

QUANTIFICATION OF GEOMETRIC UNCERTAINTIES IN SINGLE CELL CAVITIES FOR BESSY VSR USING POLYNOMIAL CHAOS *

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Abstract

The electromagnetic properties of SRF cavities are mostly determined by their shape. Due to fabrication tolerances, tuning and limited resolution of measurement systems, the exact shape remains uncertain. In order to make assessments for the real life behaviour it is important to quantify how these geometrical uncertainties propagate through the mathematical system and influence certain electromagnetic properties, like the resonant frequencies of the structure's eigenmodes. This can be done by using non-intrusive straightforward methods like Monte-Carlo (MC) simulations. However, such simulations require a large number of deterministic problem solutions to obtain a sufficient accuracy. In order to avoid this scaling behaviour, the so-called generalized polynomial chaos (gPC) expansion is used. This technique allows for the relatively fast computation of uncertainty propagation for few uncertain parameters in the case of computationally expensive deterministic models. In this paper we use the gPC expansion to quantify the propagation of uncertain geometry on the example of single cell cavities used for BESSY VSR as well as to compare the obtained results with the MC simulation.

THE BESSY UPGRADE

The BESSY II accelerator is a 1.7 GeV storage ring at the HZB in Berlin. To satisfy the demands for shorter pulse lengths in combination with relatively high beam current in the order of mA, the structure needs to be upgraded. The idea is to install additional cavities at the facility in order to fill the storage ring simultaneously with pulses of variable length [1]. This is done by adding a set of superconducting cavities with high gradient and very good HOM damping to the normal conducting ones as shown in Fig. 1. A variable

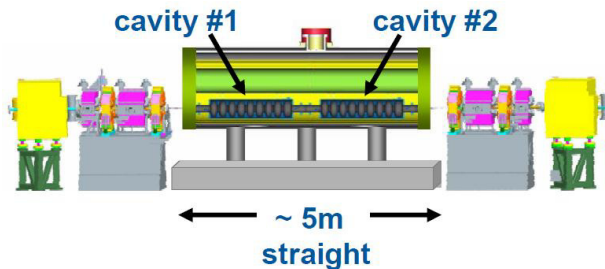


Figure 1: SRF cavities for the BESSY II upgrade (picture taken from [2]).

pulse length can be achieved by two multi-cell cavities with

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different frequencies of their π -mode. This gives a beating pattern in the longitudinal voltage and by this provides a different pulse lengths [2].

CAVITY DESIGN

For the upgrade of the BESSY accelerator a new mid-cell for the cavity needs to be designed with two times 1.5 GHz five-cell cavities having a total voltage of 20 MV. A high accuracy in the radio frequency (RF) behaviour is needed since the HOM damping of the cavities has to be as high as possible to enable high beam currents [2]. As starting point of the optimization the referenced design of the Cornell ERL [3] is used like in the BERLinPro accelerator [4]. These cells are optimized to be robust against perturbations [5]. However, for an effective design we will need a scheme that is able to get these results very efficient. It is the aim (amongst others) to find a design that is robust against geometric uncertainties. These uncertainties influence the RF-behaviour of the cavity in a way that it deviates from designed values.

DETERMINISTIC PROBLEM

The deterministic problem to solve is the Helmholtz equation for the electric field $\mathbf{E}(\mathbf{r})$:

$$\Delta \mathbf{E}(\mathbf{r}) + \left(\frac{\omega}{c}\right)^2 \mathbf{E}(\mathbf{r}) = \mathbf{0} \text{ in } \Omega, \quad (1)$$

where Δ denotes the Laplace operator acting on vector fields, and ω the resonant angular frequency of the electric field. The speed of light in vacuum is denoted by c . The surface of the iris $\partial\Omega_{\text{iris}}$ is assumed to be perfectly magnetic conducting:

$$\mathbf{n} \cdot \mathbf{E}(\mathbf{r}) = 0 \text{ on } \partial\Omega_{\text{iris}}, \quad (2)$$

whereas the remaining boundary $\partial\Omega_{\text{wall}}$ is assumed to be perfectly electric conducting:

$$\mathbf{n} \times \mathbf{E}(\mathbf{r}) = \mathbf{0} \text{ on } \partial\Omega_{\text{wall}}, \quad (3)$$

with the normal component \mathbf{n} of the boundary of the structure. The geometry is given by a cavity cell consisting of two half cells as shown in Fig. 2. Since the equator radius needs to be equivalent on both half cells the problem geometry consists of 13 independent parameters. This eigenmode problem can be solved in 2D assuming the structure to be rotational symmetric. For the solution of this eigenmode problem the 2D eigenmode solver SUPERFISH [6] is used. After the computation of the eigenmodes, the cavity geometry is changed in such a way that the frequency of the first TM monopole mode TM_{010} equals 1.3 GHz. This tuning is performed using the Brent-Decker algorithm [7]. From the TM_{010} -mode of the tuned cavity several RF-parameters

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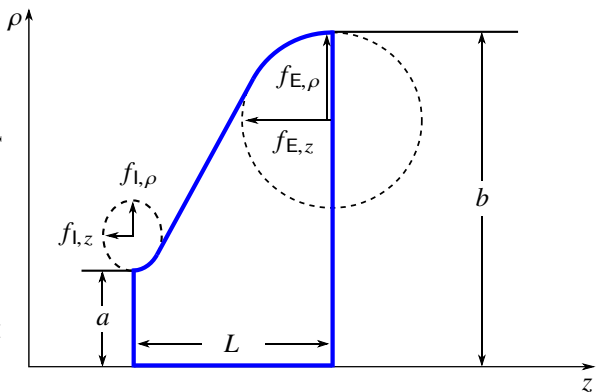


Figure 2: Parametrized geometry of an elliptical half-cell using two ellipses.

such as the shunt impedance R_s , the quality factor Q_0 , the geometry factor G and the r/Q are computed.

UNCERTAINTIES

A real life structure deviates in its behaviour from the designed one. These deviations have various reasons and can be roughly subdivided in deviations caused by the fabrication process and deviations caused by the operation. While the fabrication errors are in any case static, and thus not subject to any temporal changes, the operational deviations can be time-dependent or, respectively static. For our actual investigations we only consider rotational symmetric distortions. This has two reasons. First, the structure can be computed in 2D and by this gives a huge speed up for the computation. The second reason is the mathematical description of the geometry. In the case of perturbations breaking the rotational symmetry the structure would not be describable using the way shown in Fig. 2.

Uncertainty Quantification

Uncertainty quantification (UQ) denotes a set of mathematical techniques mostly used in the design process of a system in order to quantify the effects of uncertainties to the systems behaviour. UQ can be performed in every iteration of the design process, and its outputs can be considered as a goal function. This leads to a design that is robust against uncertainties of any kind and gives a maximum likelihood for the structure to operate in given limitations. In most cases it requires a repetitive solution of a deterministic problem with varying input parameters.

These techniques can be divided into probability sampling ones like Monte Carlo (MC) [8] or latin hypercube and non-probability sampling techniques like perturbation theory. For the application described in this article probability sampling techniques are the method of choice as they do not require any in-depth knowledge of the solution of the problem. Hence the deterministic model can be considered as black box. This gives the technological advantage that the solution of the deterministic model (in this case the specific eigenmode solver) can be easily substituted, leading to a

high variability and wide range of practical applications for the written code.

For probability sampling techniques there is a wide range of methods that fit our requirements for the solution. The first choice which is extremely easy to implement is the MC sampling technique. The drawback for this method is its low order of convergence ($\log(N)$). For this reason we picked the polynomial chaos technique which was first described by Ghanem and Spanos [9] and was previously successfully used in our working group for the investigation of brain conductor models in bio-engineering [10].

Polynomial Chaos

The technique of choice is the generalized polynomial chaos technique (gPC) which will be briefly explained in the following. For a detailed description of gPC see [11] and references therein.

The main idea is to find a surrogate model of the original one for which the UQ problem is way easier to solve. This is done by expanding the output quantity of interest Y by a multivariate polynomial expansion $\psi_k(\xi)$ of the degree P :

$$Y \approx \sum_{k=0}^P c_k \psi_k(\xi), \quad (4)$$

where Legendre-Polynomials are being used in order to get the best convergence for uniformly distributed input variables. The expansion coefficients c_k are being determined by a numerical integration which requires the evaluation of the deterministic problem at quadrature nodes determined by the used quadrature. In this case the sparse grid based on the Clenshaw Curtis rule is used. Since the computational demand scales with the input dimension, in order to reduce the computational demand a univariate gPC expansion was performed for all 13 input parameters as a preprocessing step to find the five most sensitive parameters. In the full computation with gPC only these five parameters will be accounted as uncertain while all other parameters are assigned their design value. For the probability density function of the input parameters we assume a uniform distribution with a standard deviation of 0.125 mm as measured in previous work at Cornell [5] for this specific cavity design.

RESULTS

The uncertainty quantification for the single cell cavity was performed using gPC with a polynomial order of $p = 2$ using 61 samples. To verify these results, the same computation was performed using MC. The number of samples needed is computed with the width of a 95% confidence interval for the mean value \mathbb{E} of the quality factor Q_0 :

$$\left[\mathbb{E} - \frac{1.96\sigma}{\sqrt{N}}, \mathbb{E} + \frac{1.96\sigma}{\sqrt{N}} \right], \quad (5)$$

with the standard deviation σ of the output parameter after N samples. This confidence interval is supposed to have a relative width of 0.1%, which is the case at approximately

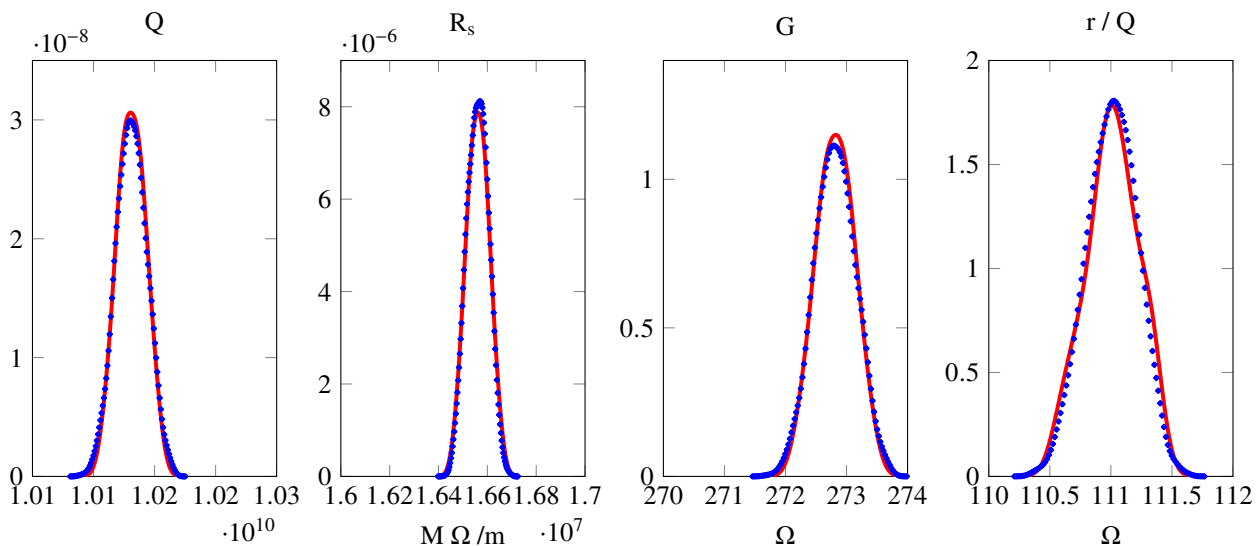


Figure 3: Probability density function of several output parameters computed with MC (blue curve) and gPC with polynomial degree $p = 2$ (red curve).

4000 samples. In Table 1 the mean values of several RF-parameters are shown computed using gPC and MC. It becomes obvious that the results of gPC and MC match in the order of below 1%. Also the results lead to the conclusion that though the cavity structure deviates from the designed value, the mean of any of the RF-parameters does hardly vary from its design value. Remarkably, the slight deviations tend to be numerical, since the error decreases if the polynomial degree of the solution is increased.

Table 1: Mean of Several RF-Parameters

	gPC	MC	Design Value
Q	$1.0181 \cdot 10^{10}$	$1.0181 \cdot 10^{10}$	$1.0181 \cdot 10^{10}$
R_s	$16.561 \cdot 10^6$	$16.560 \cdot 10^6$	$16.561 \cdot 10^6$
G	272.8168	272.8116	272.819
r/Q	111.0237	111.0084	111.04

Using gPC we were able to achieve a major reduction of the computational demand, compared to MC. This enables us to use uncertainty quantification as part of a multi dimensional goal function in an optimization process. The results also lead to the assumption that it is possible to handle the half-cells separately in case that the cell design is rotational symmetric. This is due to the fact that there seem to be nearly no non-linear effects between the geometry parameters of the half-cells regarding the output parameters. Further, the results indicate that due to the high linearity, the probability density function of the output parameter hardly depends on the distribution of the input parameter.

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