

INFLUENCE OF HIGHER ORDER PHASE SLIP FACTOR CONTRIBUTIONS ON BEAM LOSS DURING SIS-100 PROTON OPERATION

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Abstract

The projected FAIR synchrotron SIS-100 is envisaged to accelerate intense proton and heavy-ion beams. The maximum proton energy will be $E = 29$ GeV. In order to stay below transition energy a special powering scheme of the quadrupoles has been introduced which provides a maximum transition gamma $\gamma_{tr} = 45.5$. The resulting settings of the quadrupole focusing strengths generate large maxima of the horizontal beta and dispersion functions. In particle tracking simulations we observed beam loss caused by a large momentum spread in a deformed rf bucket close to transition. Application of the chromaticity correction sextupoles led to a reduction of the first-order phase slip factor term and of the beam losses. In this contribution we will analyze the effect of the sextupoles on the higher-order components of the phase slip factor. The rf bucket shape will be discussed as well as the transverse beam loss and possible longitudinal instabilities.

LATTICE PROPERTIES

Besides heavy ion operation, SIS-100 is also foreseen to deliver high intense proton beams of 2.0×10^{13} protons in a short single bunch. The particles will be injected at $E = 4$ GeV and fast extracted at $E = 29$ GeV. To keep the length of the bunch small, large momentum spreads will be put up with. The largest maximum momentum deviation will be $\delta_m = \pm 0.0051$ at $E = 7$ GeV. At $E = 29$ GeV, there will be $\delta_m = \pm 0.0043$ [2]. The working point used in this study is $(Q_x, Q_y) = (21.8, 17.7)$. This choice ensures the space charge tune spread according to the maximum proton number to fit in the mesh of resonances excited by non-linear multipole errors in magnets. However, in a former study unacceptably large beam loss was found even when high current effects are still absent and generated only by the properties of the linear optics set for operation at maximum energy [1]. These linear optics provide the transition gamma $\gamma_{tr} = 45.5$ by applying three independent families of quadrupoles and were developed in order to stay below transition energy when the extraction energy is reached. The high γ_{tr} is the consequence of a strongly oscillating dispersion function so that the momentum compaction factor

$$\alpha_c = \frac{1}{\gamma_{tr}^2} = \frac{1}{C} \oint \frac{D_x(s)ds}{\rho(s)} \quad (1)$$

is minimized. The maximum of the high- γ_{tr} dispersion function is much larger than that of the dispersion func-

Table 1: Maximum values of some lattice functions of high γ_{tr} lattice and ion-like lattice, both at the working point (21.8, 17.7). The natural chromaticities in row 6 correspond to Eq. (2).

	high γ_{tr}	ion-like
quadrupole families	3	2
γ_{tr}	45.5	18.4
$ D_{x,max} $	2.9 m	1.3 m
$(\beta_{x,max}, \beta_{y,max})$	(72 m, 29 m)	(19 m, 21 m)
$(\xi_{x,nat}, \xi_{y,nat})$	(-2.68, -1.45)	(-0.91, -1.12)

tion generated with two quadrupole families as done during heavy ion operation, see Table 1.

Furthermore, the high- γ_{tr} optics increase significantly the maximum horizontal beta function and the horizontal chromaticity. The natural chromaticities $\xi_{x,nat}, \xi_{y,nat}$ shown in Table 1 are defined by

$$\Delta Q_z = \xi_z Q_z \delta, \quad z = x, y. \quad (2)$$

The maximum horizontal beta function at the working point chosen was further increased to more than 100 m by including magnet errors. Reducing the horizontal tune, the maximum horizontal beta function more and more increased and, for $Q_x < 21.5$, MAD-X could not determine the lattice functions anymore, so that no dynamic aperture could be determined, which is shown in Figure 1.

The large horizontal chromaticity at extraction energy causes a maximum tune deviation of about $\Delta Q_{x,max} = 0.25$ so that the total horizontal chromatic tune spread, shown by the red double arrow in Figure 1, exceeds the gap between the half integer resonance at $Q_x = 21.5$ and the integer resonance at $Q_x = 22$.

In addition, the dependence of the phase slip factor on the momentum is significant under extraction conditions,

$$\eta = \eta(\delta) = \eta_0 + \eta_1 \delta + \dots, \quad (3)$$

because the zero order phase slip factor is small,

$$\eta_0 \equiv \frac{1}{\gamma_{tr}^2} - \frac{1}{\gamma^2} = -4.99 \times 10^{-4}. \quad (4)$$

The results are the non-symmetric rf bucket shape denoted by the longitudinal phase space trajectory shown in Fig. 2 and the formation of an unstable fixed point given by

$$(\phi_{FP}, \delta_{FP}) = (0, -\eta_0/\eta_1) \quad (5)$$

when the linear approximation is used in Eq. (3).

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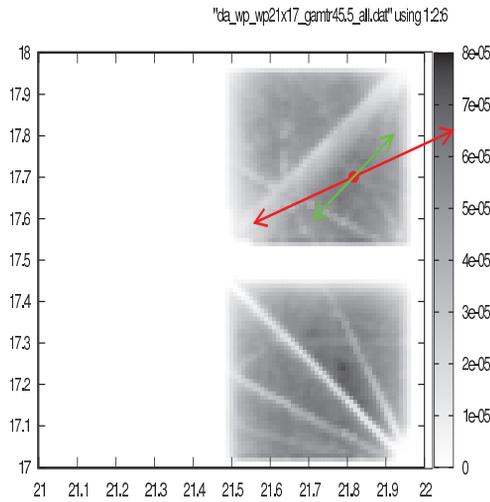


Figure 1: Dynamic aperture scan, working point and tune spread due to $\delta = \pm 0.0043$ with natural chromaticity (red arrow) and reduced to $\Delta Q_{x,max} = \Delta Q_{y,max} = \pm 0.1$ by correcting the chromaticities (green arrow) obtained for $\gamma_{tr} = 45.5$. The dynamic aperture was determined by tracking a single particle during 500 turns for different angles in the $x - y$ plane. The results were averaged to obtain the dynamic aperture shown in this figure.

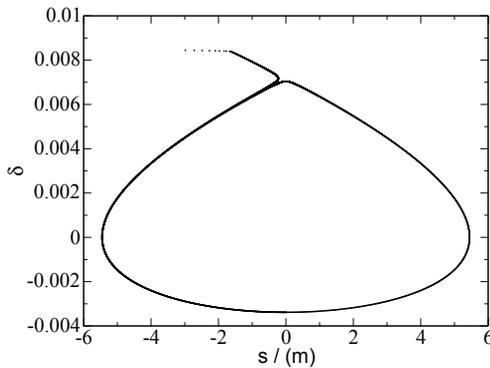


Figure 2: Stable and unstable longitudinal phase space trajectories close to the separatrix simulated for extraction conditions: $E = 29$ GeV, $\gamma_{tr} = 45.5$, $V_{rf} = 300$ kV. The synchronous phase $\phi_s = 0$ corresponding to constant energy and the lattice without magnet imperfections and chromaticity correction were used.

The large horizontal chromatic tune spread arising from large horizontal chromaticity and tune spread, and the non-symmetric rf bucket shape turned out to be major sources of particle loss.

MAGNET ERRORS

Real magnets are with imperfections. For this study, systematic and random non-linear multipole errors in dipole and quadrupole magnets were taken into account. Multipoles of order $N + 1$ up to 15 were used, where $N = 0$ for

dipoles and $N = 1$ for quadrupoles. In addition, a random error to the quadrupole focusing strengths was introduced.

The systematic errors are results of simulations and described in [3]. They influence the dynamic aperture. Resonances are driven by random multipole errors. In our modelling, they are assumed to follow a Gaussian distribution truncated at 2σ . In case of the random errors to the focusing strengths of the quadrupoles, $\sigma = 0.001 \text{ m}^{-1}$ was used what approximately corresponds to 0.3% of the total integrated strength $k_1 L$ of a quadrupole. For the non-linear random normal and skew errors, the σ values were set to 30 % of the systematic normal component of same order.

CHROMATICITY CORRECTION

In the first step of the study, the chromaticities were corrected using all 52 sextupoles of SIS-100 to reduce the chromatic tune spread. Owing to the large number of sextupoles, additional constraints could be applied in order to diminish the non-linear influence of the sextupoles. Three constraints were introduced,

$$\sum_i (k_2 L)_i^2 \rightarrow \min. \quad (6)$$

$$\sum_i [(k_2 L)_i \beta_{x,i}]^2 \rightarrow \min. \quad (7)$$

$$\sum_i (k_2 L)_i^2 (\beta_{x,i}^2 - D_{x,i}^2) \rightarrow \min., \quad (8)$$

where only sextupoles in one sector are independently set. In the 2nd and the 3rd constraint is regarded that the non-linear impact of a sextupole grows with the beta function at its location. Additionally, the ability to contribute to the chromaticity correction is large in case of a large dispersion function [4]. Note, that the third constraint could be applied because the dispersion function is smaller than the horizontal beta function at the location of each sextupole.

PARTICLE TRACKING RESULTS

Particle tracking simulations were performed in order to study the effect of the different chromaticity correction scheme, in particular, on the longitudinal particle motion and to determine beam loss.

The study on synchrotron motion affected by different chromaticity correction schemes, basically, consisted in the search for the separatrix with single particle tracking with the initial momentum coordinate found within a trial-and-error procedure. The simulation interval was 16000 turns what corresponds to a synchrotron period at extraction energy. No lattice non-linearities besides the sextupoles were included. The initial particle coordinates were $(x_{ini}, x'_{ini}, y_{ini}, y'_{ini}, -ct_{ini}, \delta_{ini}) = (x_{co}(\delta), x'_{co}(\delta), 0, 0, 0, -\delta)$, where $-ct = \phi\beta C/N_h$ with the circumference of SIS-100, $C = 1083.6$ m, and the harmonic number of the rf cavity, $N_h = 5$. Separatrix and unstable fixed point obtained with natural chromaticities are shown in Figure 2. We found that, by correcting the chromaticities, momentum components of the unstable fixed

Table 2: Initial momentum deviations and momentum coordinates of the unstable fixed point obtained with the chromaticities not corrected and corrected applying the constraints given in Eqs. (6)-(8), respectively. The desired maximum chromatic tune deviations were $\Delta Q_{x,max} = \Delta Q_{y,max} = \pm 0.1$.

constraint Eq.	δ_{FP}
natural chromaticities	0.0071
(6)	0.0079
(7)	0.0101
(8)	0.0101

Table 3: Particle losses found in multi-particle simulations with chromaticities not corrected and corrected applying the constraints given in Eqs. (6)-(8), respectively. Multipole errors in dipoles and quadrupoles were taken into account. The desired maximum chromatic tune deviations were $\Delta Q_{x,max} = \Delta Q_{y,max} = \pm 0.1$.

constraint Eq.	P_{loss}
natural chromaticities	8.8 %
(6)	3.4 %
(7)	0.04 %
(8)	0.02 %

point, δ_{FP} are increased what is a sign of a decreased momentum dependence of the phase slip factor. The δ_{FP} values for the maximum chromatic tune deviations reduced to $\Delta Q_{x,max} = \Delta Q_{y,max} = \pm 0.1$, represented by the green double arrow in Figure 1, are shown in Table 2. It turned out that the momentum dependence of the phase slip factor could be most reduced by applying the constraints where the lattice functions at the locations of the sextupoles had been taken into account.

In order to estimate beam loss, five simulations per sextupole powering scheme with 1000 test particles with different samples of initial coordinates, tracked during 32000 turns corresponding to two synchrotron periods at $E = 29$ GeV, were performed. The transverse initial particle coordinates were distributed according to bi-Gaussian functions truncated at 2σ determined from the rms emittances $(\epsilon_{x,rms}, \epsilon_{y,rms}) = (2.1, 0.65) \mu$ m rad. The initial momentum deviations were distributed according to a Gaussian function truncated at 2σ with $2\sigma = \delta_{max} = 0.0043$. In the first step, by neglecting magnet errors, we obtained acceptable particle loss of $\approx 1\%$ or less when the constraints to the sextupole settings of Eqs. (7) and (8), where the lattice functions at the sextupole locations were taken into account, were applied, see Table 3.

Including the multipole errors introduced in the section on magnet errors, the strongly increased particle losses shown in Table 4 were obtained. Correcting the chromaticities to a degree so that the maximum chromatic tune is fur-

Table 4: Particle losses found in multi-particle simulations with chromaticities not corrected and corrected applying the constraints given in Eqs. (6)-(8), respectively. The desired maximum chromatic tune deviations were $\Delta Q_{x,max} = \Delta Q_{y,max} = \pm 0.1$.

constraint Eq.	P_{loss}
natural chromaticities	41.4 %
(6)	18.0 %
(7)	12.8 %
(8)	13.0 %

Table 5: Particle losses found in multi-particle simulations with chromaticities not corrected and corrected applying the constraints given in Eqs. (6)-(8), respectively. The desired maximum chromatic tune deviations were $\Delta Q_{x,max} = \Delta Q_{y,max} = \pm 0.05$.

constraint Eq.	P_{loss}
natural chromaticities	41.4 %
(6)	14.5 %
(7)	1.7 %
(8)	1.5 %

ther reduced $\Delta Q_{x,max} = \Delta Q_{y,max} = \pm 0.05$, the increase in beam loss due to the multipole errors could be partially compensated so that beam loss of nearly acceptable level could be found by applying the constraints of Eqs. (7) and (8), which one can see in Table 5. Nevertheless, the reduction of beam loss by applying these constraints was found rather by chance, what is somewhat unsatisfactory. On the other hand, the attempt to apply the formula [5]

$$\Delta\eta_1 = -\frac{1}{3C} \sum_i (k_2 L)_i (D_{x,i}^3 - 3D_{x,i} D_{y,i}^2) \quad (9)$$

as an additional constraint to the sextupole settings in order to explicitly change the linear term of the phase slip factor in Eq. (3) did not reduce the obtained beam loss which is, probably, due to a simultaneous reduction of the dynamic aperture due to stronger sextupoles. This point should be clarified and requires further investigation.

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