Phase Space Tomography Research at Daresbury

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The Cockcroft Institute of Accelerator Science and Technology
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Overview

Introduction

Measurements and techniques

Space charge studies

4D reconstruction

Conclusions

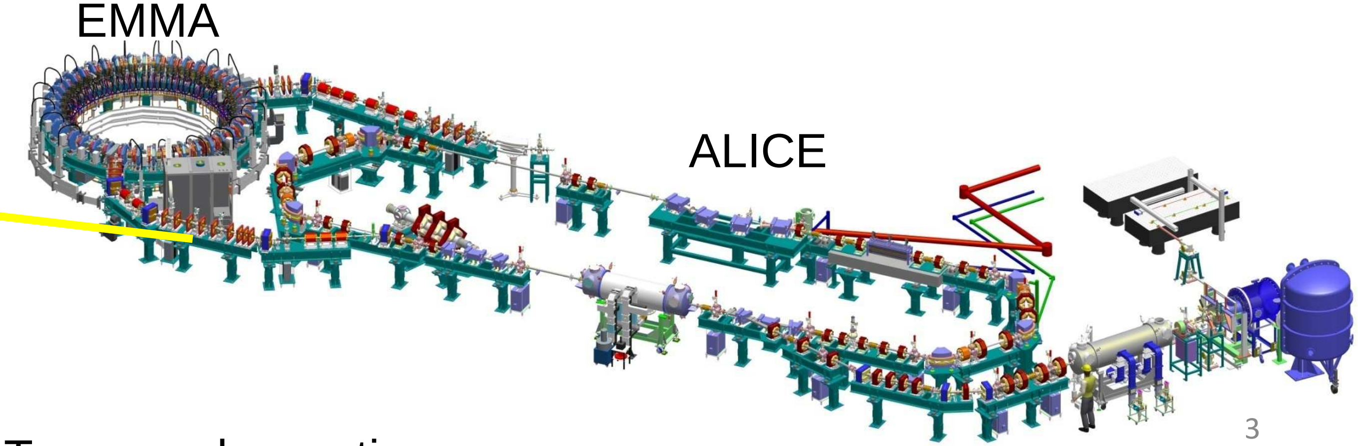
ALICE Tomography Section

ALICE tomography section is part of the ALICE-EMMA injection line.

ALICE (Accelerators and Lasers In Combined Experiments) is an accelerator test facility which has been designed and built at Daresbury Laboratory.

EMMA (Electron Model for Many Applications) is the first demonstration of acceleration in a non-scaling FFAG (Fixed Field, Alternating Gradient) machine.

Electron beam parameters in this work	
Energy	12 MeV
Repetition rate	5 Hz
Bunch charge	20 - 80 pC



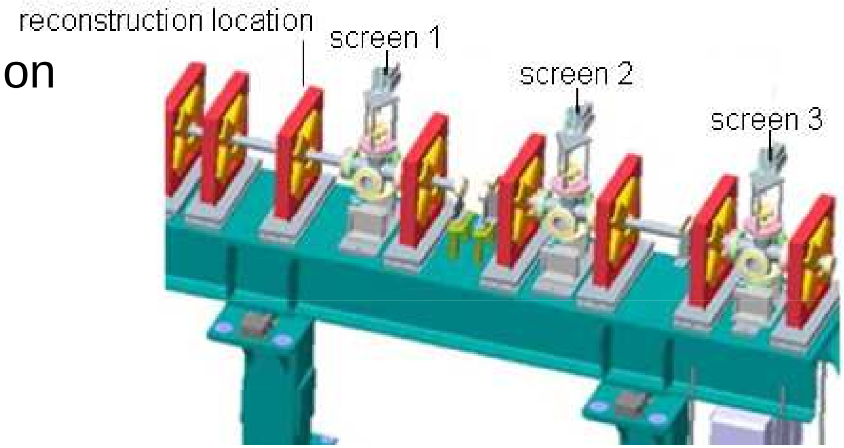
Tomography section

ALICE Tomography Section

Purposes

- for transverse phase space measurements

- to keep the beamwidth small for EMMA injection



Design

- multiple screens with FODO cells in between, similar to PITZ, DESY

- 3 screens here instead of the 4 screens at PITZ

Two different measurement methods

1. Record one image at each screen and reconstruct phase space distribution with Maximum Entropy Technique, MENT (used in PITZ, SNS, PSI, etc).

2. Record images on 1 screen for different strengths of quadrupoles and reconstruct with Filtered Back Projection, FBP (used in UMER, BNL, ...) .

Research and Development

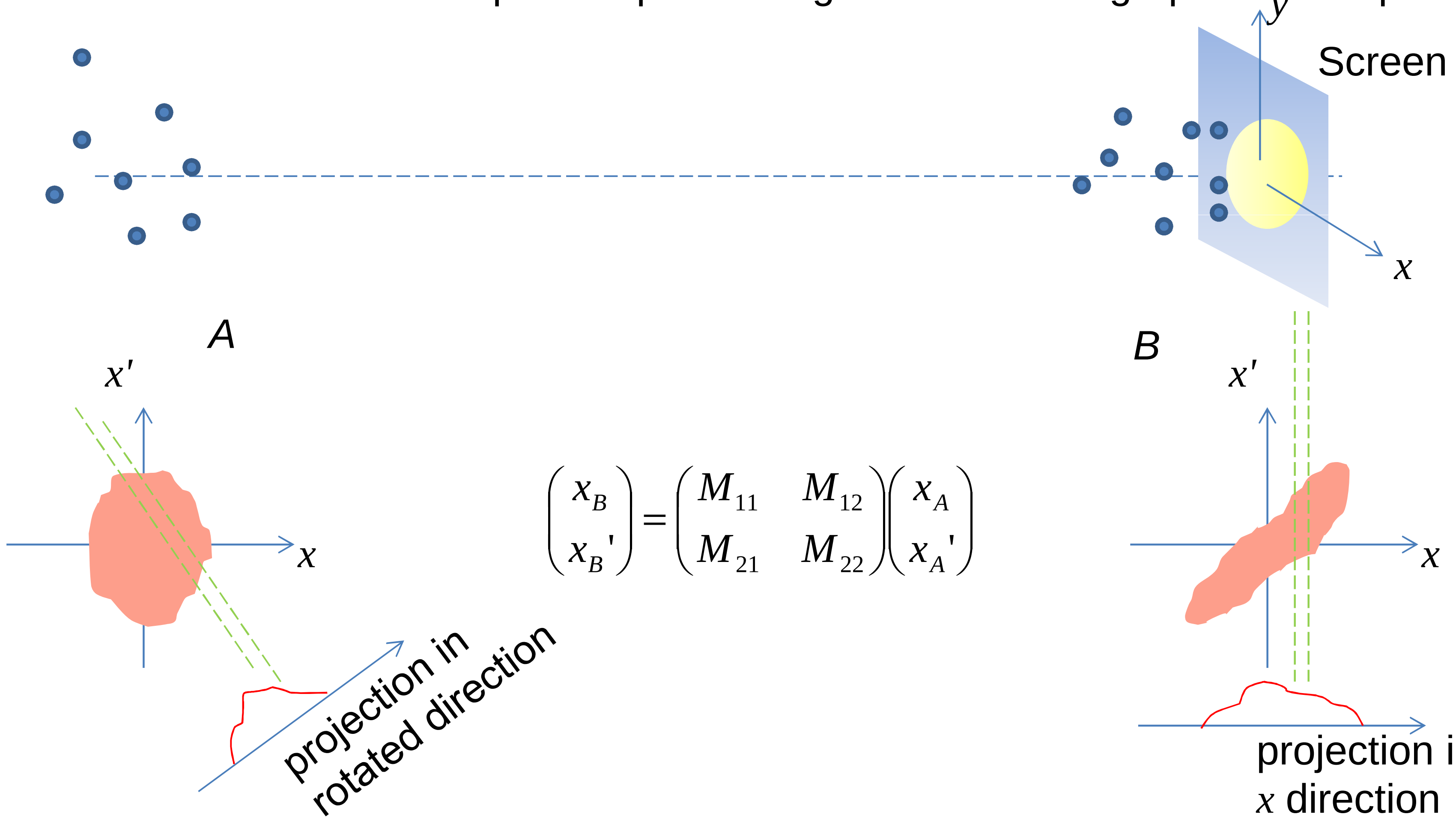
Challenges (3 years ago)

- To establish phase space measurement from scratch.
- To get reliable results with limited data (due to limited beam time, only 3 screens)
- To study the onset of space charge effect.
- To develop novel phase space measurement techniques.

Phase space tomography

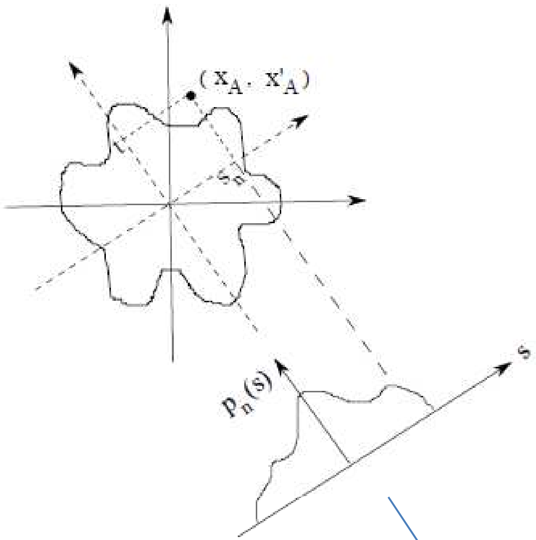
Theory: 1. Screen image projection = phase space projection in x direction.

2. This is related to a projection at reconstruction location - in a rotated direction.
3. The rotation angle can be obtained from the transfer matrix.
4. By changing the transfer matrix, we can get projections for different angles.
5. Then reconstruct phase space using standard tomographic technique.



$$\begin{pmatrix} x_B \\ x_B' \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} x_A \\ x_A' \end{pmatrix}$$

Filtered Back Projection (FBP)



To reconstruct original distribution from projections:

1. Fourier transform each projection, filter, invert transform.
2. Spread filtered projection over the phase space.
3. Repeat for each angle and add over the previous one.

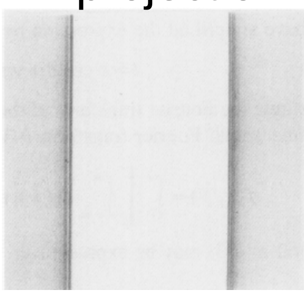
- Spreading out the filtered projection creates streaking artefacts.
- These cancel out with enough angles – ideally ALL angles over 180° range.

FFT, filter, invert

Streaking artefact

1 projection

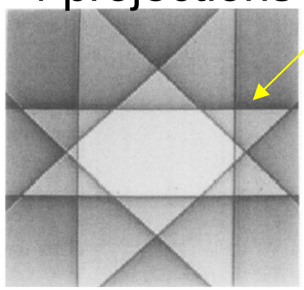
spread



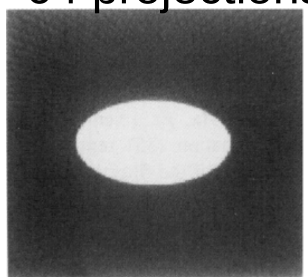
Kak and Slaney (1999)

next

4 projections



64 projections



(close to original)

simulations

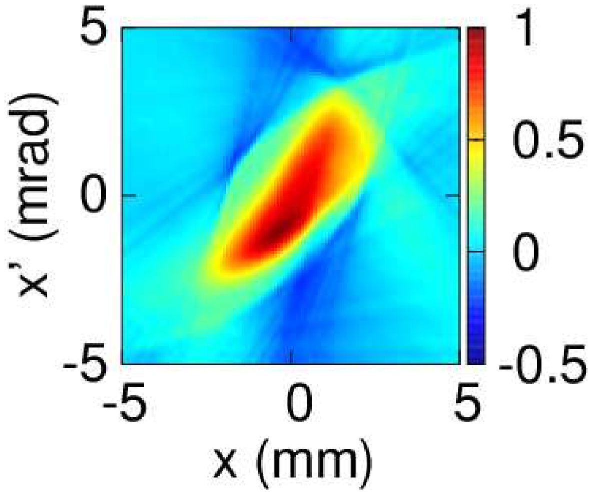
Removing streaking artefacts

Streaking artefacts appear when either the range of angles or the number of angles is insufficient. They tend to confuse the real signal.

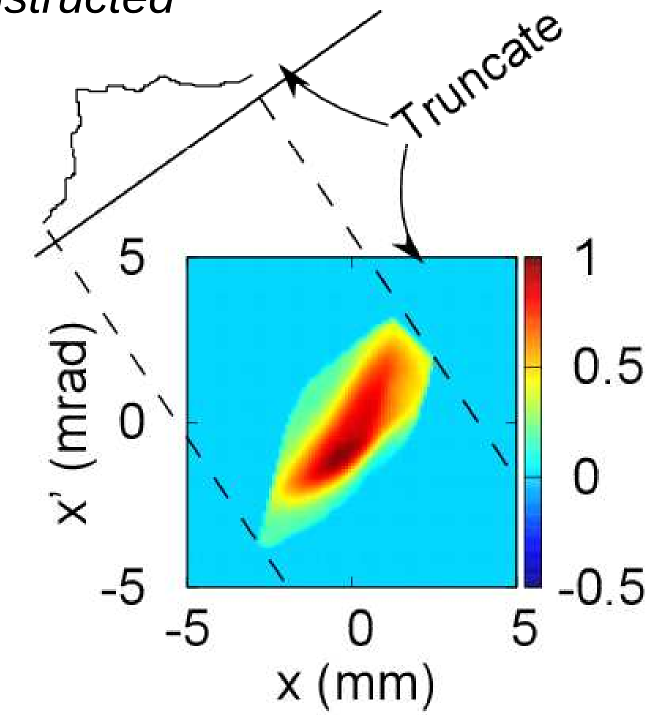
A simple way to remove them is to impose a threshold to each projection (e.g. 10% of peak value). Below this threshold, the projection is truncated.

The corresponding region in the reconstructed phase space is then set to zero.

Quad scan – angle range 160^{o}



measurements



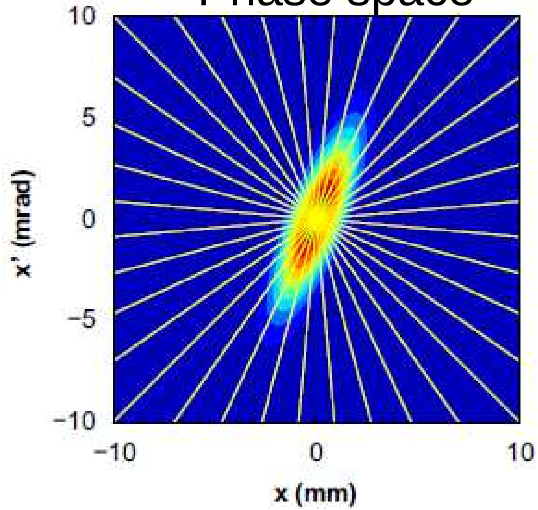
Angle sampling problem

Phase space distributions tend to be long and narrow because of long drift spaces. Angles along distribution may not be optimally sampled.

In normalised phase space where distribution looks more circular, the corresponding intervals are not uniform.

This could affect the quality of the reconstructed image.

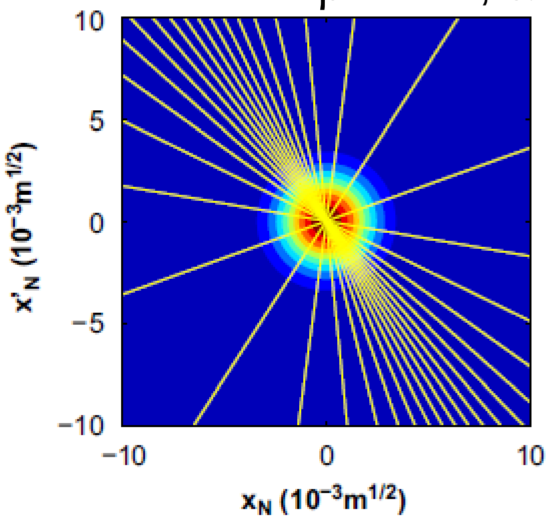
Phase space



$$\begin{pmatrix} x_N \\ x'_N \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{\beta}} & 0 \\ \frac{\alpha}{\sqrt{\beta}} & \sqrt{\beta} \end{pmatrix} \cdot \begin{pmatrix} x \\ x' \end{pmatrix}$$

simulations

Normalised: β = 1 m, α = 0



Sampling at uniform intervals in normalised phase space may overcome this problem.

Normalised phase space

This can be implemented by combining the linear mapping to screen with the normalising matrix – during measurement and reconstruction:

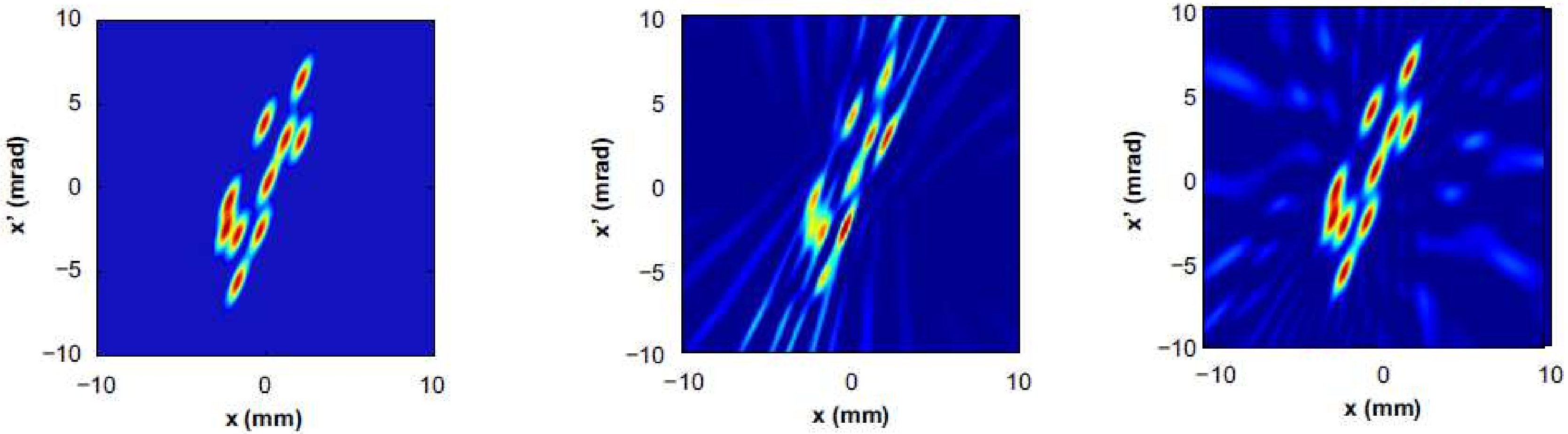
$$\begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix} \Rightarrow \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix} \cdot \begin{pmatrix} \sqrt{\beta_1} & 0 \\ -\frac{\alpha_1}{\sqrt{\beta_1}} & \frac{1}{\sqrt{\beta_1}} \end{pmatrix}$$

Additional step needed: Estimated values of the Twiss parameters must be obtained first (e.g using quadrupole scan).

Hypothetical distribution

Reconstructed from uniform angle intervals in real phase space

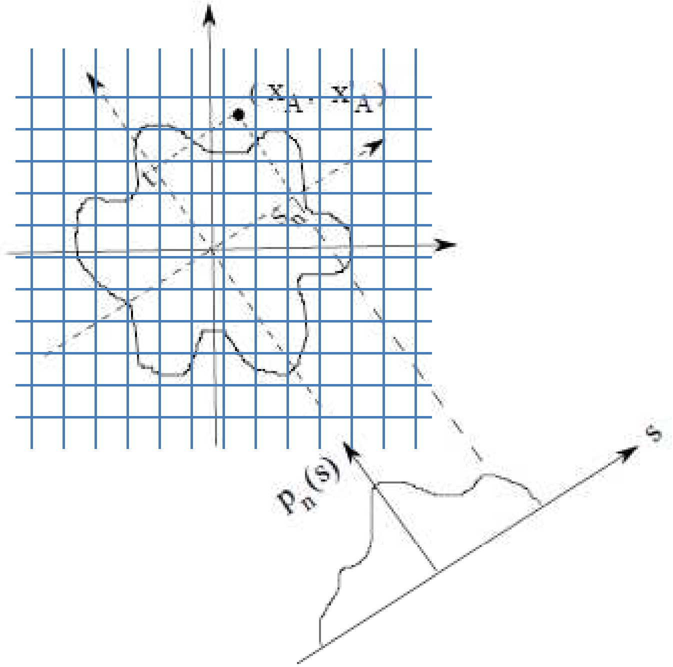
Reconstructed from uniform angle intervals in normalised phase space



simulations

Simulations suggest that this could give better results.

Maximum Entropy Technique



If we have only a few projections, MENT can give us the most likely answer:

Problem: To find the distribution in which the particles can have the greatest number of arrangements among square elements.

Constraints: Projections from this distribution must agree with the measured projections. Particle number is constant.

Using Lagrange multiplier, the solution obtained is:

A product of functions in which each function is a function of one variable.
This variable must be a coordinate of a measured projection (s in the figure).

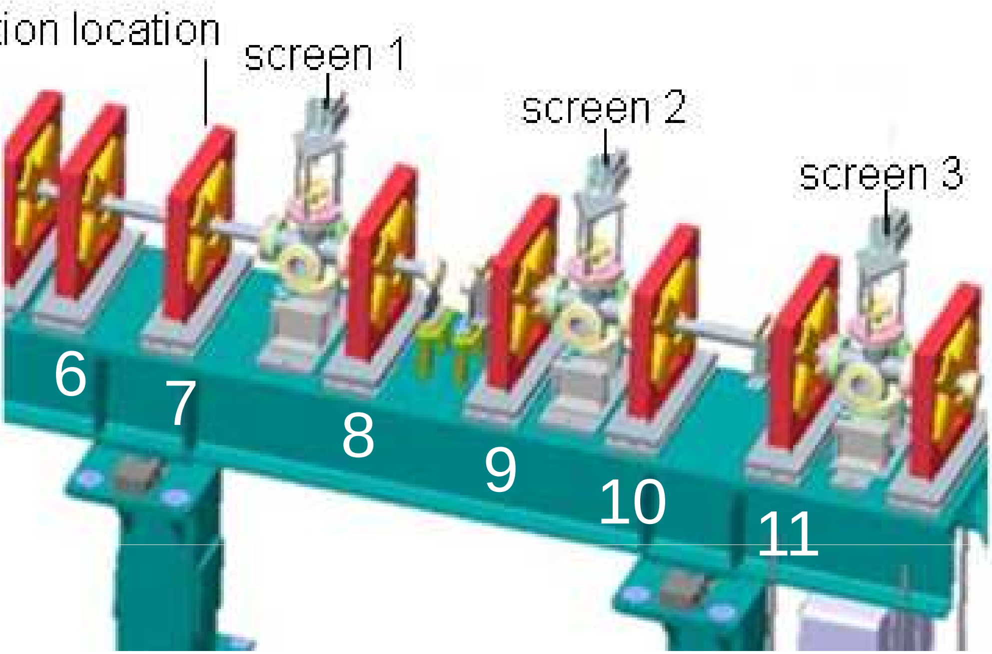
With this information and the measured projections, the distribution can be computed iteratively.

3-screen measurement

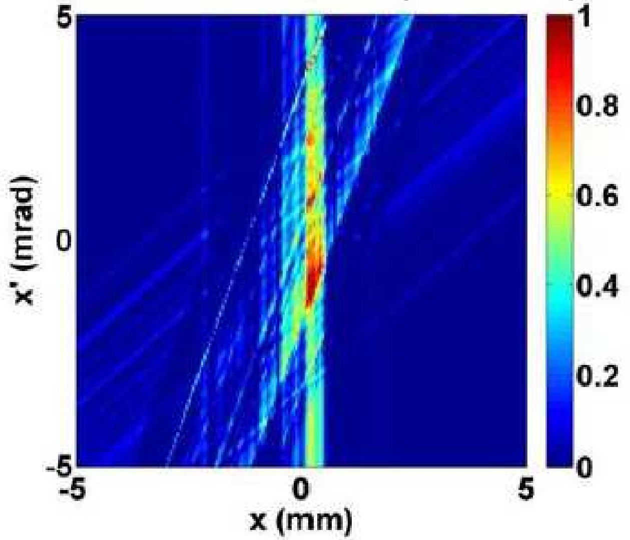
With only 3 angles, FBP result is clearly unphysical.

MENT appears to produce a much better result than FBP.

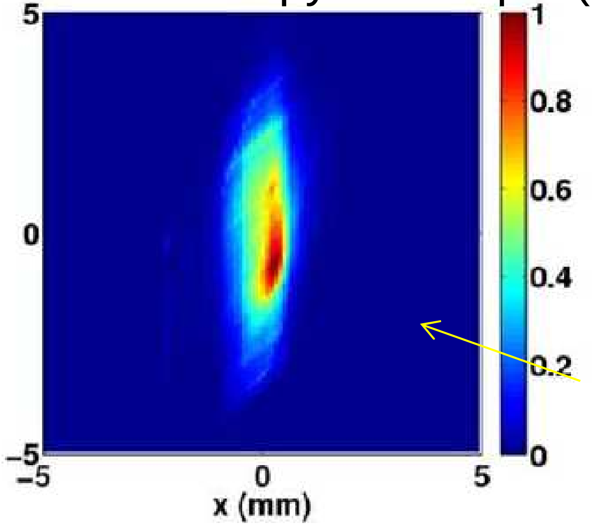
But with so little information, is MENT really better?



Filtered Back Projection (FBP)



Maximum Entropy Technique (MENT)



Measurements

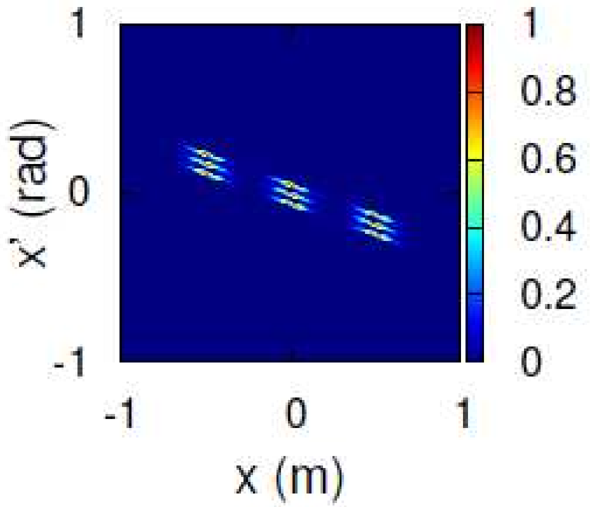
FBP artefact removal technique inspired by MENT

Problem and improvement

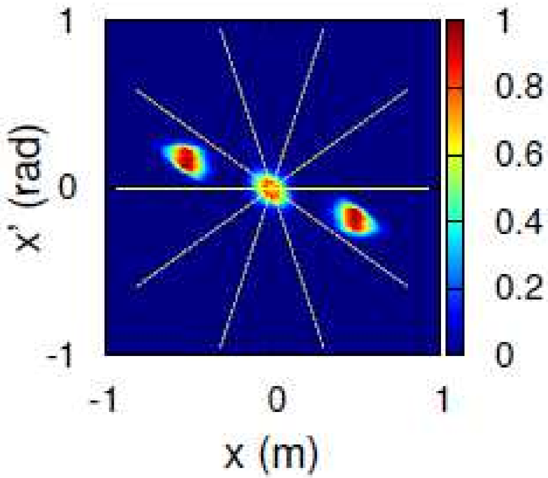
Simulations show that sampling real phase space using uniform angle intervals could produce distorted results for MENT.

Sampling at equal intervals in normalised phase space could improve reconstruction.

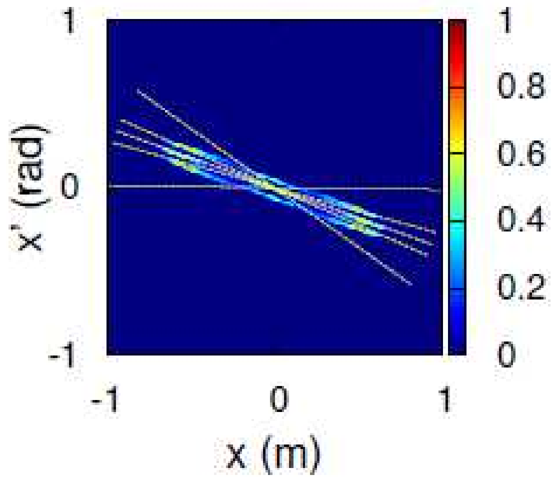
Hypothetical distribution



MENT recontruction with 5 angles



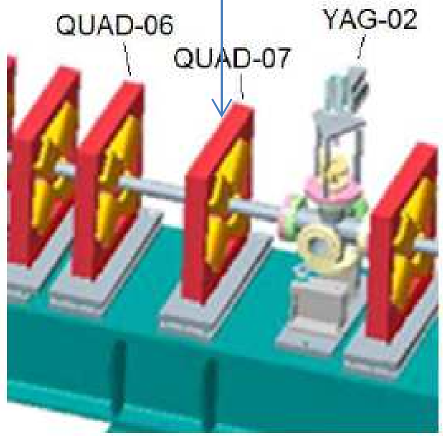
Equal angle intervals in normalised space



Simulations

Test on measured data

Original from quad scan



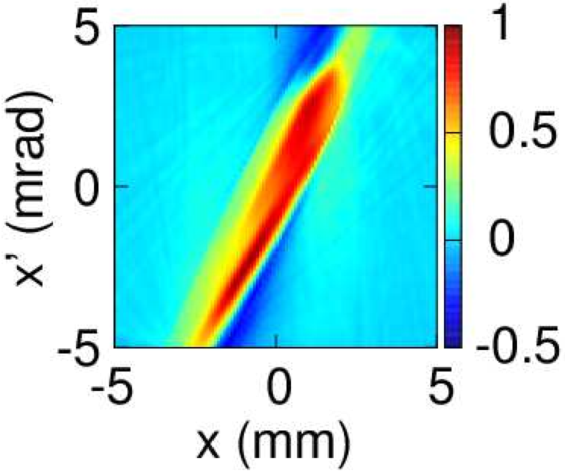
ALICE tomography section

An FBP reconstruction measured with 159 angles is taken as the original distribution.

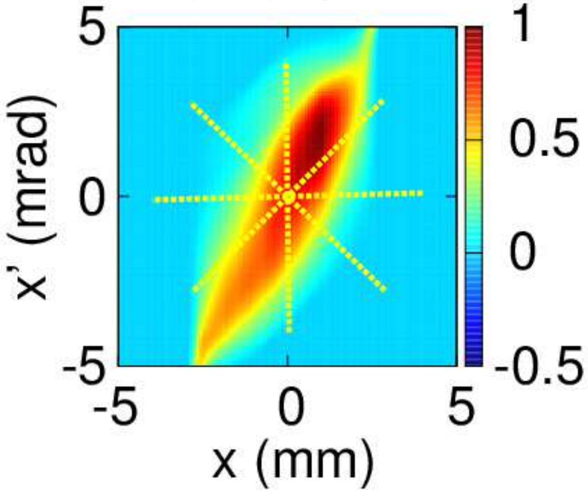
To reconstruct with MENT, select 4 angles at equal intervals in:

1. real phase space - clearly distorted
2. normalised phase space - better agreement

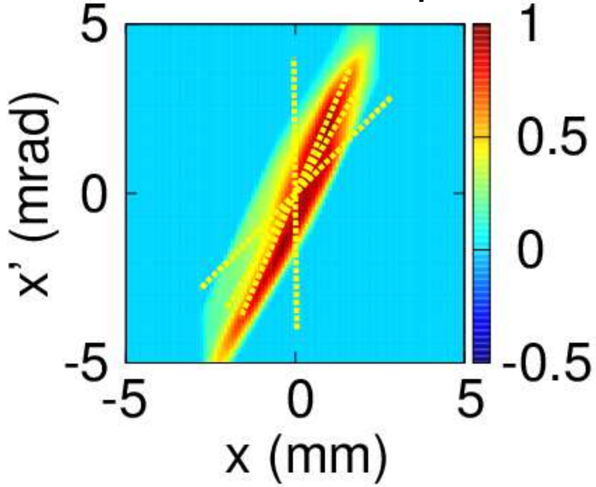
"Original" (actually FBP result)



Equal angle intervals in (x,x') space



Equal angle intervals in normalised space

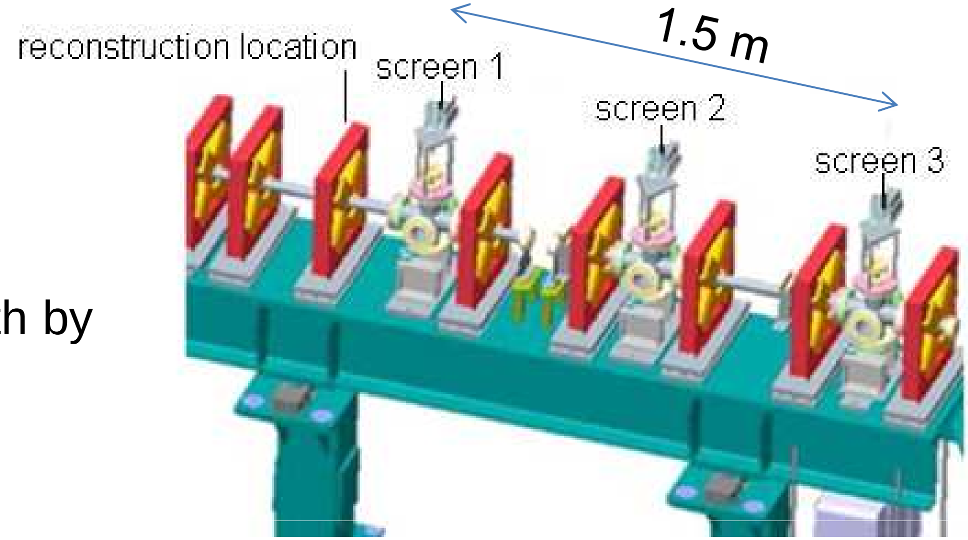


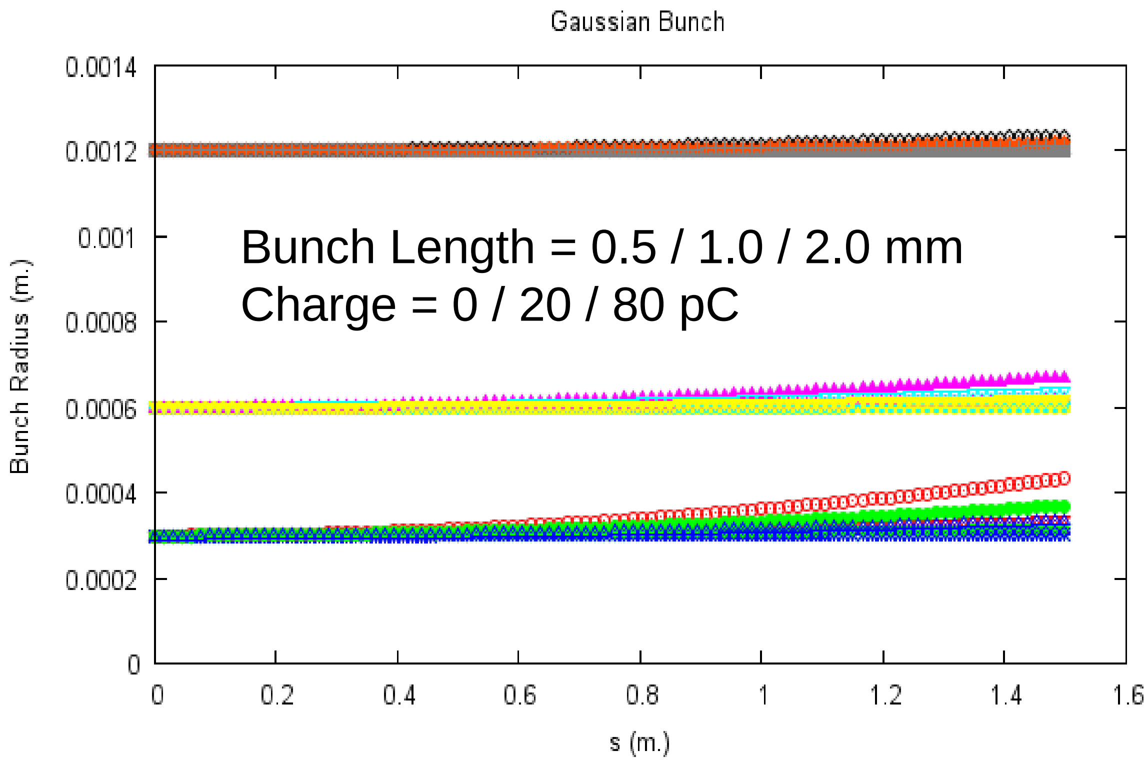
Measurements

Search for space charge

Analytic calculations with Gaussian beam, assuming drift space only shows:

Typical ALICE beam likely to expand in width by a few per cent over a 1.5 m drift space.





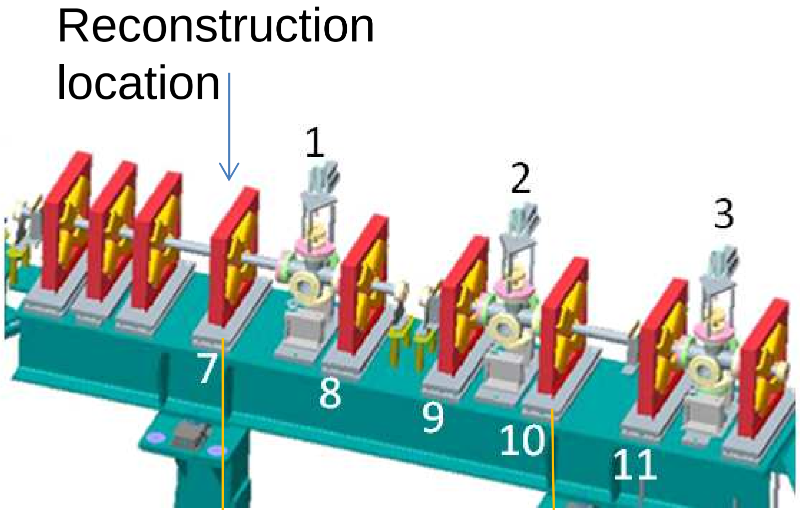
Suggests that it may be possible to observe space charge effect in the ALICE tomography section.

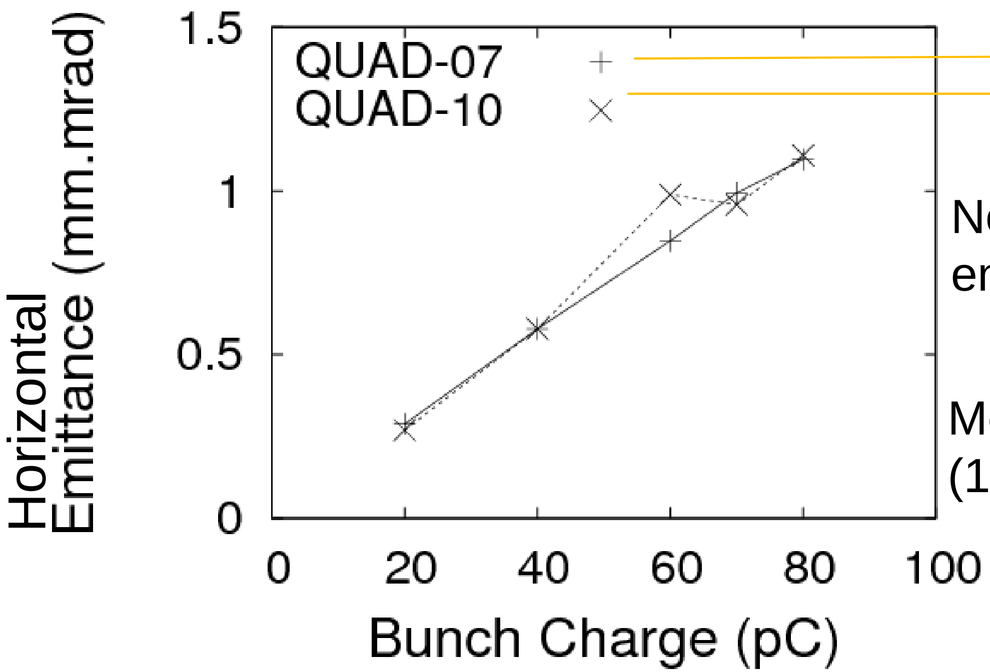
Search for space charge

Method:

1. Quadrupole scan at screen 1.
2. Quadrupole scan at screen 3.
3. Reconstruct both data at same location.

They should give the same results.



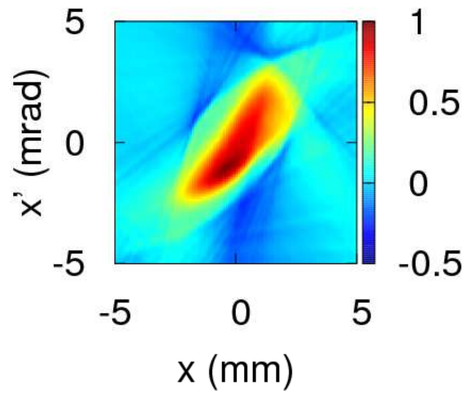


No consistent difference in measured emittances observed.

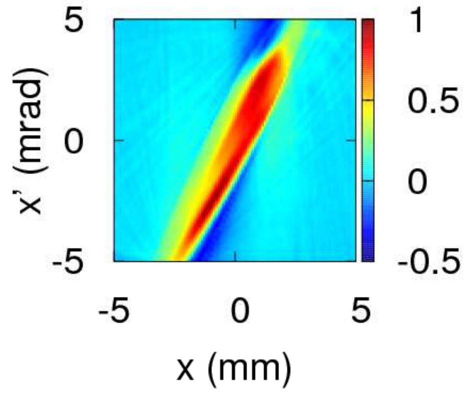
Measurements
(12 MeV, bunch length ~ 1.2 mm)

Search for space charge

From Screen 1

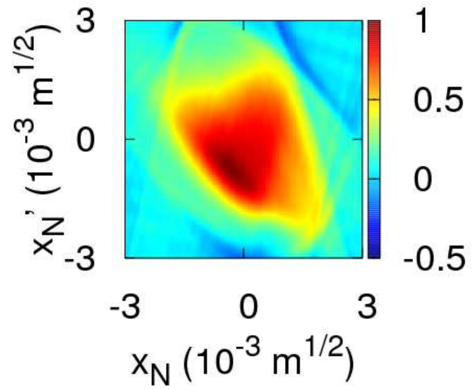


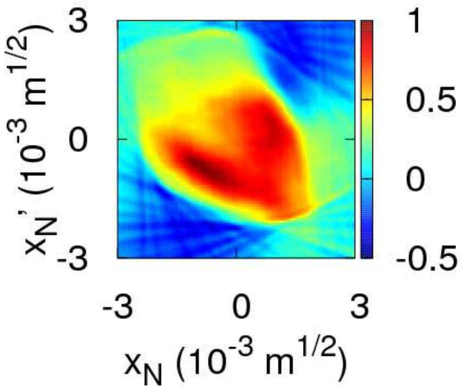
From Screen 3



Reconstructions look different.

Normalised Phase Space





In normalised phase space - shapes look similar but rotated.

This suggests linear effects. Space charge is a possible explanation.

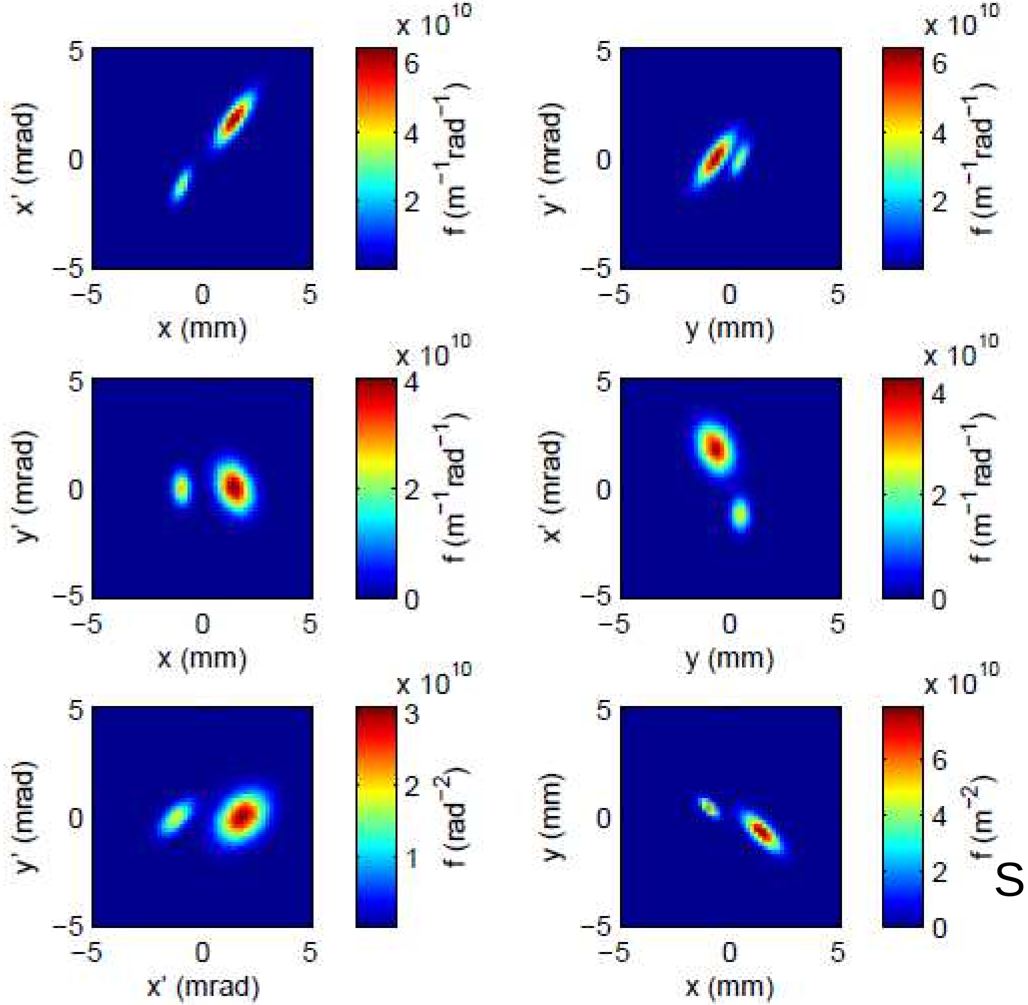
Measurements

4D phase space tomography

Standard phase space tomography works for 2D phase space (x,x'), (y,y') or (z,δ) only.

Is it possible to measure and reconstruct a full 4D phase space like (x,x',y,y')?

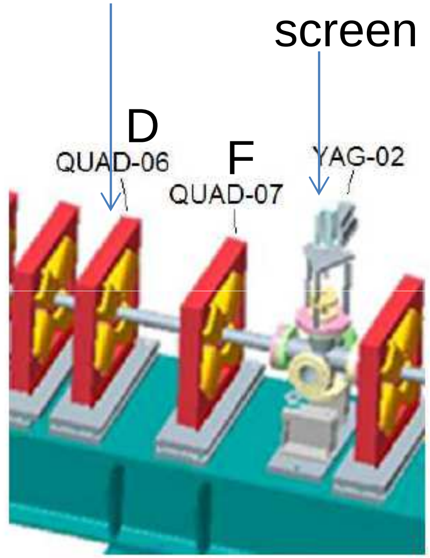
Projections from hypothetical 4D distribution.



Simulations

4D measurement proposed

Reconstruction location



ALICE tomography section

1. Scan a focussing quadrupole (F) to rotate in (x,x'). Keep the (y,y') angle fixed by adjusting a defocussing quad (D).

2. For each (y,y') angle, record the images for different (x,x') angles.

3. Process the recorded images to reconstruct the 4D phase space.

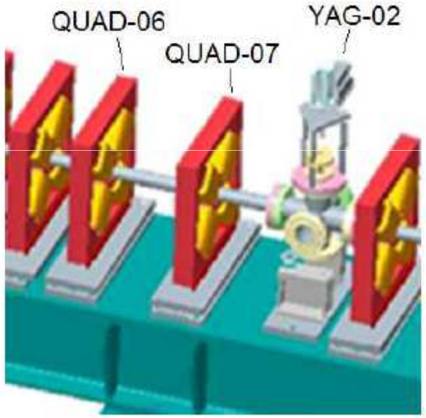
Details, equations and algorithm are given in:

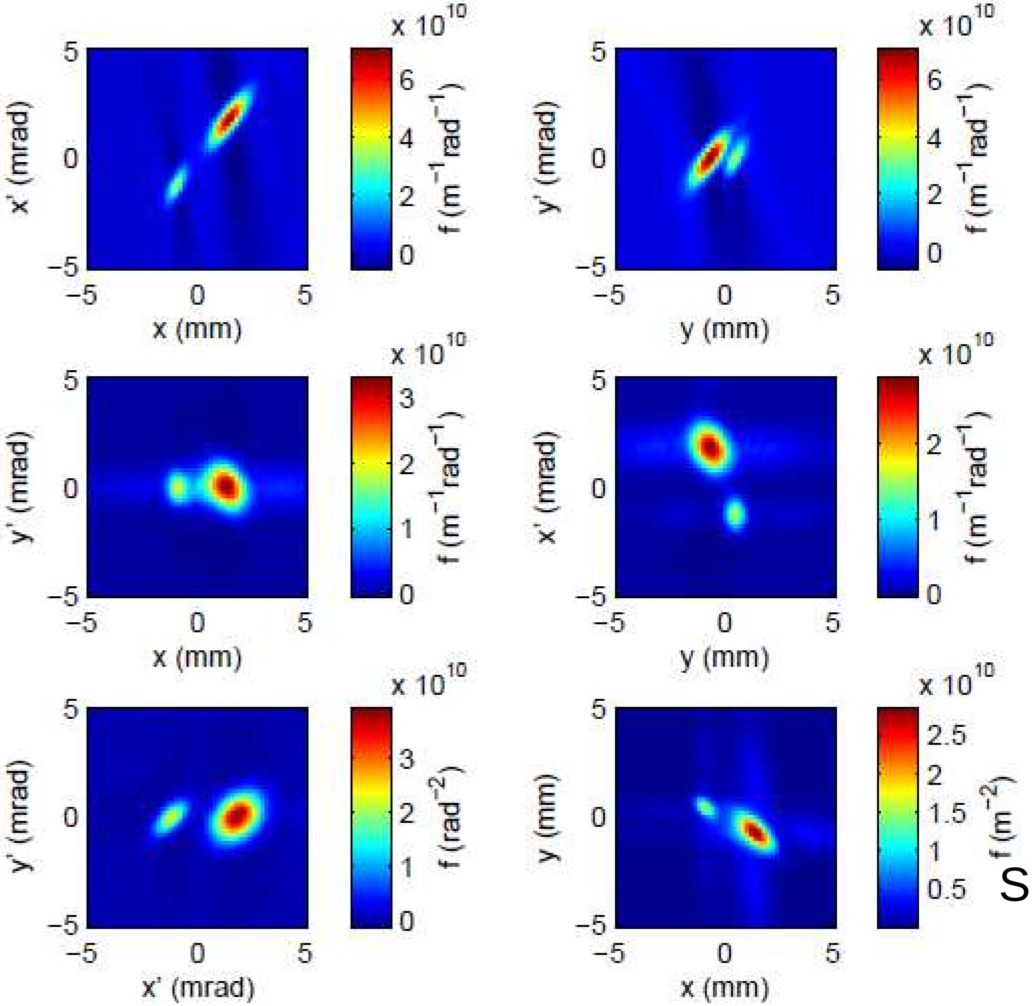
K. M. Hock and A. Wolski. "Tomographic Reconstruction of the Full 4D Transverse Phase Space", NIM A, in press (2013).

Simulation of a measurement

Assuming 2 typical quadrupoles in ALICE, a 4D measurement is simulated.

Angle ranges are limited to 150º/160º – reconstruction still shows good agreement.





Simulations

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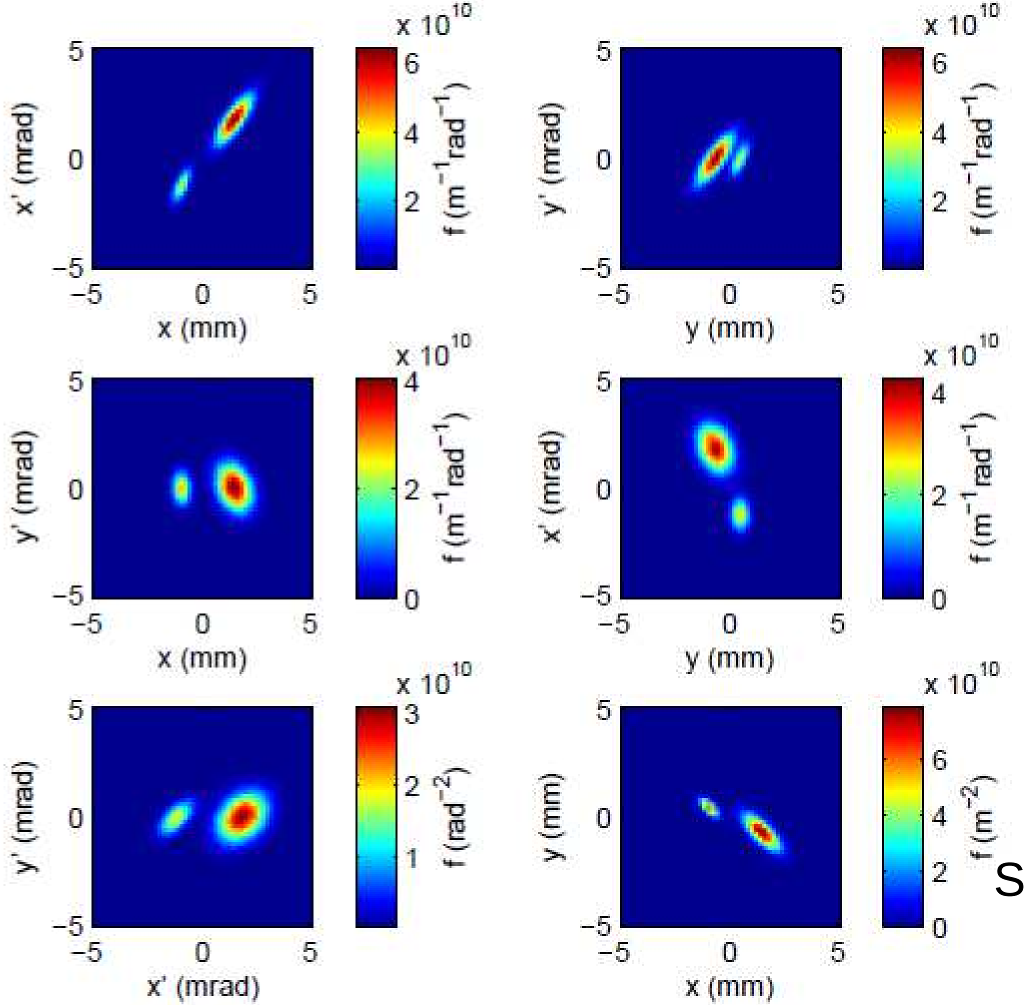

Simulations

Conclusions

1. Set up and demonstrated phase space measurement at ALICE.

2. Demonstrated improvements using normalised phase space technique.

3. Preliminary result on space charge studies not yet conclusive.

4. Developing method for 4D reconstruction of full transverse phase space.

Publications

K. M. Hock, M. G. Ibison, D.J. Holder, A. Wolski, B.D. Muratori, "Beam tomography in transverse normalised phase space," Nuclear Instruments and Methods A, vol. 642, (2011), pp. 36-44.

M. G. Ibison, K. M. Hock, D. J. Holder, B. D. Muratori and A. Wolski, "ALICE Tomography Section: Measurements and Analysis," Journal of Instrumentation, 7, P04016 (2012).

K. M. Hock and M. G. Ibison, "A study of the Maximum Entropy Technique for phase space tomography", Journal of Instrumentation, 8, P02003 (2013).

K. M. Hock and A. Wolski, "Tomographic Reconstruction of the Full 4D Transverse Phase Space", Nuclear Instruments and Methods A, in press (2013).

Thank you