{-0.0307s} + 0.62$.

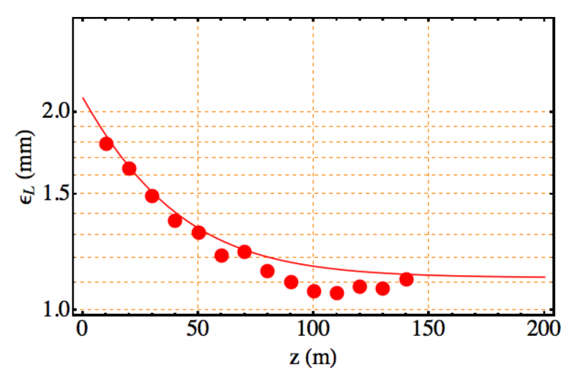

Figure 6: Estimated longitudinal emittance evolution. Red line is the predicted emittance evolution curve, $\varepsilon_L(s) = (2.1 - 1.12)e^{-0.0307s} + 1.12$.