

APPLICATION OF Z-TRANSFORM TO NOISE RESPONSE MODELING OF A BUNCH-BY-BUNCH FEEDBACK SYSTEM*

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Abstract

The APS storage ring has a beam energy of 7 GeV and a single current of up to 16 mA. Transverse beam instability is controlled with combination of chromatic correction and the bunch-by-bunch feedback system [1]. Noises in the pickup circuits of the feedback system are processed and transferred to the beam, and contribute to beam motions when the loops are closed. By analysis of the input data stream of the feedback system, we can passively obtain useful information, such as the tunes, loop stability, noise spectrum, etc. This approach has been reported by J. Klute and D. Teytelman [2, 3]. We implemented a passive tune monitoring application at the APS storage ring. In order to understand the underlying principle, we applied z-transform analysis to the noise-response model of a bunch-by-bunch feedback system. Our analysis shows a direct relationship between the spectrum of the noise response and the open-loop response of the beam. Other applications of the noise-response model include assessing the stability of feedback loop, and identifying noise sources in such feedback systems. This report presents our analysis and some experimental data.

NOISE RESPONSE MODEL OF A BUNCH-BY-BUNCH FEEDBACK SYSTEM

Figure 1 shows a block diagram of the noise response model of a feedback system. Beam motion is detected by the pickup stripline (PU). Pickup noises and detected beam signal superpose together as the input to the system. The processed signal is sent to the amplifiers and kicker stripline (KICK) to drive the beam.

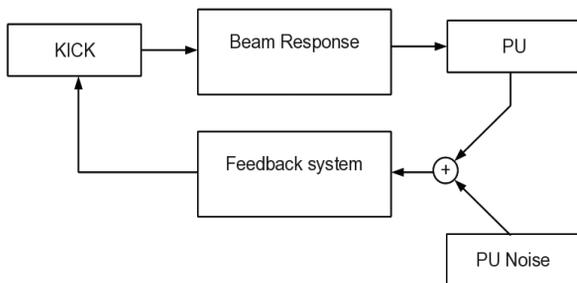


Figure 1: A block diagram of the P0 feedback system noise model.

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In the time domain beam response can be expressed with one-turn matrix as

$$\begin{bmatrix} x(n) \\ y(n) \end{bmatrix} = R \begin{bmatrix} x(n-1) \\ y(n-1) \end{bmatrix} + \begin{bmatrix} 0 \\ \delta(n) \end{bmatrix}, \quad (1)$$

where $x(n)$, $y(n)$, and $\delta(n)$ are beam position, slope, and kick, respectively applied to the beam on the n^{th} turn. M is the one-turn matrix of the ring lattice. The output of the feedback process to the KICK is expressed as

$$\delta(n) = g \sum_{k=1}^M c_{FB}(k) [x(n-k) + w(n-k)], \quad (2)$$

where $c_{FB}(n)$ are FIR filter coefficients of a feedback system, g is the gain factor, and $w(n)$ represents PU noises.

Replacing the KICK term in Eq. (1), we have the following difference equation:

$$\begin{aligned} x(n) &= r_{11}x(n-1) + r_{12}y(n-1) \\ y(n) &= r_{21}x(n-1) + r_{22}y(n-1) \\ &+ g \sum_{l=1}^M c_{FB}(k) [x(n-k) + w(n-k)]. \end{aligned} \quad (3)$$

with the solution

$$\begin{aligned} x(n) &= (r_{11} + r_{22})x(n-1) - x(n-2) \\ &+ r_{12}g \sum_{k=1}^M c_{FB}(k)x(n-k-1) \\ &+ r_{12}g \sum_{k=1}^M c_{FB}(k)w(n-k-1). \end{aligned} \quad (4)$$

This shows that beam response has IIR character.

APPLICATION OF Z-TRANSFORM

Assuming that the system is fully relaxed, apply the properties of z-transform to both sides of Eq. (4):

$$F(z)X(z) = r_{12} \sum_{k=1}^M C_{FB}(k)W(z)z^{-k-1}. \quad (5)$$

The factor $F(z)$ is defined as

$$\begin{aligned} F(z) &= 1 - (r_{11} + r_{22})z^{-1} + z^{-2} \\ &- r_{12} \left(g \sum_{l=1}^M C_{FB}(k)z^{-k-1} \right). \end{aligned} \quad (6)$$

Solving Eq. (5) we have

$$X(z) = \frac{r_{12} g \sum_{k=1}^M C_{FB}(k) W(z) z^{-k-1}}{F(z)} \quad (7)$$

This is the beam response to noises in the z-transform. Similar expressions can be worked out for more general cases where the PU and KICK are at different locations. Figure 2 shows a direct-form diagram of Eq. (5).

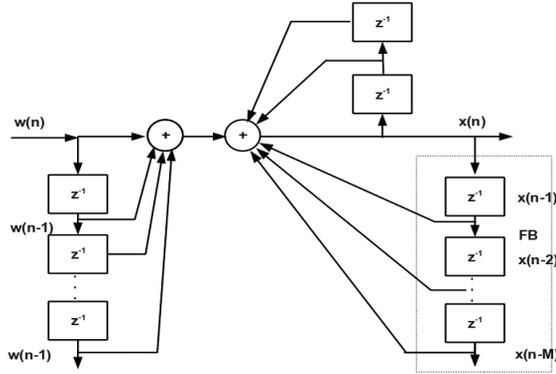


Figure 2: Direct-form diagram of noise response model for a bunch-by-bunch feedback system.

ANALYSIS OF FEEDBACK STABILITY

Replacing matrix elements in Eq. (7) with lattice functions at the PU and KICK we get

$$X(z) = R(z) W(z) \quad (8)$$

where $R(z)$ is defined as

$$R(z) = \frac{g \beta \sin 2\pi \nu z^{-1} \sum_{l=1}^M c_{FB}(k) z^{-k-1}}{P(z)}$$

and

$$P(z) = 1 - 2 \cos 2\pi \nu z^{-1} + z^{-2} - g \beta \sin 2\pi \nu z^{-1} \sum_{l=1}^M c_{FB}(k) z^{-k-1}.$$

A stable system response must be bounded output for bounded input (BOBI). A sufficient condition is that all poles of the response function $R(z)$ are within the unit circle of the z-plane. We solve the following equation for the poles of $R(z)$:

$$z^{M+1} - 2 \cos 2\pi \nu z^M + z^{M-1} - g \beta \sin 2\pi \nu \sum_{k=1}^M c_{FB}(k) z^{M-k} = 0. \quad (9)$$

We substitute the lattice functions with that of the APS storage ring model lattice, and the FIR coefficients with that of the APS feedback system, and then solve the poles. Figures 3 and 4 show x- and y-plane poles-versus-gain

plots of the APS current model lattice with different filter turn numbers in imaginary-real coordinates. The poles located close to the unit circle represent oscillation modes that are closer to instability threshold and are important to system stability. The poles close to the origin are parasitic modes that can only survive very few turns and do not affect system stability. It is clear that both turn numbers and gains play important roles in the stability. The x- and y-tunes in this case are 0.1689 and 0.2374, respectively.

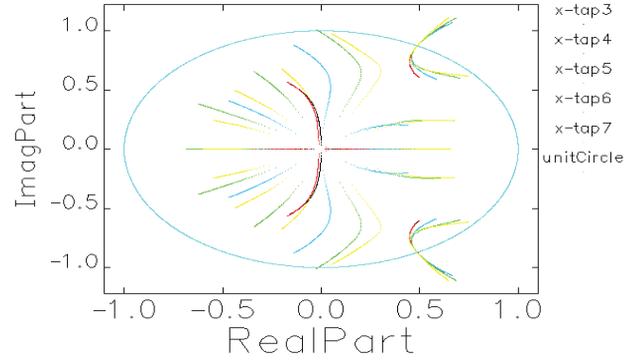


Figure 3: X-plane pole diagram of the APS storage ring feedback system.

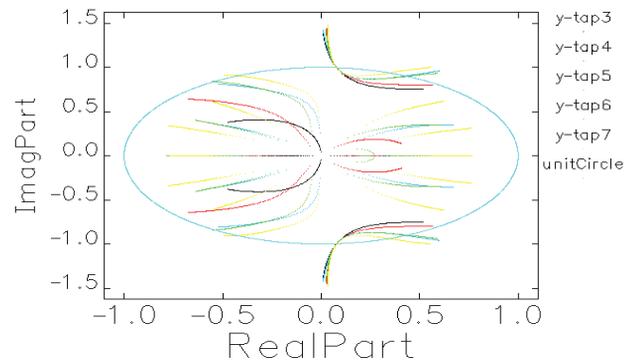


Figure 4: Y-plane pole diagram of the APS storage ring feedback system.

NOISE RESPONSE-BASED PASSIVE TUNE MEASUREMENT

The input data are superpositions of both beam response $X(z)$ and noise data $W(z)$ and are expressed as

$$U(z) = X(z) + W(z)$$

or:

$$U(z) = \frac{1 - 2 \cos 2\pi \nu z^{-1} + z^2}{P(z)} W(z) \quad (10)$$

We obtain the amplitude frequency response (R_u) by evaluating the following on unit circle $z=e^{j\omega}$:

$$R_u(\omega) = \frac{1 - 2 \cos 2\pi \nu e^{-j\omega} + e^{-2j\omega}}{P(\omega)} \quad (11)$$

and take the amplitude and inverse of both sides:

$$\frac{|R_u^{-1}(\omega)|}{|P(\omega)|} = \frac{1}{|1 - 2\cos 2\pi \nu e^{-j\omega} + e^{-2j\omega}|} \quad (12)$$

The right side is the open-loop beam response spectrum. Since an M-turn FIR filter generally has relatively broad frequency response, the effect of the $P(\omega)$ term on the location of spectrum peaks is very small. For tune measurement purposes we can simply use the R_u^{-1} term only.

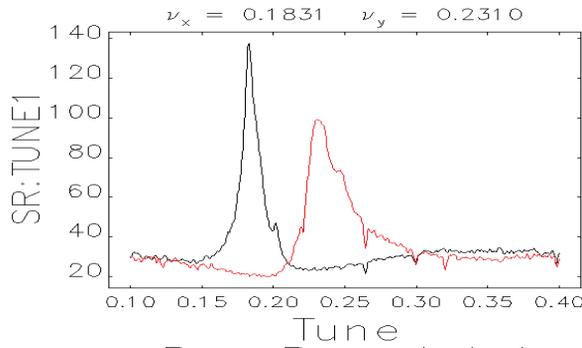


Figure 5: x- (black) and y-tunes (red) spectrum of 24-singlet fill pattern.

We developed a passive tune measurement application for the APS storage ring based on this idea and incorporated it in our data logger system. Figure 5 shows a tune plot of a 24-singlet fill with this method. Figure 6 shows a plot of monitored tunes and peaks of the APS storage ring during a user run period with RHB (reduced horizontal beta function) lattice. The logged data and waveforms help to monitor variation of both tunes and feedback loop performances. Presently the tune resolution is limited to ~ 0.0015 . We plan to increase the data record length to improve the resolution in the future.

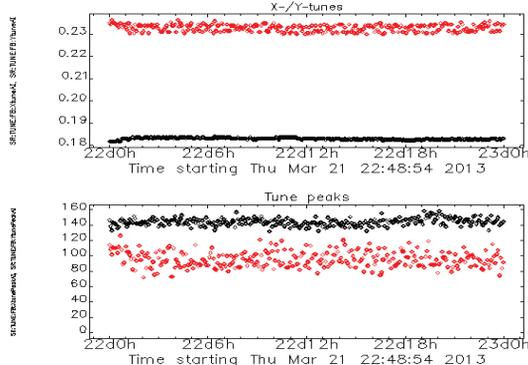


Figure 6: Logged tunes and peaks.

In the real system the detected spectrum not only contains the tune spectrum, but also internal and external noise spectra. These spectrum lines are aliased to a frequency span of revolution frequency. By scanning the rf frequency of the storage ring, which also changes the sample rate proportionally, we can deduce the real

frequency of these noises. This provides a way of identifying the noise sources. So far we have identified a 1-MHz AM modulation in the rf master synthesizer, and a 300-MHz interference from the 3rd harmonic of a crystal clock source of the FPGA board.

We used this method to measure the chromaticity of the storage ring, and the result is comparable to that of using a network analyzer with reduced loop gains. The peak amplitude fluctuates when the tunes change with rf frequency. However that does not affect the result. Figure 7 shows a plot of measured chromaticity result.

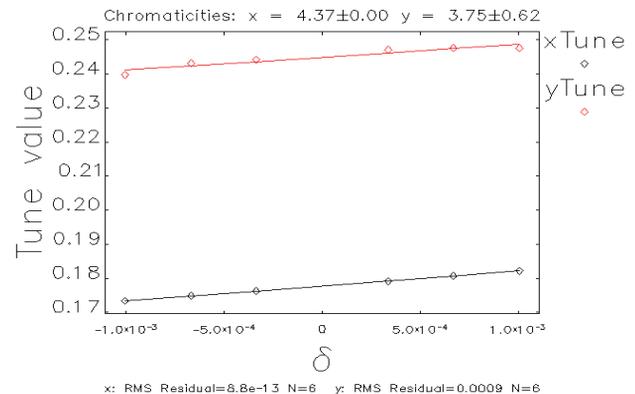


Figure 7: Chromaticity data of the APS storage ring with the passive tune measurement method.

CONCLUSION

Our analysis concludes that the tune spectrum derived from input data of a bunch-by-bunch feedback system directly reflects the open-loop transverse beam tune spectrum. A tune monitoring application has been developed at the APS storage ring to monitor both tune variations and feedback loop performances. The method can also be applied to the analysis of the stability of a transverse feedback system and identification of both internal and external noise sources.

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