LONGITUDINAL STABILITY OF MULTITURN ERL WITH SPLIT ACCELERATING STRUCTURE

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Abstract

Some modern projects of the new generation light sources use the conception of multipass energy recovery linac with split (CEBAF-like) accelerating structures [1 - 4]. One of the advantages of these light sources is the possibility to obtain a small bunch length. To help reduce it, the longitudinal dispersion should be non-zero in some arcs of the accelerator. However small deviations in voltages of the accelerating structures can be enhanced by induced fields from circulating bunches due to the dependence of the flight time on the energy deviation and the high quality factor of the superconducting radio-frequency cavities. Therefore, instabilities caused by interaction of electron bunches and fundamental modes of the cavities can take place. The corresponding stability conditions are discussed in this paper. Numerical simulations were performed for two projects – MARS [4] and FSF [3].

INTRODUCTION

The proposed scheme of the fourth generation light source project based on multiturn accelerator-recuperator with two accelerating structures is shown in Fig. 1.

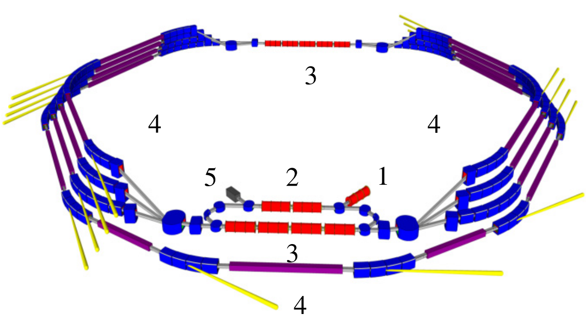

Figure 1: Scheme of MARS - ERL with two linacs.

Electrons from injector 1 pass through the preliminary accelerating structure 2 to the main accelerator. Obtaining necessary energy in the main linacs 3 electrons are used in undulators 4. After deceleration, electrons are dropped to the dump 5.

There are four electron beams in each main linac simultaneously. Each beam induces large voltage in the linac, but the sum is not so large. If the phases of the beams vary, the accelerating voltages also change, and initially small phase deviation may increase due to the dependence of flight times through arcs on the particle energy. This longitudinal instability is considered in our paper.

THEORY

To simplify the picture, consider each linac as one RF cavity. Taking the effective voltage on the linac with number α in the form $\mathrm{Re}(U_\alpha e^{-i\omega t})$ (ω is the frequency of the RF generator), one obtains:

$$\frac{2}{\omega}\frac{dU_\alpha}{dt} = \frac{i\xi_\alpha - 1}{Q_\alpha}U_\alpha + \rho_\alpha(I_{b\alpha} + I_{g\alpha}), \qquad (1)$$

where $\omega_\alpha = 1/\sqrt{L_\alpha C_\alpha} = (1 - \xi_\alpha / 2Q_\alpha)\omega$ is the resonant frequency, $Q_\alpha = R_\alpha \big/ \sqrt{L_\alpha / C_\alpha}$ >>1 is the loaded quality of the cavity, $\rho_\alpha = R_\alpha / Q_\alpha = \sqrt{L/C}$ and R_α are the characteristic and the loaded shunt impedances for the fundamental (TM_{010}) mode, $I_{g\alpha}$ and $I_{b\alpha}$ are the complex amplitudes of the beam and (reduced to the gap) generator currents correspondingly. We are interested in the case of constant $I_{g\alpha}$. The beam currents $I_{b\alpha}$ depend on all U_α due to the phase motion. Linearization of Eq. (1) near the stationary solution gives:

$$\frac{2}{\omega}\frac{d\delta U_\alpha}{dt} = \frac{i\xi_\alpha - 1}{Q_\alpha}\delta U_\alpha + \qquad (2)$$

$$+\rho_\alpha \sum_\beta \left(\frac{\partial I_{b\alpha}}{\partial \mathrm{Re} U_\beta} \mathrm{Re}\,\delta U_\beta + \frac{\partial I_{b\alpha}}{\partial \mathrm{Im} U_\beta} \mathrm{Im}\,\delta U_\beta \right)$$

Considering the exponential solutions $\exp(\omega\lambda t/2)$ of system of linear differential equations Eq. (4), one can find the stability conditions. Indeed, the system Eq. (2) corresponds to the system of the linear homogeneous equations $\lambda\delta\mathbf{U} = \mathbf{M}\delta\mathbf{U}$ with the consistency condition $|\mathbf{M} - \lambda\mathbf{E}| = 0$. Then Re($\lambda$) < 0 for all roots of the last equation (i. e., eigenvalues of the matrix **M**) is the stability condition. The explicit expression for **M** is

$$\mathbf{M} = \begin{pmatrix} \rho_1 \frac{\partial \mathrm{Re} I_{b1}}{\partial \mathrm{Re} U_1} - \frac{1}{Q_1} & \rho_1 \frac{\partial \mathrm{Re} I_{b1}}{\partial \mathrm{Im} U_1} - \frac{\xi_1}{Q_1} & \rho_1 \frac{\partial \mathrm{Re} I_{b1}}{\partial \mathrm{Re} U_2} & \rho_1 \frac{\partial \mathrm{Re} I_{b1}}{\partial \mathrm{Im} U_2} \\ \rho_1 \frac{\partial \mathrm{Im} I_{b1}}{\partial \mathrm{Re} U_1} + \frac{\xi_1}{Q_1} & \rho_1 \frac{\partial \mathrm{Im} I_{b1}}{\partial \mathrm{Im} U_1} - \frac{1}{Q_1} & \rho_1 \frac{\partial \mathrm{Im} I_{b1}}{\partial \mathrm{Re} U_2} & \rho_1 \frac{\partial \mathrm{Im} I_{b1}}{\partial \mathrm{Im} U_2} \\ \rho_2 \frac{\partial \mathrm{Re} I_{b2}}{\partial \mathrm{Re} U_1} & \rho_2 \frac{\partial \mathrm{Re} I_{b2}}{\partial \mathrm{Im} U_1} & \rho_2 \frac{\partial \mathrm{Re} I_{b2}}{\partial \mathrm{Re} U_2} - \frac{1}{Q_2} & \rho_2 \frac{\partial \mathrm{Re} I_{b2}}{\partial \mathrm{Im} U_2} - \frac{\xi_2}{Q_2} \\ \rho_2 \frac{\partial \mathrm{Im} I_{b2}}{\partial \mathrm{Re} U_1} & \rho_2 \frac{\partial \mathrm{Im} I_{b2}}{\partial \mathrm{Im} U_1} & \rho_2 \frac{\partial \mathrm{Im} I_{b2}}{\partial \mathrm{Re} U_2} + \frac{\xi_2}{Q_2} & \rho_2 \frac{\partial \mathrm{Im} I_{b2}}{\partial \mathrm{Im} U_2} - \frac{1}{Q_2} \end{pmatrix}.$$

The sufficient conditions are given by the Liénard-Chipart criterion [5]. It requires the positivity of the coefficients of the characteristic equation

$$\lambda^4 - S_1\lambda^3 + S_2\lambda^2 - S_3\lambda + S_4 = 0 \qquad (3)$$

and the third Hurwitz minor: $S_1 < 0,\ S_2 > 0,\ \ S_4 > 0$, $\Delta_3 = S_1(S_2S_3 - S_1S_4) - S_3^2 > 0$.

Coefficients of the matrix $\partial I_{bi}/\partial U_j$ can be found from dependence of beam current amplitude on phase deviations

$$I_{b\alpha} = -2I\sum_{n=0}^{N-1}\left(e^{i\varphi_{2n+\alpha-1}+i\psi_{2n+\alpha-1}} + e^{i\varphi_{4N-2n-\alpha}+i\psi_{4N-2n-\alpha}}\right),$$

where φ_n - reference phase of the n-th arc and $\psi_n = e\sum_{k=0}^{n-1} S_{nk}\,\mathrm{Re}\left[\delta U_{\alpha(k)}e^{-i\varphi_k}\right]$ - its phase deviation, $\alpha(k)$ - number of the linac after the the arc k. $S_{n,k}$ is the longitudinal sin-like trajectories. $S_{n,k}$ and its derivation $S'_{n,k}$ are the solutions of the homogeneous system of equations with the initial conditions $S_{k,k} = 0$, $S'_{k,k} = 1$:

$$S'_{n+1,k} = S'_{n,k} + e\,\mathrm{Im}\left[U_{0\alpha(n)}e^{-i\varphi_n}\right]S_{n,k} \qquad (4)$$

$$S_{n+1,k} = S_{n,k} + \omega\left(\frac{dt}{dE}\right)_{n+1} S'_{n+1,k} \qquad (5)$$

In the simplest case of the isochronous arcs the conductivity matrix is zero. Then it is easy to proof, that all stability conditions are satisfied.

For the longitudinal stability it also needs to have longitudinal focusing for most of passes through the linac. It means, that bunch should be accelerated and decelerated on the same slope of the voltage curve. If $(dt/dE)_n > 0$, it can be expressed as $\varphi_{4N-n-1} = \pi - \varphi_n + 2\arg\left(eU_{0\alpha(n)}\right)$ for $n < 2N$.

To make the stability conditions more explicit, let us consider a simple example. Assume, that φ_{2n}-arg(eU_{01}) = Φ_1, and φ_{2n+1}-arg(eU_{02}) = Φ_2 for $n < N$. Then for high enough detunings ξ_α the threshold current is

$$I_{th} < \frac{1/Q_1 + 1/Q_2}{e\rho_1\sin(2\Phi_1)\sum_{n=0}^{N-1}\sum_{k=0}^{N-1}S_{4N-2n-1,2k} + e\rho_2\sin(2\Phi_2)\sum_{n=0}^{N-1}\sum_{k=0}^{N-1}S_{4N-2n-2,2k+1}}$$

More detail description of the theory was presented in [6].

SIMULATIONS

The proposed scheme of the multiturn ERL has also the preliminary acceleration/deceleration system to reduce beam induced radiation and RF power consumption. For this case, it is necessary to make numerical simulations.

Considering the bunch as the point with charge q one can calculate it induced voltage

$$V_{\parallel}(t) = q\frac{\omega_r R_s}{Q}\cos(\omega_r t)\exp\left(-\frac{\omega_r}{2Q}t\right)H(t),$$

where ω_r is the resonance frequency of the cavity, R_s is the shunt impedance, Q is the loaded quality factor, $H(t)$ is Heaviside's step-function. Each bunch induces the voltage on the cavity resonance frequency. As the recirculating frequency of the generator is close to resonance, but not equal, electrons will see the phase of induced voltage by previous bunch slightly changed. The total voltage is the superposition of induced voltages with appropriate phases.

For each point-like bunch the energy phase system is

$$\Delta E_n = \Delta E_{n-1} + e\Delta U(\delta\phi_n) + eW_{n-1}(\delta\phi_n)$$

$$\delta\phi_n = \delta\phi_{n-1} + \frac{\omega_g}{c}(R_{56})_n\frac{\Delta E_n}{E_n}$$

where $\Delta U(\delta\phi_n)$ is additional energy received by electron with phase deviation $\delta\phi_n$, $W_{n-1}(\delta\phi_n)$ - the induced voltage by previous *n-1* turns is seen by electron.

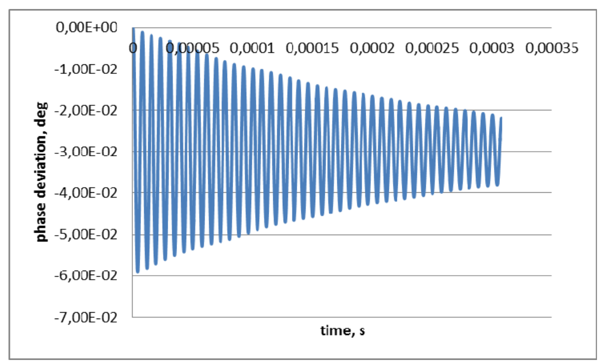

Figure 2: Example of the relaxation of the bunch's phase after the last deceleration to the equilibrium value.

Simulations start with filling the accelerator trajectory by array of bunches without initial deviation. Bunches with appropriate numbers interact with cavities. After that, the unperturbed bunch is injected in the facility and the last one goes to the dump. An example of the phase to time dependence is shown in Fig. 2. The deviation of the bunch phase is decreasing exponentially.

MARS STRUCTURE

Numerical calculations were made for proposed structure of MARS (the scheme is shown in Fig. 1). Parameters of the main accelerating structures are: $Q_1 = Q_2 = 10^6$, $\rho_1 = 40\,\mathrm{k\Omega}$, $\rho_2 = 90\,\mathrm{k\Omega}$, $\omega = 2\pi\cdot 1.3\cdot 10^9\,\mathrm{Hz}$, $U_1 = 0.8\,\mathrm{GV}$, $U_2 = 1.8\,\mathrm{GV}$. The transport matrix elements $R_{56} \sim 1\,\mathrm{m}$ at all arcs. The dependence of the threshold currents calculated by stability condition (3) and by numerical simulation on accelerating phases $\Phi_1 = \Phi_2$ is shown on Fig. 3.

ISBN 978-3-95450-122-9

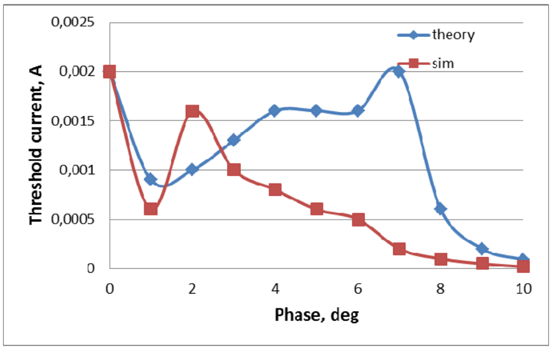

Figure 3: MARS threshold current for equal accelerating phases. $\xi_1 = \xi_2 = 10$.

Preliminary accelerating system consist of two linacs with energy gain 350 MeV and 40 MeV. Parameters of the linacs are:

$$Q_{in1} = Q_{in2} = 10^6,\ \rho_{in1} = 2\ \text{k}\Omega,\ \rho_{in2} = 15\ \text{k}\Omega.$$

On the Fig. 4 is shown the comparison of threshold currents calculated in the three cases: by stability condition (3), by numerical simulation without preliminary acceleration and with preliminary accelerating structure.

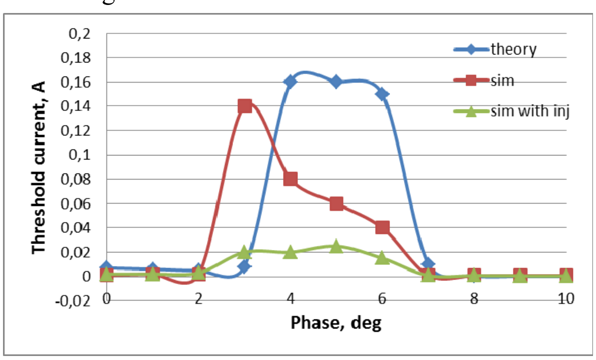

Figure 4: MARS threshold current for equal accelerating phases. $\xi_1 = \xi_2 = 1000$.

FSF STRUCTURE

The Femto-Science Factory [3] (Fig. 5) is another fourth generation light source project.

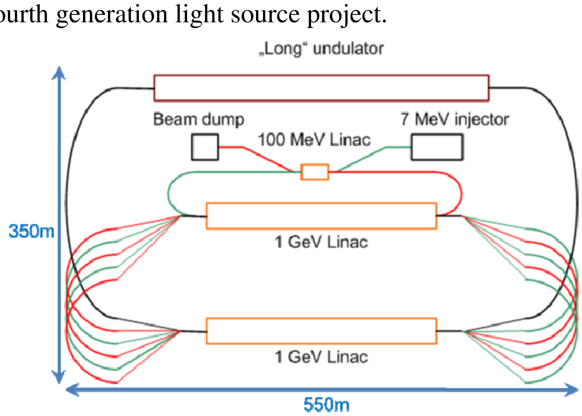

Figure 5: Scheme of Femto-Science Factory (FSF)

For the longitudinal stability, the main difference of FSF from MARS is much lower longitudinal dispersion; the first two arcs have small transport matrix elements ($R_{56,1} = 0.2$ m, and $R_{56,2} = 0.05$ m) in the short pulse mode [3]. Parameters of accelerating structures: $Q_1 = Q_2 = 10^6$, $\rho_1 = 50$ kΩ, $\rho_2 = 50$ kΩ, $\omega = 2\pi \cdot 1.3 \cdot 10^9$ Hz, $U_1 = U_2 = 1$ GV. The dependence of threshold current on accelerating phases is shown on Fig. 6.

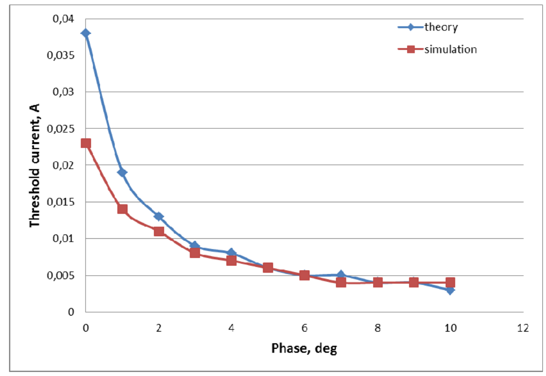

Figure 6: FSF Threshold current for equal accelerating phases. $\xi_1 = 100$, $\xi_2 = -100$.

CONCLUSION

In this paper we considered the criterion of the longitudinal stability for the ERL with two accelerating structures. Numerical simulations were made for two light source projects based on multiturn ERLs. The simulated threshold current is lower than the theoretical lower limit of the threshold current. To increase the threshold current, it is necessary to develop a proper feedback system. Such scheme was proposed in paper [7].

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