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WAKEFIELD-BASED DECHIRPER STRUCTURES FOR ELBE

F. Reimann, U. van Rienen, University of Rostock, Rostock, Germany
U. Lehnert, P. Michel, Helmholtz-Zentrum Dresden-Rossendorf, Rossendorf, Germany

Abstract

The efficient reduction of the pulse length and the energy width of electron beams plays a crucial role in the generation of short pulses in the range of sub-picoseconds at future light sources. At the radiation source ELBE in Dresden-Rossendorf short pulses are required for coherent THz generation and laser-electron beam interaction experiments such as X-ray Thomson scattering.

Energy dechirping can be carried out passively by wakefields generated when the electron beam passes through suitable structures, namely corrugated and dielectrically lined cylindrical pipes or dielectrically lined rectangular waveguides. All structures offer the possibility to tune the resulting wakefield and therefore the resulting energy chirp through a variation of purely geometrical or material parameters.

In this paper we present a semi-analytical approach to determine the wakefield in dielectrically lined rectangular waveguide, starting with the expression of the electric field in terms of the structure's eigenmodes.

THE STRUCTURE

The concept of wakefield dechirpers has been studied in the last years for application at various particle accelerators. The forms of these dechipers range from corrugated cylindrical pipes [1] over corrugated rectangular waveguides [2] to dielectrically lined rectangular waveguides [3], which all have been shown to induce a reduction of the energy width in the particle beam. The major advantages of these structures are their passive mode of operation and their simplicity, while they nonetheless remain tunable to a multitude of possible applications due to their adaptable geometry.

At the radiation source ELBE such a wakefield dechirper in form of a dielectrically lined rectangular waveguide is planned, its schematic layout can be seen in figure 1. The structure is surrounded by a perfect electric conductor (PEC) and has a length L, the rectangular cross-section has a width a and a height b. This waveguide is lined with to symmetrically placed slabs of a dielectric of relative permittivity $\bar{\varepsilon}_r$ with the thickness $b-d$. Note that the relative permittivity in this waveguide changes according to a piecewise constant function, $\varepsilon_r(y)$. To determine the geometrical parameters of the dechirper in order to meet the requirements of ELBE, the longitudinal wakefield in such a structure is to be expressed in a semi-analytical way. Therefore a semi-analytical description of the electric field through an expansion in the eigenmodes of the waveguide will be derived first.

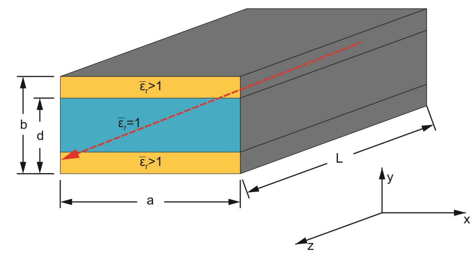

Figure 1: Geometry of a dielectrically lined rectangular waveguide. The red arrow indicates the moving direction of the particles.

THE EIGENMODES

In [4] the eigenmodes of rectangular waveguides loaded with dielectric slabs are introduced as LSE (Longitudinal Section Electric) and LSM (Longitudinal Section Magnetic) modes. Together they form a complete orthogonal set in which an arbitrary electromagnetic field can be expanded.

The presence of the dielectric slabs complicates a closed analytical description of the modes, so that both mode types are rather derived semi-analytically following a Fourier-expansion of the unknown behaviour in y-direction and determining the expansion coefficients by solving Maxwell's equations. This procedure is called Rayleigh-Ritz method in [4] and is applied here to the given situation shown in figure 1. The Rayleigh-Ritz method represents an approximation, and its accuracy is mainly defined by the number of expansion functions taken into account. Nonetheless, comparisons of Rayleigh-Ritz obtained fields and eigenfrequencies to results from numerical simulations with CST Studio [5] show a very good agreement of the two methods with a use of only a few expansion functions (10-30).

Both mode types present in dielectrically lined rectangular waveguides shall be described shortly. LSE modes do not possess an electric field component along the direction of the changing permittivity, so in this case in y-direction. They can be obtained with the Rayleigh-Ritz method using a Fourier-sine expansion. The Hertzian potential of this mode type is

$$\mathbf{\Pi}_H = \mathbf{e}_y \cos(k_x x) \sum_{m=1}^{N} q_m \sin(k_{ym} y) \cos(k_z z),$$

with $k_x = \frac{n\pi}{a}$, $k_z = \frac{l\pi}{L}$ and $k_y = \frac{m\pi}{b}$; as well as n, l and m being non-negative integers. The spatial electric and magnetic fields (indicated with μ from now on) are obtained via

$$\mathbf{E}_\mu = \omega_\mu \nabla \times \Pi_H, \tag{1}$$

$$\mathbf{B}_\mu = -i\nabla \times \nabla \Pi_H, \tag{2}$$

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with ω_μ being the mode's vacuum frequency.

The electric field of the lowest LSE mode of an example geometry is depicted in figure 2 on a cut plane at $z = L/2$. As the field does not possess a y-component, the field lines are directed out of the cut plane. LSM modes on the contrary do

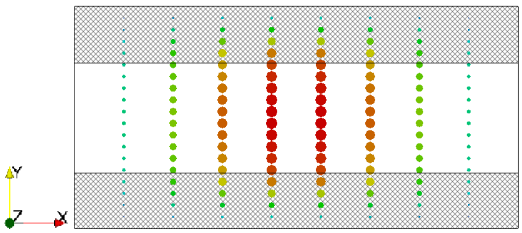

Figure 2: Electric field of the lowest LSE mode. The shaded areas indicate the dielectrics. The field is not dependent on the changing permittivity, there is no visible jump in the field strength between the vacuum region and the dielectrics.

not possess a magnetic field component along the direction of the changing permittivity. The Rayleigh-Ritz method uses a Fourier-cosine expansion, and the Hertzian potential reads

$$\mathbf{\Pi}_E = \mathbf{e}_y \sin(k_x x) \sum_{m=0}^{N} q_m \cos(k_{ym} y) \sin(k_z z).$$

The spatial fields (depicted with an index λ) derive from the potential according to

$$\mathbf{E}_\lambda = \varepsilon_r(y)^{-1} \nabla \times \nabla \times \Pi_E, \tag{3}$$

$$\mathbf{B}_\lambda = -i \frac{\omega_\lambda}{c_0^2} \nabla \times \Pi_E, \tag{4}$$

with ω_λ again the mode's vacuum frequency.

The electric field of the lowest LSM mode of the same example structure is shown in figure 3, again on a cut plane at $z = L/2$. The figure confirms that the field is, as expected from (3), weaker in the dielectric regions. It is important to

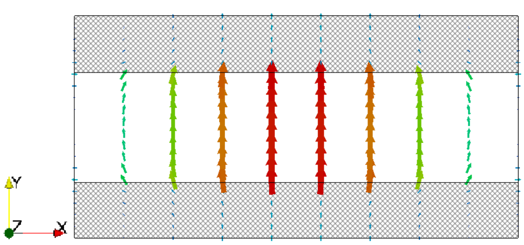

Figure 3: Electric field of the lowest LSM mode. The shaded areas indicate the dielectrics. The field strength shows jumps between the vacuum and the dielectric region, and is weaker in the dielectric.

note that both mode types again divide into two subtypes, namely symmetric and antisymmetric modes, meaning that either all expansion coefficients corresponding to an even index m are zero, or all expansion coefficients corresponding to odd m.

Concerning the orthogonality of these modes it remains to be mentioned that while their magnetic field fulfills the usual orthogonality relation (with $i = \mu, \lambda$)

$$\int \mathbf{B}_i \mathbf{B}_{i'} \, dV = V_i \delta_{i,i'}, \tag{5}$$

the electric fields are orthogonal with respect to the permittivity function,

$$\int \varepsilon_r(y) \mathbf{E}_i \mathbf{E}_{i'} \, dV = U_i \delta_{i,i'}. \tag{6}$$

Both mode types are not normalised here, for it is unnecessary for the next steps.

THE ELECTRIC FIELD OF A POINT CHARGE

As stated before, the LSE and LSM modes of a dielectrically lined rectangular waveguide form a complete set, and the given orthogonality relations can be used to expand the field inside such a structure when it is passed by a single point-like charge. It enters the waveguide at $x = a/2, y = b/2, z = 0$ and leaves the waveguide at $x = a/2, y = b/2, z = L$. For simplicity reasons the waveguide is still fully enclosed by PEC. The charge and current density read

$$\rho = Q\delta\left(x - \frac{a}{2}\right)\delta\left(y - \frac{b}{2}\right)\delta(z - ct), \tag{7}$$

$$\mathbf{J} = c\rho\mathbf{e}_z, \tag{8}$$

where the speed of light in the waveguide is dependent on y, $c(y)^2 = \frac{1}{\varepsilon_0 \varepsilon_r(y) \mu_0} = c_0^2 \varepsilon_r(y)^{-1}$.

The fields in this waveguide structure now obey the following set of Maxwell's equations,

$$\nabla \mathbf{D} = \varepsilon_r \varepsilon_0 \nabla \mathbf{E} = \rho \qquad \nabla \mathbf{B} = 0$$
$$\nabla \times \mathbf{E} = -\frac{\partial}{\partial t}\mathbf{B} \qquad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \varepsilon_r \frac{\partial}{\partial t}\mathbf{E}.$$

Note that these equations only hold true for a piecewise constant relative permittivity, where $\nabla \varepsilon_r(y) = 0$, and for a constant permeability throughout the waveguide, meaning that the dielectric is required to have a permeability of μ_0.

Applying the curl to Faraday's law, inserting both Ampere's and Gauss's law and using some vector algebra the following equation is obtained,

$$\nabla \times \nabla \times \mathbf{E} = -\mu_0 \frac{\partial}{\partial t}\mathbf{J} - \mu_0 \varepsilon_0 \varepsilon_r \frac{\partial^2}{\partial t^2}\mathbf{E}. \tag{9}$$

This equation describes the electric field of the point charge in the waveguide.

The electric field is now expanded in LSM and LSE modes, where the expansion coefficients completely carry the time dependence, whereas the spatial part is delivered by the modes. The general expansion reads

$$\mathbf{E}(\mathbf{r}, t) = \sum_\lambda \chi_\lambda(t) \mathbf{E}_\lambda(\mathbf{r}) + \sum_\mu \chi_\mu(t) \mathbf{E}_\mu(\mathbf{r}). \tag{10}$$

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With this ansatz put into equation (9), multiplying once by $\mathbf{E}_{\lambda'}$ and once by $\mathbf{E}_{\mu'}$, integrating over the volume and by the utilization of the orthogonality relations stated before the expansion coefficients can be determined. They read as

$$\chi_\lambda(t) = -\frac{Qk_xk_zc_0^2}{U_\lambda\varepsilon_0}\sin\left(\frac{n\pi}{2}\right)\sum_{m=1}^{N} q_m \sin\left(\frac{m\pi}{2}\right) \cdot \int_0^t \sin\left(k_zc_0t'\right)\sin\left(\omega_\lambda\left(t-t'\right)\right)dt', \quad (11)$$

and

$$\chi_\mu(t) = -\frac{Qk_z^2c_0^2}{U_\mu\omega_\mu\varepsilon_0}\sin\left(\frac{n\pi}{2}\right)\sum_{m=1}^{N} q_m k_{ym}\sin\left(\frac{m\pi}{2}\right) \cdot \int_0^T \sin\left(k_zc_0t'\right)\sin\left(\omega_\mu\left(t-t'\right)\right)dt', \quad (12)$$

where the time integral can be truncated at $\frac{L}{c_0}$ if $t > \frac{L}{c_0}$, because there will be no further contributions from the charge, since it has left the structure.

The expansion coefficients clearly show that all modes with an even n do not contribute to the electric field due to the $\sin\left(\frac{n\pi}{2}\right)$ term in both coefficients, which vanishes for even n. The same holds for modes with purely even m. This significantly reduces the number of participating modes for the electric field, and thus also the number of modes that will contribute to the longitudinal wakefield.

As an example of the resulting fields, the electric field inside the generic waveguide from above is expanded in 500 modes at two different times and shown in figure 4. Again, the fields are shown on a cut plane at $z = L/2$.

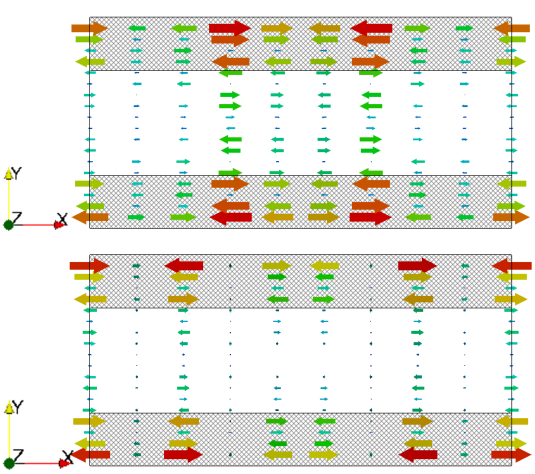

Figure 4: Electric field of a dielectrically lined rectangular waveguide with a point charge passing centrally through it. In the upper panel, the charge is located at $z = L/2$, in the lower at $z = 3L/4$.

CONCLUSION AND OUTLOOK

It has been shown that using the Rayleigh-Ritz method the eigenmodes of dielectrically lined rectangular waveguides, namely LSE and LSM modes, can be derived semi-analytically to high precision with little effort. Expanding the electric field inside such a waveguide in presence of a point charge into a set of these eigenmodes enables a semi-analytical description of the field inside the waveguide.

Providing this field expansion it becomes now possible to derive an expression for the longitudinal wakefield, since only one more integration has to be carried out,

$$W_z(s) = -\frac{1}{Q_{test}}\int_0^L E_z\left(\frac{a}{2}, \frac{b}{2}, z, \frac{(z+s)}{c}\right)dz.$$

The wakefield obtained in this manner serves as a Green's function for the wakefield of any form of particle beam, as it can be convolved with the particle beam's shape function to gain the longitudinal wakefield of the beam. Thus it becomes possible to obtain a semi-analytical expression of the wakefield in the dielectrically lined rectangular waveguide. This expression can and will be used to perform parameter studies and to gain insight into the influence of the geometrical and material parameters on the wakefield. This finally will serve as a basis to determine suiting parameters for the dechirping of the ELBE radiation source.

ACKNOWLEDGMENT

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