Halo Mitigation in Nonlinear Integrable Lattices

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Overview

- Linear lattices
- Nonlinear decoherence versus Landau damping
- Nonlinear integrable optics
- Halo mitigation
- Questions & future work

Linear Lattices

Why & Why Not

Why...

Integrable behavior

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The Strong-Focusing Synchroton—A New High Energy Accelerator*

ERNEST D. COURANT, M. STANLEY LIVINGSTON,† AND HARTLAND S. SNYDER
Brookhaven National Laboratory, Upton, New York

(Received August 21, 1952)

Strong focusing forces result from the alternation of large positive and negative n-values in successive sectors of the magnetic guide field in a synchrotron. This sequence of alternately converging and diverging magnetic lenses of equal strength is itself converging, and leads to significant reductions in oscillation amplitude, both for radial and axial displacements. The mechanism of phase-stable synchronous acceleration still applies, with a large reduction in the amplitude of the associated radial synchronous oscillations. To illustrate, a design is proposed for a 30-Bev proton accelerator with an orbit radius of 300 ft, and with a small magnet having an aperture of 1×2 inches. Tolerances on nearly all design parameters are less critical than for the equivalent uniform-n machine. A generalization of this focusing principle leads to small, efficient focusing magnets for ion and electron beams. Relations for the focal length of a double-focusing magnet are presented, from which the design parameters for such linear systems can be determined.

Courant-Snyder invariant

This creates...

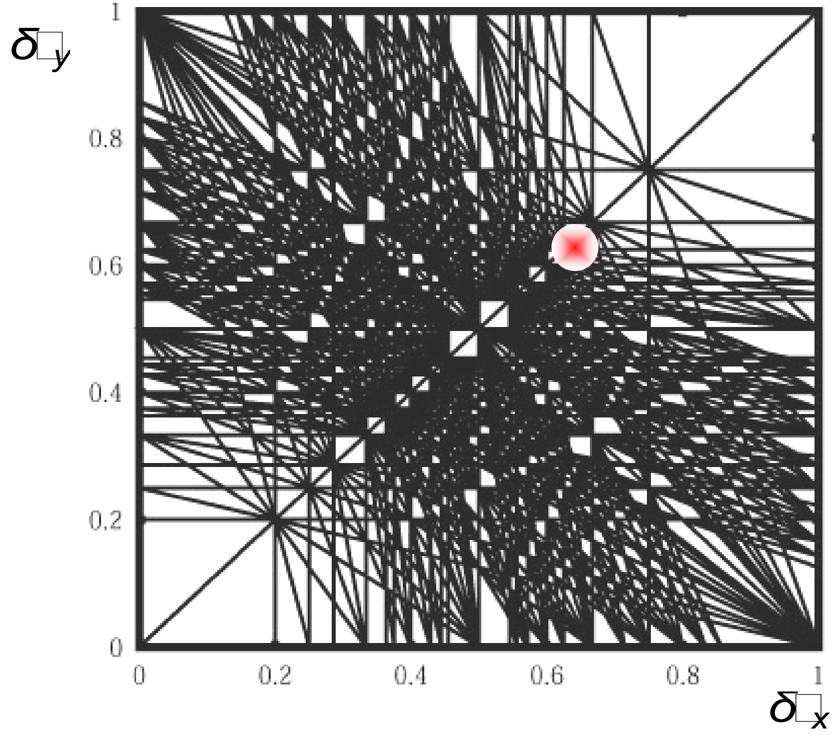
- Tunes
- Beta functions
- Dispersion

$$\mathcal{J}_i = \frac{1}{2\beta_i(s)}\left[z_i^2 + \left(\beta_i(s) z_i' + \alpha_i(s) z_i\right)^2\right]$$

Why Not...

In a word: **Resonances**

- tune spread



Nonlinear Decoherence:

It’s not Landau Damping

Landau damping

“There is no clear agreement as to which effects can be labeled as Landau damping.”
~ A. Hofmann, “Landau Damping”, 1987 CERN Accelerator Physics Course

Landau damping (n.) - The process by which a spread of bare frequencies in an <u>ensemble</u> of harmonic oscillators prevents a resonant perturbation from adding energy coherently.

A toy model --

$$\frac{d^2 x_\beta}{dt^2} + \omega_\beta^2 x_\beta = G \sin(\omega t)$$

$$\langle \omega_\beta \rangle = \omega$$

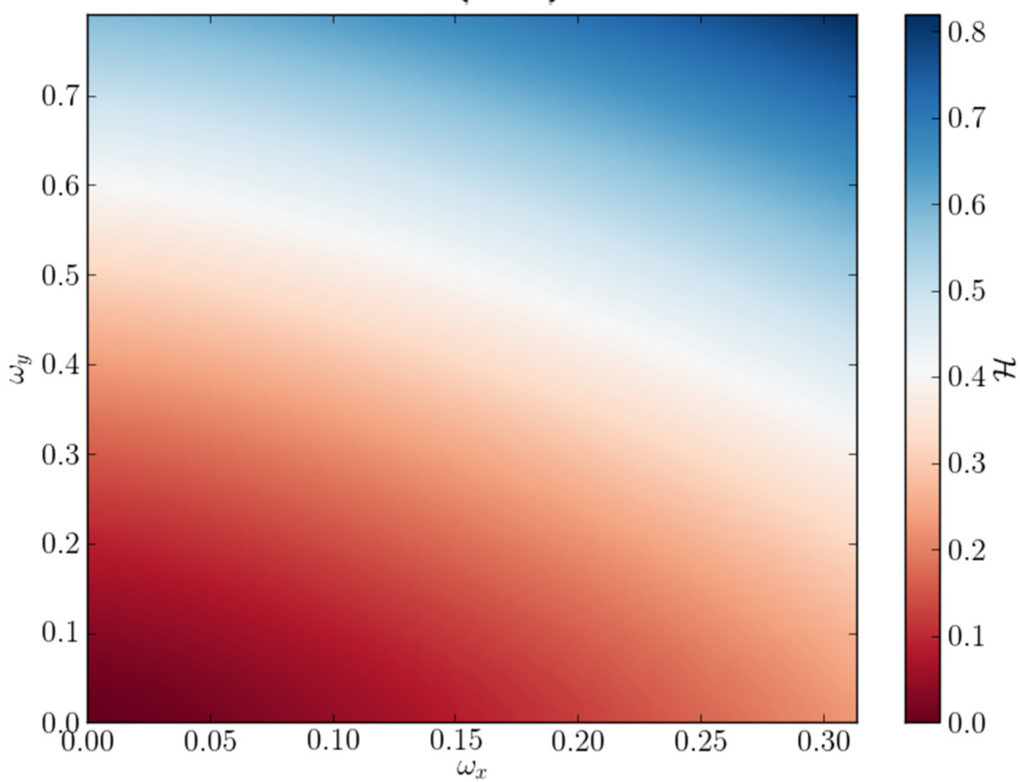
An example problem...

Completely integrable Hamiltonian

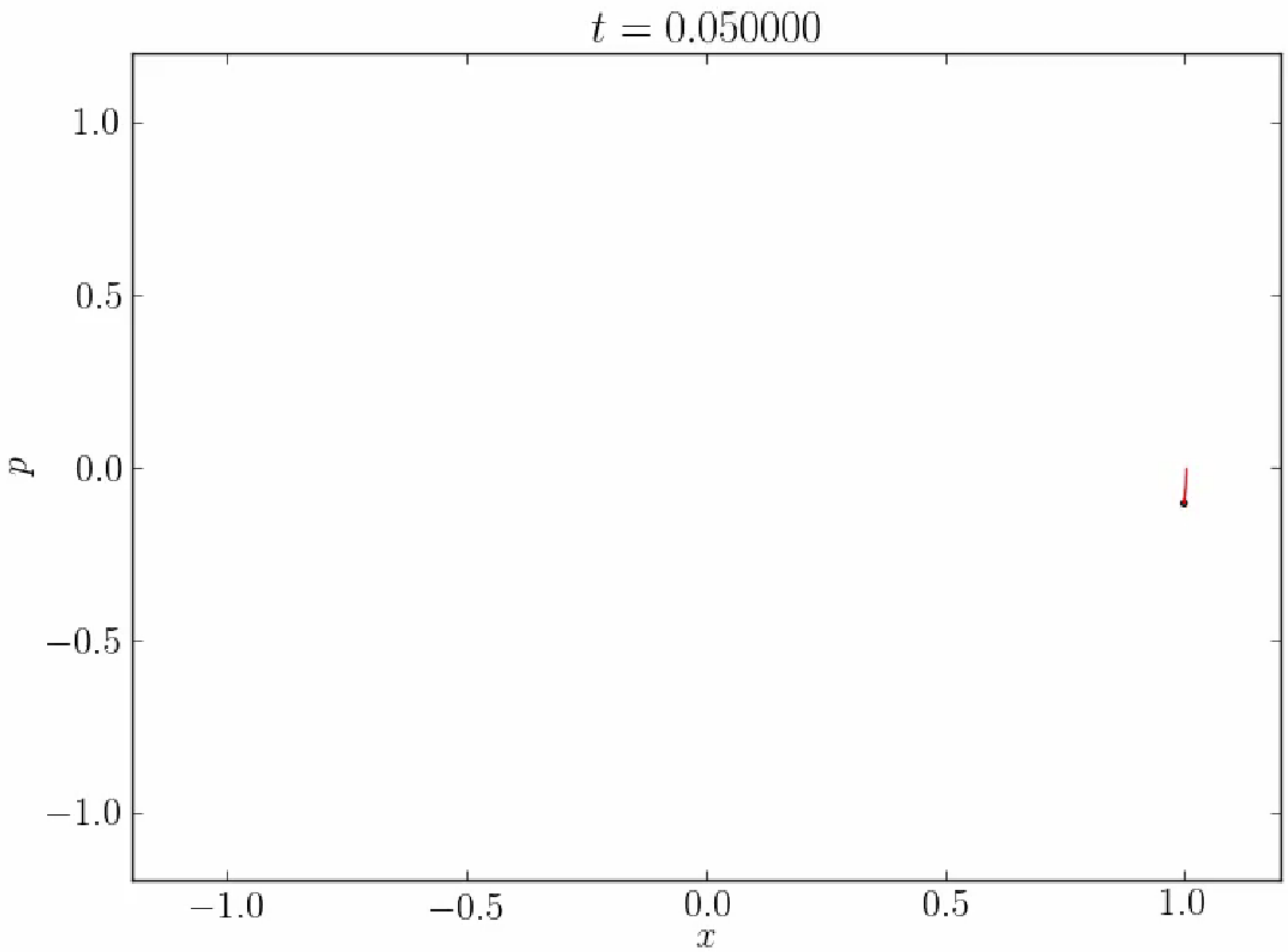
$$\mathcal{H} = \underbrace{\frac{p_x^2}{2} + \frac{1}{4}\lambda_x^4 x^4}_{H_x} + \underbrace{\frac{p_y^2}{2} + \frac{1}{4}\lambda_y^4 y^4}_{H_y}$$

In action-angle variables

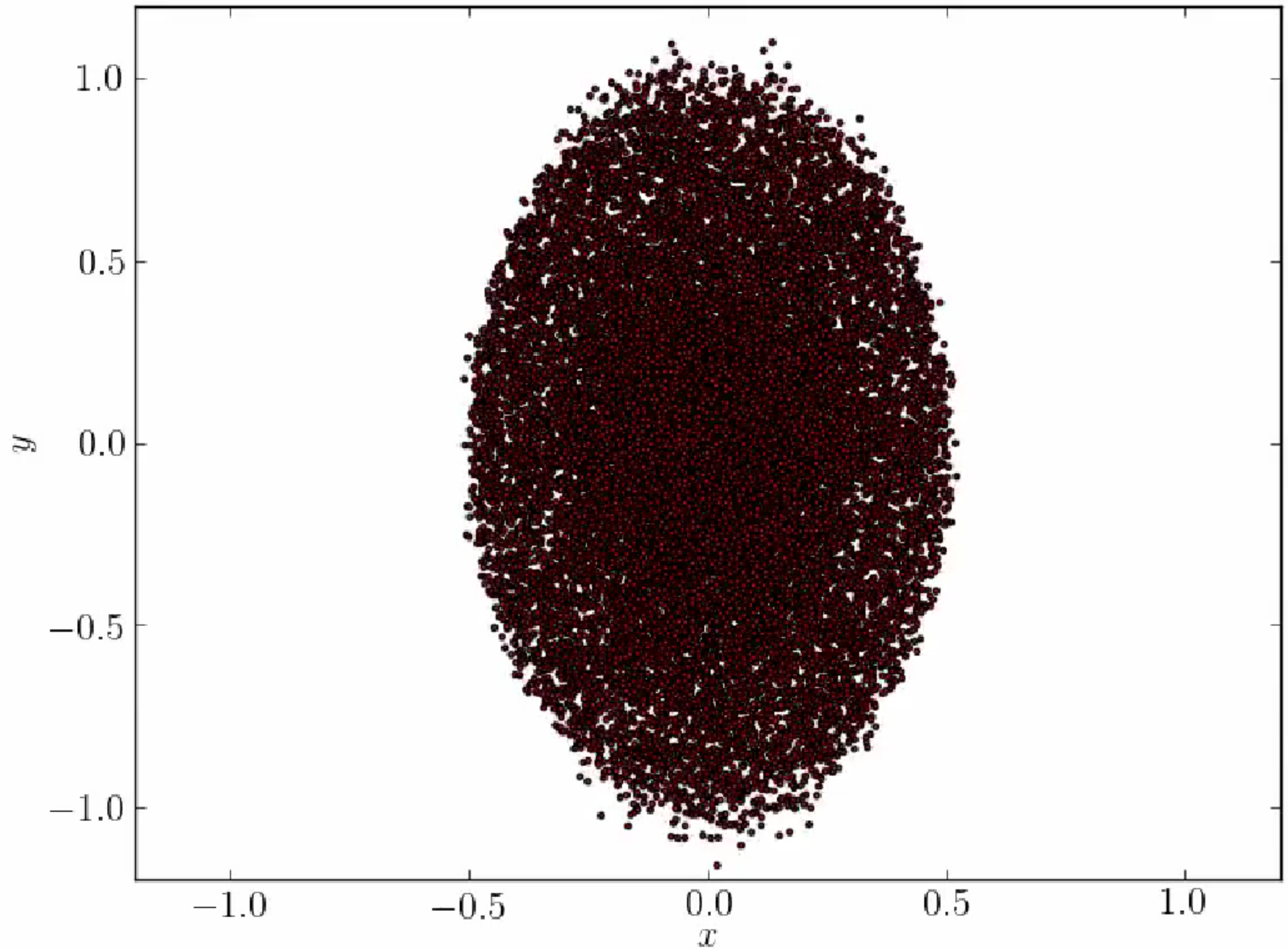
$$\mathcal{K} = \left(\frac{1}{2\alpha}\right)^{4/3} \left\{ (\mathcal{J}_x\lambda_x)^{4/3} + (\mathcal{J}_y\lambda_y)^{4/3} \right\}$$



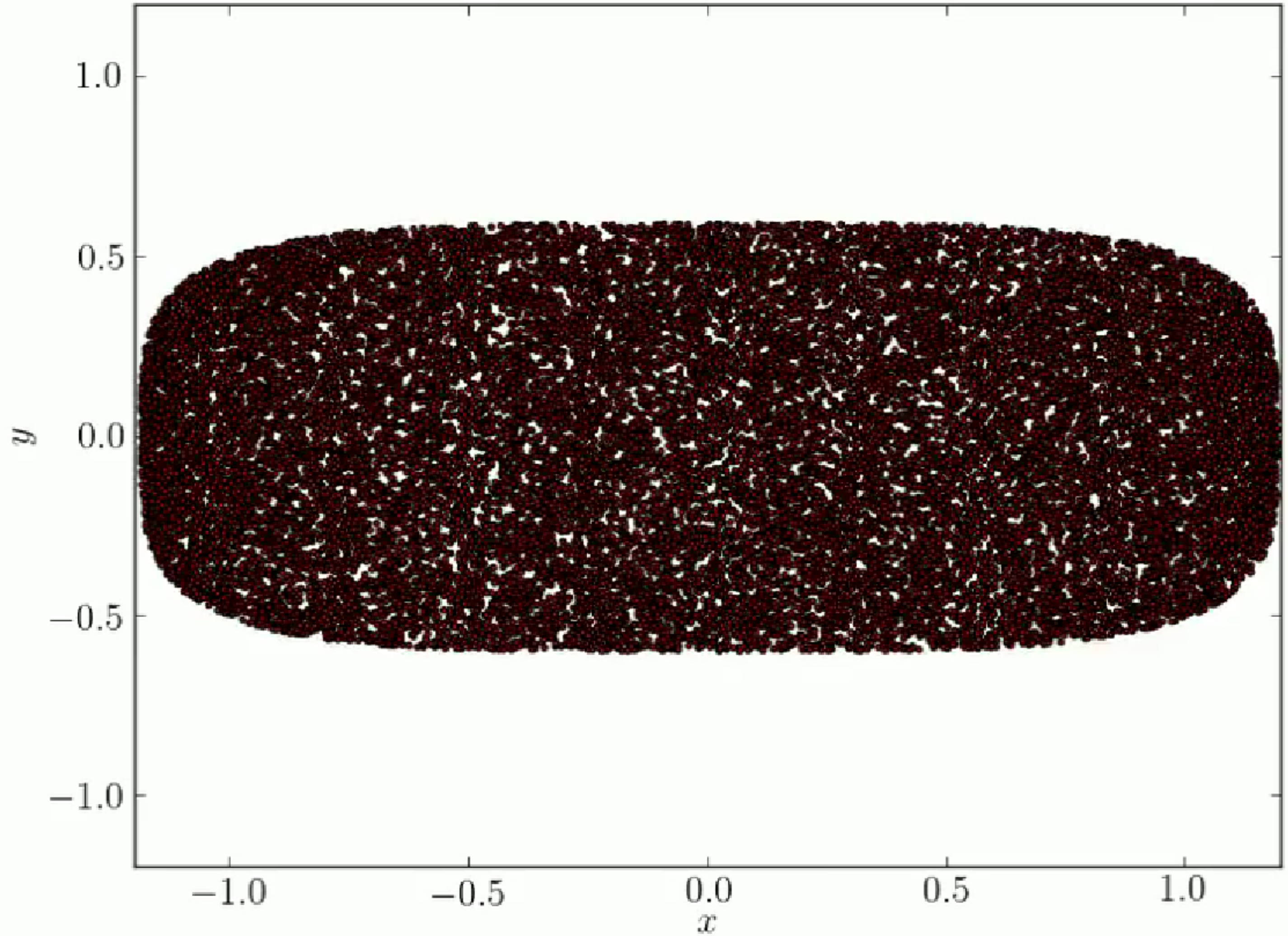
An example problem...



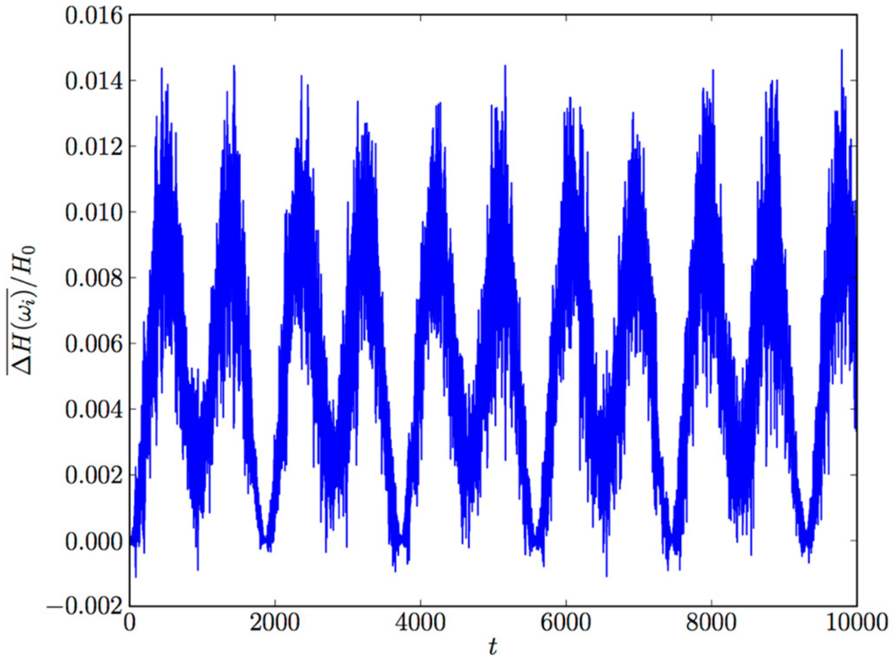
Landau damping



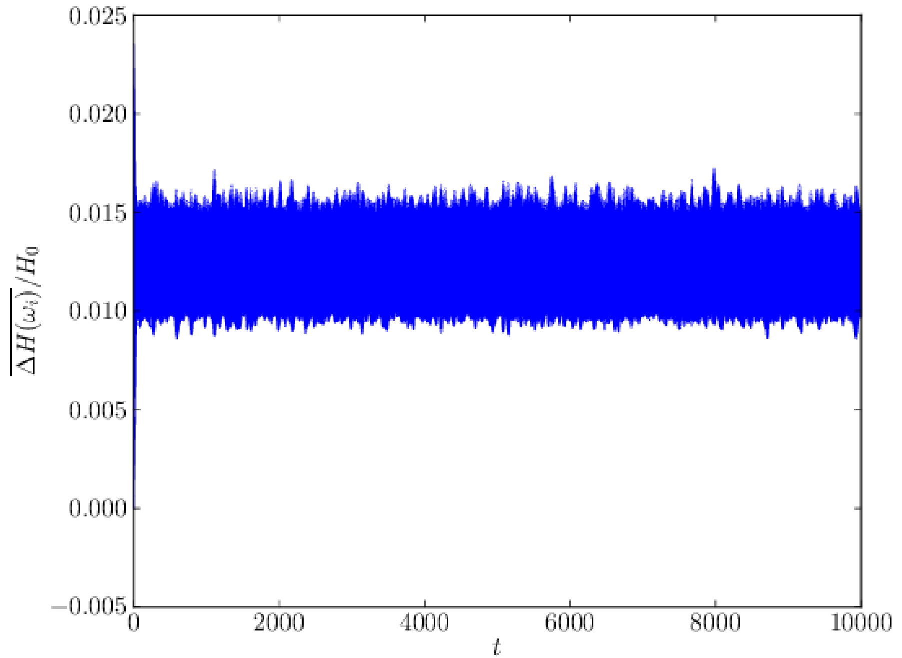
Not Landau damping



Nonlinear Decoherence vs.Landau Damping Energy Growth



Nonlinear decoherence



Landau damping

Nonlinear Integrable Optics

Controlled Nonlinear Lattices can have Bounded Motion

Normalized coordinates

$$H = \frac{1}{2}\vec{p}^{\,2} + \vec{q}^{\,T}\tilde{K}(s)\,\vec{q} + U(\vec{q}, s)$$

$$\begin{cases} z_N = \dfrac{z}{\sqrt{\beta(s)}} \\ p_N = p\sqrt{\beta(s)} - \dfrac{\beta'(s)z}{2\sqrt{\beta(s)}} \\ \psi'(s) = \dfrac{1}{\beta(s)} \end{cases}$$

canonical transformation

$$\mathcal{H} = \frac{1}{2}\vec{p}_N^{\,2} + \frac{1}{2}\vec{q}_N^{\,2} + \beta(\psi)U\left(\sqrt{\beta_x(\psi)}x_N, \sqrt{\beta_y(\psi)}y_N, s(\psi)\right)$$

Controlled nonlinearity

$$\beta(\psi)U\left(\sqrt{\beta_x(\psi)}x_N, \sqrt{\beta_y(\psi)}y_N, s(\psi)\right) = V\left(x_N, y_N\right)$$

Hamiltonian becomes a conserved quantity

Nonlinear Integrable Optics §

Bertrand-Darboux Eqn.

$$xy\left(\partial_{xx}U - \partial_{yy}U\right) + \left(y^2 - x^2 + c^2\right)\partial_{xy}U + 3y\,\partial_x U - 3x\,\partial_y U = 0$$

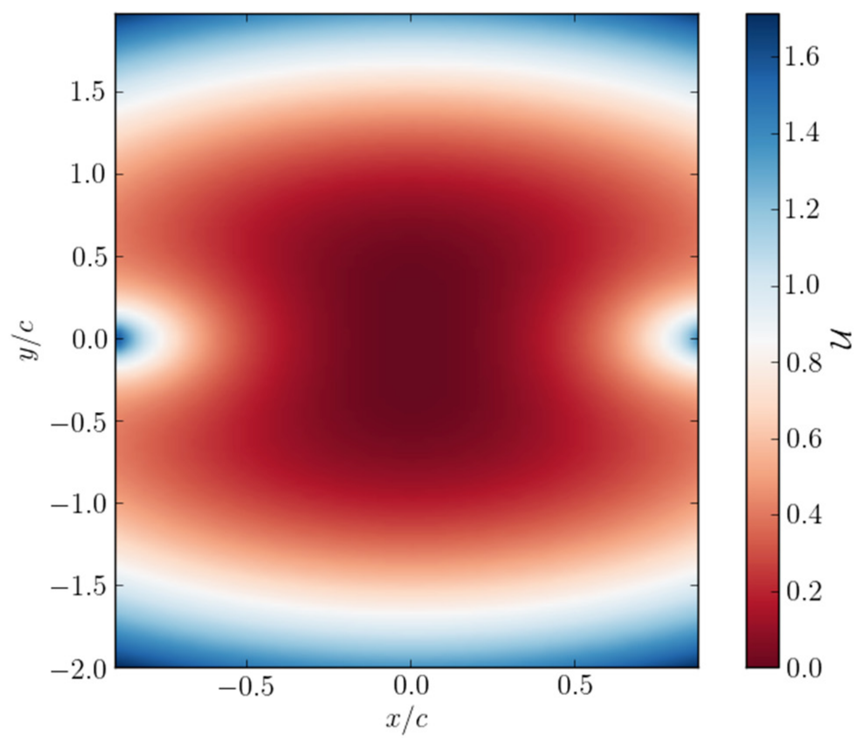
Self-consistently with Maxwell's equations yields...

$$U(x,y) = \frac{f(\xi) + g(\eta)}{\xi^2 - \eta^2}$$

$$\left\{\begin{array}{c} \xi \\ \eta \end{array}\right\} = \frac{\sqrt{(x+c)^2 + y^2} \pm \sqrt{(x-c)^2 + y^2}}{2c}$$

$$f(\xi) = -\xi\sqrt{\xi^2 - 1}\cosh^{-1}(\xi)$$

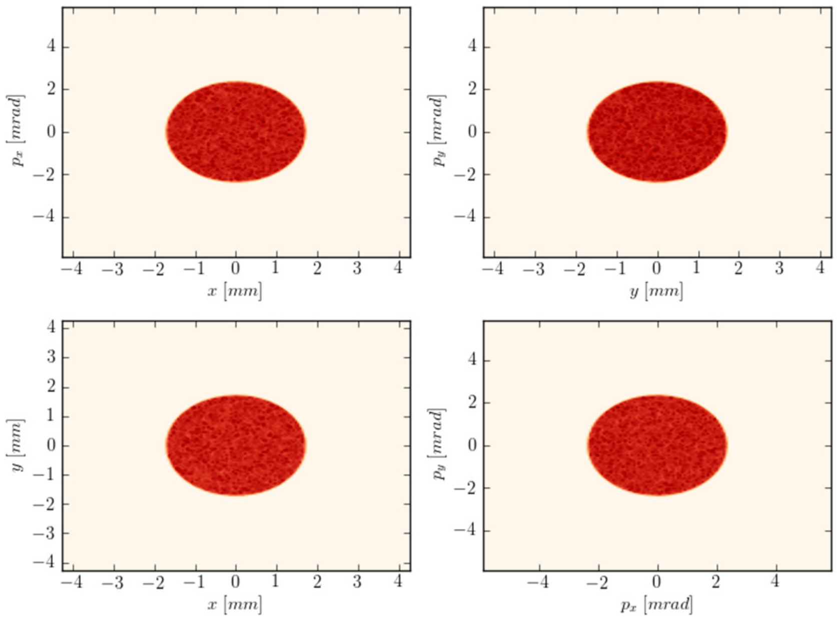
$$g(\eta) = \eta\sqrt{1-\eta^2}\left(\frac{\pi}{2} + \cos^{-1}(\eta)\right)$$



§ V. Danilov and S. Nagaitsev, "Nonlinear lattices with one and two analytic invariants", Phys. Rev. ST - Acc. Beams 13, 084002 (2010).

Nonlinear Integrable Optics

Mismatch is a problem...



Nonlinear Integrable Optics

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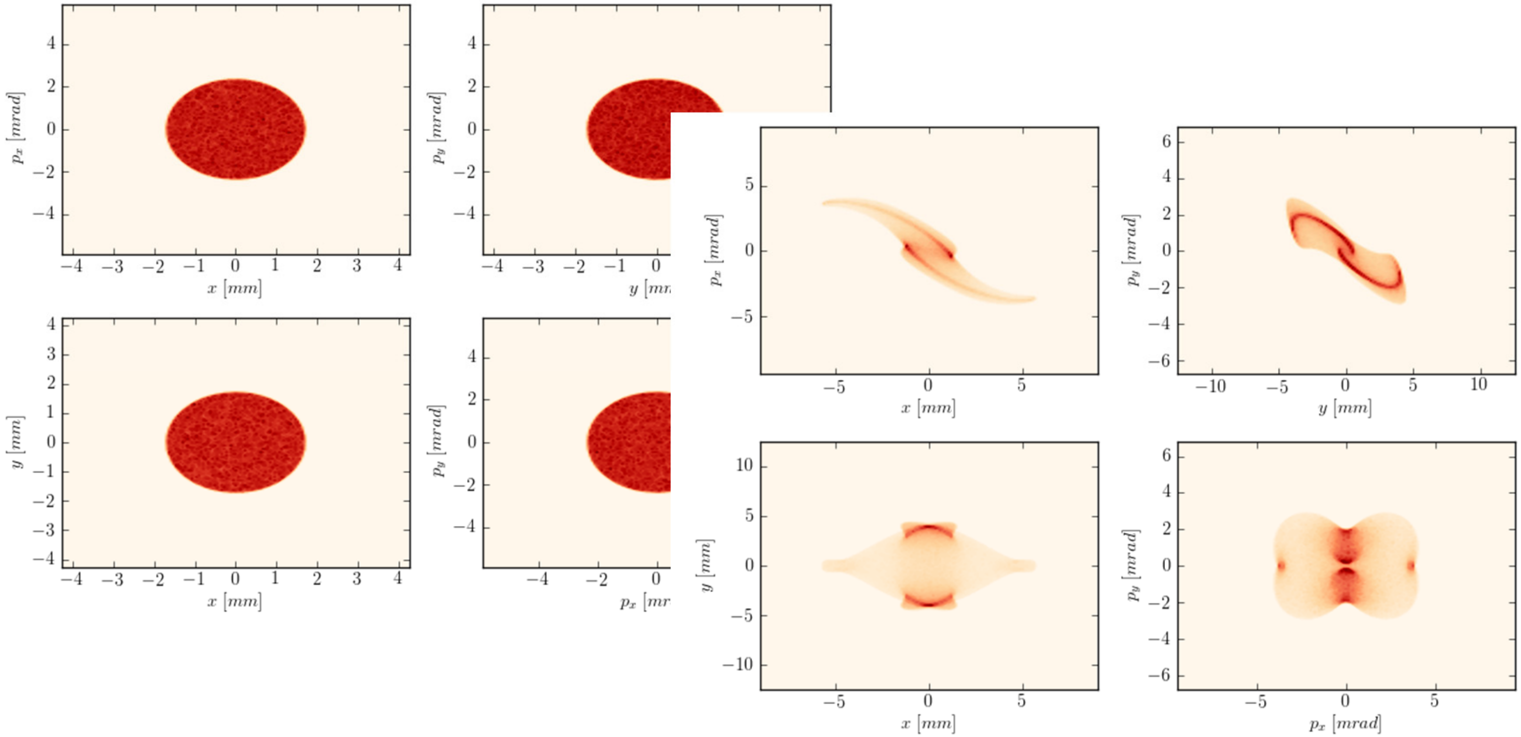

Nonlinear Integrable Optics

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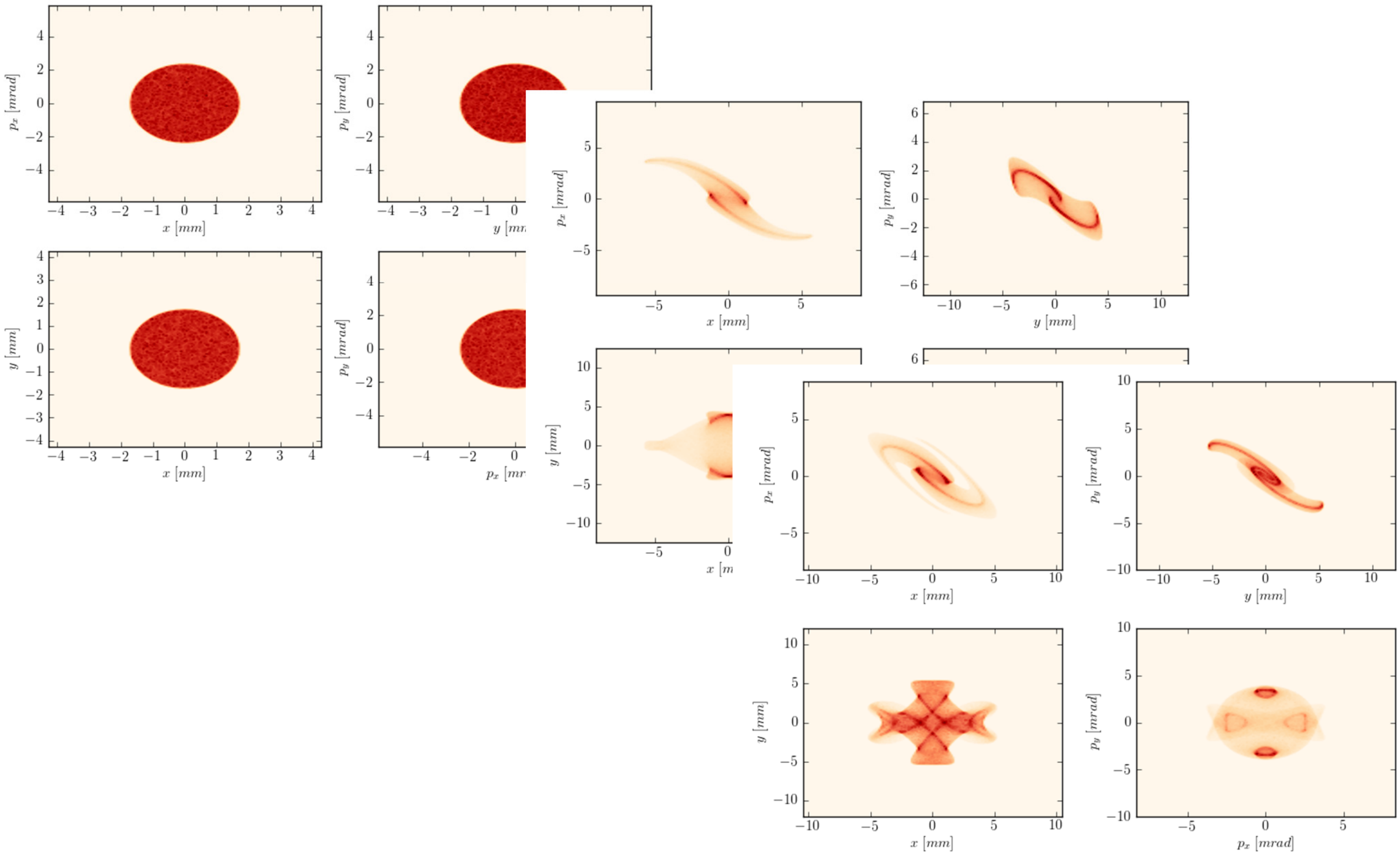

Nonlinear Integrable Optics

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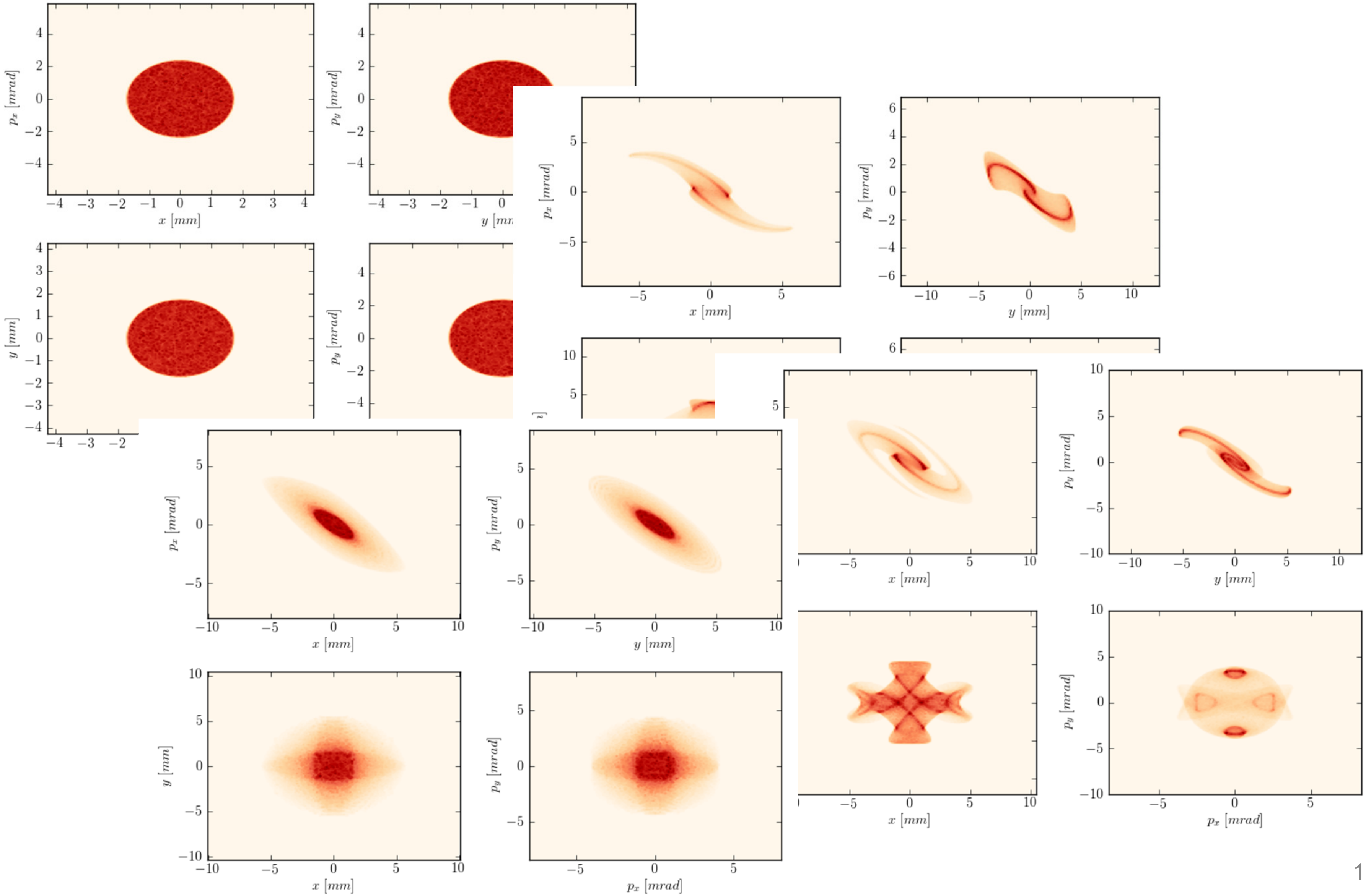


Generalized Matching Creates Stable Beams

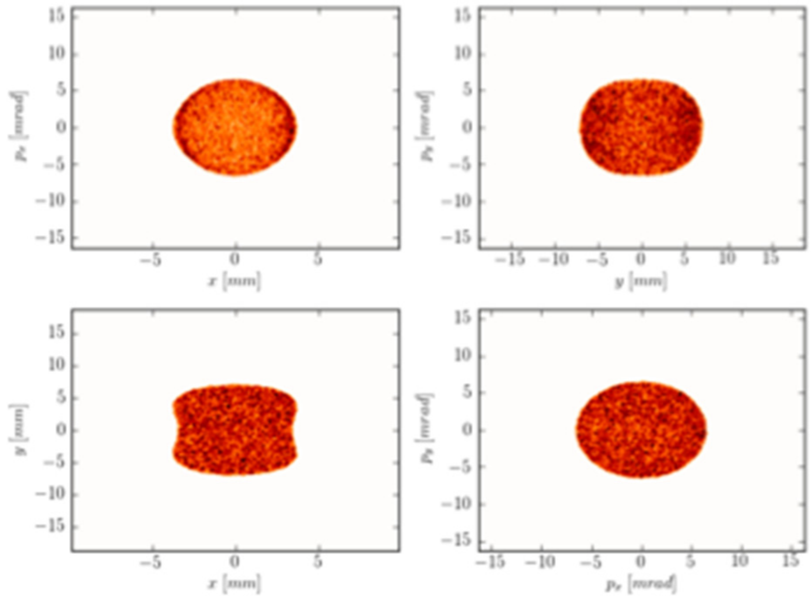
Beam Matching & Fixed Points of the Single Particle Hamiltonian

$$\hat{\mathcal{H}} = \frac{\hat{p}_x^2}{2} + \frac{\hat{p}_y^2}{2} + \frac{\hat{x}^2}{2} + \frac{\hat{y}^2}{2} + U\left(\hat{x}, \hat{y}\right)$$

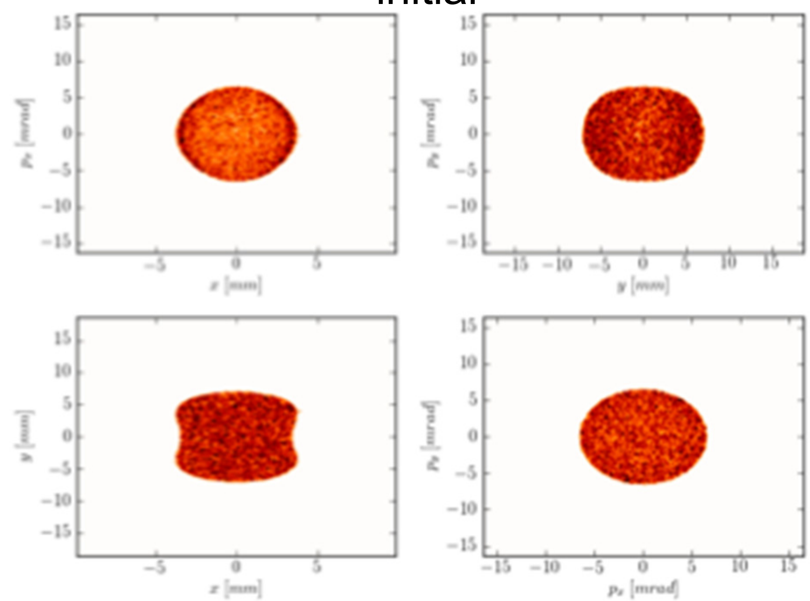
General KV-type Distribution:

$$f\left(\hat{\mathcal{H}}\right) = \delta\left(\hat{\mathcal{H}} - \epsilon\right)$$

$$F\left(\hat{\mathcal{H}}\right) = \int d\epsilon' F(\epsilon')\delta\left(\hat{\mathcal{H}} - \epsilon'\right)$$



initial



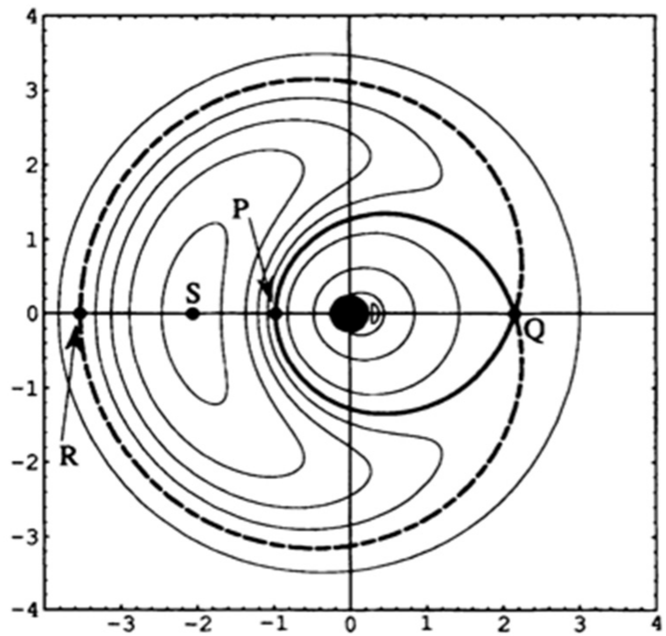
10000 turns

Halo Formation Mitigation

Beam Halo Overview

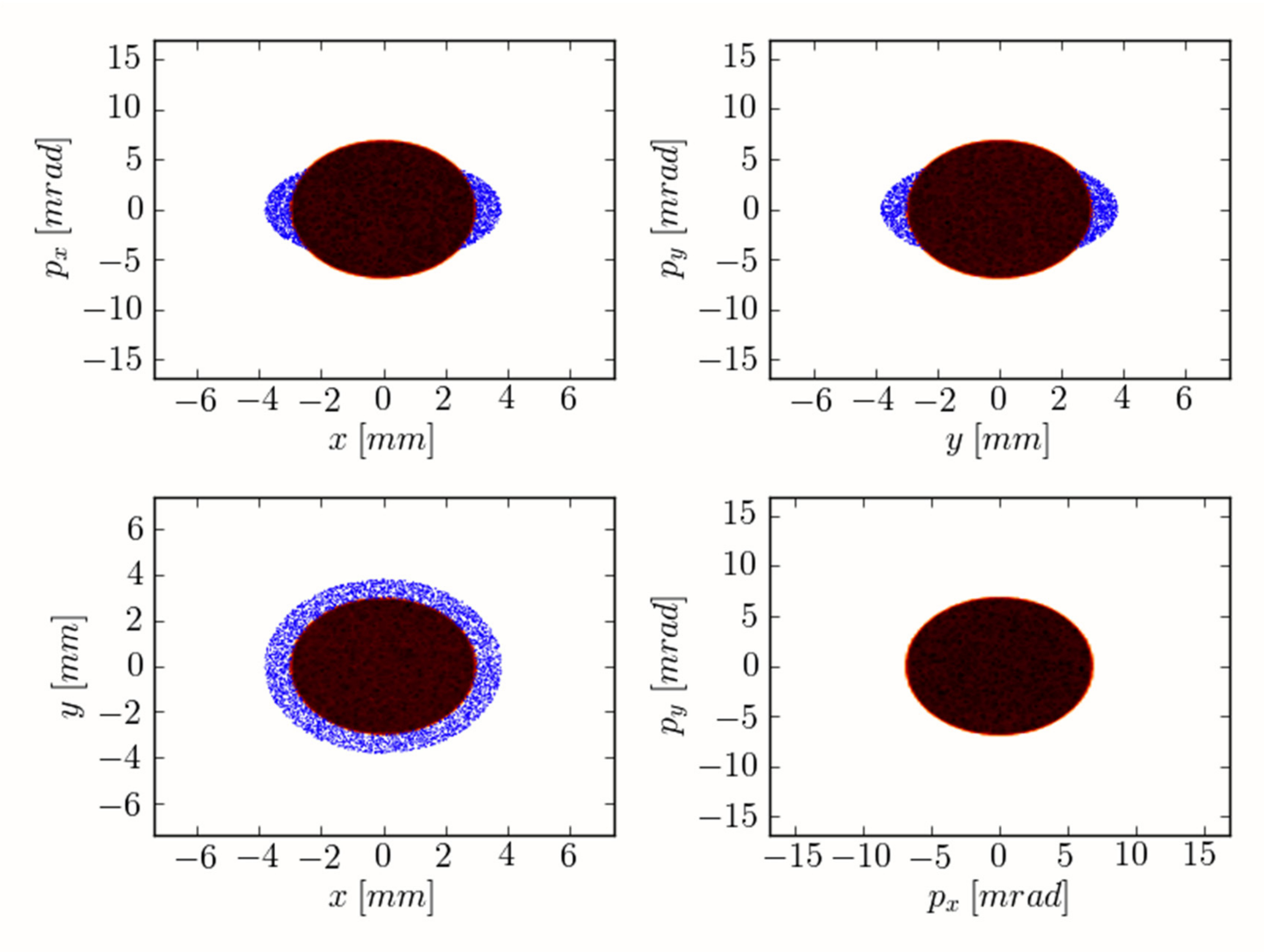
Mismatched KV core “breaths”, driving a parameteric space charge driven resonance §

$$\tilde{H} = {}^{\kappa}\!/_{qa^2} \left(w\epsilon \cos\Psi - \Delta\ w + \frac{3}{8}w^2 \right)$$

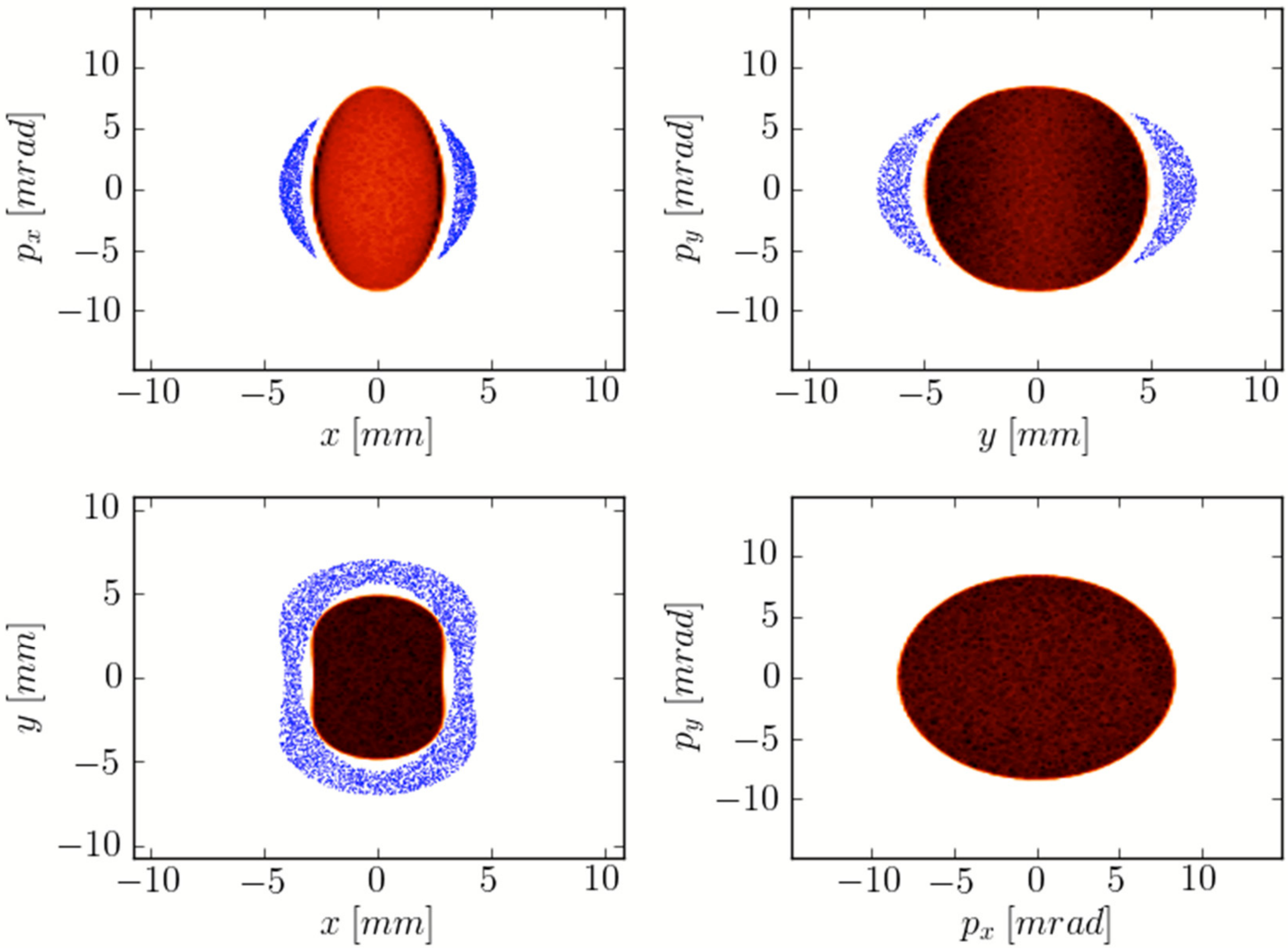


§R. Gluckstern, “Analytic Model for Halo Formation in High Current Ion Linacs”. Phys. Rev. Lett. 73 **9** 1994

Linear Lattice Forming Beam Halo



Integrable Elliptic Lattice Suppresses Beam Halo



Nonlinear Decoherence Prevents Halo Formation

- Beam halo is a major issue for intense beam transport and storage
- Properly matched beams in properly designed nonlinear lattices prevent halo formation
- Questions:
  - The limits of nonlinear decoherence
  - Effects of broken integrability
  - Preserving integrability against collective effects

Thank you

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