NONLINEAR MODEL CALIBRATION IN ELECTRON STORAGE RINGS VIA FREQUENCY ANALYSIS*

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Abstract

Frequency analysis of the turn-by-turn BPM(Beam Position Monitor) data is a very useful numerical method for analysing the detrimental effect of the nonlinear resonances in storage rings, which has been used for nonlinear resonances measurement and correction by numerical fitting of the lattice components. The method is applied in HLSII storage ring for nonlinear model calibration by numerical fitting of the sextupole components with the effect of radiation damping and decoherence in this paper.

INTRODUCTION

The calibration of the nonlinear model of the storage ring is crucial to making sure the accelerator system running at the design performance, which has been researched for many years but still lacks an comprehensive method. A important step forward was made with the introduction of FMA(Frequency Map Analysis)[1] which was the first successful attempt to use the approach of frequency analysis of turn-by-turn BPM data in analysing nonlinear dynamics. As a result of the development of perturbation theory [2] and Normal Form techniques [3], the relationships between the resonance driving terms which include the information of nonlinear resonance and the spectral lines of the turn-by-turn BPM data were builded, and it realized that the nonlinear resonance driving terms can be directly measured from Fourier analysis of the betatron oscillations of the beam in the storage ring. Within the theoretical formalism, a new method has been developed for the calibration of the nonlinear model of the storage ring[4]. The basic idea of the method is to connect the amplitude and phase of the Fourier coefficients of the spectral lines with the amplitude and phase of the driving terms of a given resonance. Then a fit algorithm similar to LOCO[5] which is powerful in calibrating linear optics can be builded to reconstruct the nonlinear model entirely by comparing the amplitude and phase of the spectral lines. This LOCO-type algorithm has been successful used in Diamond storage ring. But it did not consider the effect of radiation damping and decoherence of the betatron oscillation.

After the undergoing upgrade project, the beam position monitor (BPM) system at Hefei Light Source (HLS) will have the capability to measure turn-by-turn

beam position. To achieve the design performance, the nonlinear dynamics will be studied with the method of frequency analysis and the nonlinear model will be tried to be calibrated. As a second generation accelerator, the nonlinear optics are created mainly in sextupoles and the third order resonances will be our main research target. In this paper, the strength of sextupoles are calibrated with the LOCO-type algorithm by the numerical fit of the sextupole components. As a synchrotron radiation source, the effect of radiation damping cannot be ignored and the effect of decoherence on the spectral lines is studied. The results of the simulation and optimization of the sextupole components with the code of AT(accelerator Toolbox)[6] are reported in this paper.

ALGORITHM

According to the perturbation theory [2] and Normal Form techniques [3], the frequency analysis of betatron motion can be used to measure directly the resonance driving terms that help to understand the nonlinear dynamics quantitatively, and each spectral line of the single particle motion is proportional to a specific resonance driving term, which make it the possible to calibrate the nonlinear model by the measurement of the spectral lines. The LOCO-type algorithm is based on a least-squares fit of the lattice elements strength to minimize the difference between the spectral content measured from all beam position monitors (BPMs) in the machine and the ideal model[4], which can be write in the form

$$\chi^2 = \sum_{j=1}^{2\,Nbpm} \left[A_{model}^{(-2,0)}(j) - A_{meas}^{(-2,0)}(j) \right]^2 \qquad (1)$$

where $Nbpm$ is the total number of BPMs in the storage ring.

The target vector $A_{meas}^{(-2,0)}$ whose components are the amplitude and/or the phase of the spectral measured at each BPM can be built as follows

$$A_{meas}^{(-2,0)} = (A_1, A_2, ..., A_{Nbpm}; \phi_1, \phi_2, ..., \phi_{Nbpm}) \qquad (2)$$

where the spectral lines at the frequency $(-2,0)$ are excited by the third resonance $(3,0)$ which is mainly created in sextupole if each spectral line $mQ_x + nQ_y$ is defined by (m,n). Therefore each spectral will depend

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on the particular distribution of sextupoles in the storage ring, which can be write

$$A_{meas}^{(-2,0)} = A_{meas}^{(-2,0)}(S_1, S_2, ..., S_N) \qquad (3)$$

where N is the number of sextupoles.

As show in Eq.(1), the target vector $A_{meas}^{(-2,0)}$ is compared with the model $A_{model}^{(-2,0)}$ which can be computed form a simulation machine or the real accelerator. The strength of the sextupole components can be fitted by a least-squares minimization procedure.

RADIATION AND DECOHERENCE

As we all know, the radiation loss is the primary characteristic in a synchrotron radiation light source, which even become our main goal to design a synchrotron radiation accelerator. The betatron motion can be easily affected by the momentum change resulting from recoil of synchrotron radiation in despite of the energy gaining from a RF system. The result is that the radiation damping arises from the combination of energy loss in the direction of betatron orbit and energy gain in the longitudinal direction from RF systems, which can be simply expressed[6]

$$z' \rightarrow z' + \Delta z' = z'(1 - \delta p / p) \qquad (4)$$

where z is the vertical betatron coordinate, and p is the momentum of the particles.

The radiation damping affect not only the transverse betatron motion but also the horizontal motion. The change in betatron amplitude A becomes

$$\frac{\Delta A}{A} = -(1 - \mathscr{D})\frac{U_0}{2E} \qquad (5)$$

where $\mathscr{D}$ is the damping partition.

So the turn-by-turn phase-space coordinates in a real synchrotron radiation machine or a simulation model are sure of being affected by radiation damping, which cannot be ignored in the algorithm of calibrating the sextupole components in HLSII.

The turn-by-turn data obtained from each BPM is the position of the centroid of the particles in the beam. if all of the particles have the same betatron tune, the motion can be harmonic. But due to the effect of chromaticity and nonlinearity, the beam will contain a spread of tunes and the motion will decohere as the individual betatron phases of the particles disperse. Therefore the observed centroid of the beam from each BPM will show a decaying oscillation. This phenomena is called decoherence [7].

Both of the two sources of betatron tune spread, chromaticity and nonlinearity, will directly change the amplitude of the beam position, which can be simply expressed

$$\bar{a}(N) = ZA_c(N) = Ze^{-\alpha^2/2} \qquad (6)$$

and

$$\bar{a}(N) = ZA_n(N) = \frac{Z}{1+\theta^2}\exp[-\frac{Z^2}{2}\frac{\theta}{1+\theta^2}] \qquad (7)$$

where $A_c(N)$ and $A_s(N)$ are the decoherence factor due to chromaticity and nonlinearity. The details can be found in Ref. [7].

The fitting algorithm mentioned in this paper does not need to calculate the radiation and decoherence, but only pay attention to the spectral lines transformed from the phase space coordinates, which of cause should be obtained from a tracking code with the effect of radiation and decoherence. So in order to make sure the sextupole components can be calibrated accurately, the multiple particles should be tracked in a lattice with the affection of radiation.

SIMULATION

AT(Accelerator Toolbox) is a collection of tools to model storage rings and beam transport lines in the MATLAB environment[8]. The lattice of the HLSII storage ring is modelled with the AT code, and the turn-by-turn data can be tracked easily with the tracking code. The beam parameters at HLSII are showed in Table 1 and the spectral lines transformed from a single particle tracking data are compared between lattices with radiation and without radiation in Fig. 1.

Table 1: Parameters at HLSII

Beam energy [MeV]	800
Energy lost per turn [MeV]	1.673
Main RF peak voltage [kV]	250
Main RF frequency [MHz]	204
Energy spread	0.00047
Harmonic number	45
Harmonic frequency [MHz]	816
Transverse tune	4.4141, 3.2234
Synchrotron tune	0.00677

we can see obviously that the amplitudes of main spectral lines get smaller due to the effect of the radiation damping.

The effect of decoherence is also analysed with the comparison of the spectral lines between the single particle tracking data and the centroid of the multiple particles tracking data. Just as the theory tell, the amplitudes of the spectral lines transformed from the centroid coordinates of the multiple particles tracking data are much smaller than that from the single particle tracking data, which can be seen in Fig. 2.
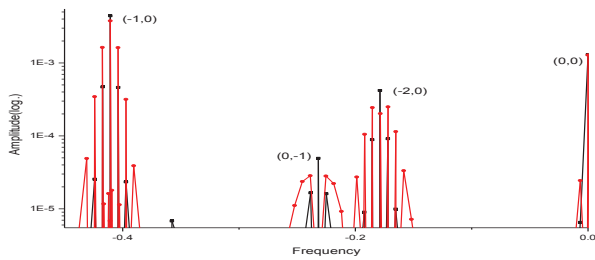
Figure 1: (Colour) The spectral lines in black is transformed from a single particle tracking data without the effect of radiation. The red ones are obtained from a single particle tracking data too, but under the effect of radiation damping.
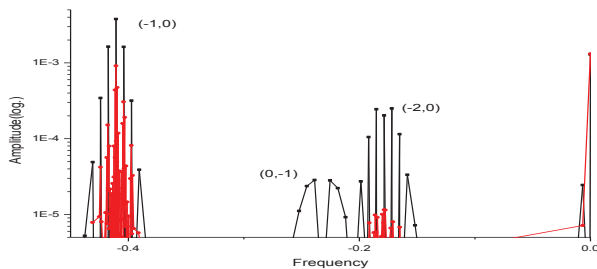

Figure 2: (Colour) The spectral lines in black is transformed from a single particle tracking data and the red ones are obtained from the centroid coordinates of the multiple particles tracking data with the effect of radiation damping.

Then the simulation model with the effect of radiation damping and decoherence of the betatron oscillation can be builded, and the vector of the model $A_{model}^{(-2,0)}$ can be obtained from the centroid of multiple particles tracking data with a series of random errors in the sextupole components. The Hefei storage ring has 32 BPMs and 32 sextupoles. In standard mode, only 16 sextupoles are running to correct the chromaticity and they are divided into two sets equally. Each set of sextupoles is controlled by one power source and has the same strength. So the two strength of the sextupoles that the spectral vector depend on will be our target to calibrate. With the amplitude of the spectral line at the frequency (-2,0) at each BPM, we can get the vector $A_{model}^{(-2,0)}$ with a particular strengths of the sextupoles, which are showed in Figure 3.

Now we will calculate the strength of the sextupoles by minimizing the quantity expressed in Eq. (1) with a least-squares minimization procedure. The MATLAB optimization toolbox is very powerful to solve nonlinear least-squares problems. We can directly use the optimization function with the Levenberg-Marquardt algorithm to obtain the error strength of the sextupole components. The result of the optimization is showed in Fig. 3 and Fig.4.

We can see that the two vectors match each other very well after 5 iterations and the strength of the sextupole components obtained from the optimization can fit that set in the error model.
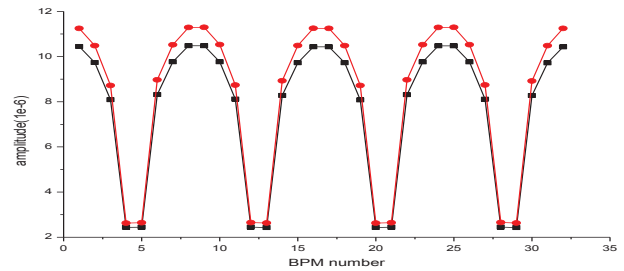

Figure 3: (Colour) Comparison of the spectral lines at each BPM between the measured vector in black and the error model vector in red before correction.
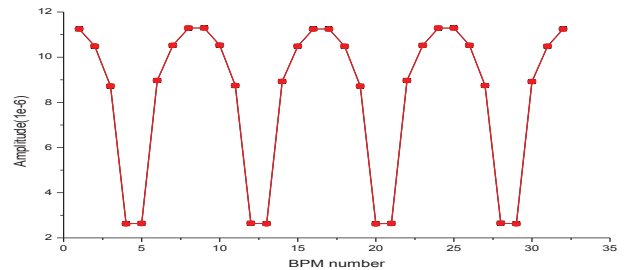

Figure 4 : (Colour) Comparison of the spectral lines at each BPM between the measured vector in black and the error model vector in red after optimization.

CONCLUSION

Two sets of sextupole components are calibrated in this paper by the method of minimizing the distance between two vectors of the amplitude of the spectral lines at the frequency (-2,0) drove by the third order resonance (3,0) in HLSII storage ring. The effect of radiation damping and decoherence of the betatron oscillation on the amplitude of the spectral lines has been explained in this paper and they have been put into considered in the fitting algorithm. The result is quite good when the errors of the sextupoles components are not too large.

The phases information of the spectral lines are not took into account in this paper, which will be analysed in future work. To make it more accurate, each of the 16 sextupoles errors should be calibrated with the algorithm mentioned in this paper in future work too.

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