

# DIPOLE FRINGE FIELD EFFECTS IN THE ThomX RING\*

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## Abstract

ThomX is a compact machine based on the Compton back-scattering, which is being built by LAL in France. The ThomX ring has a short circumference of 16.8 m and a low design energy of 50 MeV, and the dipole bending radius of 0.352 m. With such a short bending radius, the nonlinear effects due to the second order dipole fringe field in the ring become non-trivial. In this paper, we discuss the appropriate models of the dipole fringe field in the particle tracking, and then compare the vertical chromaticity of the ThomX ring calculated using four different codes: Tracy3, BETA, MADX and ELEGANT.

## INTRODUCTION

A compact machine ThomX is being built in LAL, France [1]. This machine aims to produce hard X-rays with an average flux of  $10^{11}$  -  $10^{13}$  ph/s, which will be dedicated to the medical applications, imagining, and especially the cultural heritage recovery in the museums such as the Louvre museum in Paris.

In the Thom-X machine, the electrons are generated by a laser electron gun, then accelerated up to 50 MeV by a Linac, and finally transferred and injected into the ring to collide with a laser to produce X-ray. The storage duration is planned to be 20 ms. Table 1 shows the main parameters of the ThomX ring.

Table 1: Main Parameters of the ThomX Ring

Parameters	values
Circumference [m]	16.80
Nominal energy [MeV]	50
Betatron tunes $\nu_x / \nu_y$	3.175 / 1.64
Natural chromaticities	-10.522 / -11.255
Momentum compaction factor	0.0148
RF frequency [MHz]	500
RF harmonic	28
Beta / dispersion @ IP	0.1 / 0.1 / 0
Number of dipoles	8
Number of quadrupoles	24
Number of sextupoles	12

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## VERTICAL CHROMATICITY AND DIPOLE FRINGE FIELD

Normally the dipole fringe field is modeled using the K. Brown's second order transfer matrix [2] and the vertical chromaticity is calculated using an analytical formula, such as in the simulation codes BETA [3], MADX [4] and ELEGANT [5]. The ThomX ring is designed using the code BETA.

Although it is important to correctly calculate the beam properties using an analytical method, the beam dynamics need to be further studied using the particle tracking, such as to investigate the details of the dynamic aperture using the Frequency Map Analysis.

In the particle tracking code Tracy3 [6], the initial kick map of the dipole fringe field was the first order K. Brown transfer matrix [2] with the correction of  $1/1+\delta^i$  [7],

$$p_x^f = p_x^i + \frac{\tan \theta}{\rho} x^i \quad (1)$$

$$p_z^f = p_z^i - \frac{1}{1 + \delta^i} \frac{\tan(\theta - \psi)}{\rho} z^i \quad (2)$$

where  $i$  and  $f$  respectively denote the particle enters and exits the fringe field region;  $p_{x,y}$  are the canonical momentums; the horizontal coordinate  $x$ , the vertical coordinate  $z$ , the longitudinal coordinate  $-ct$ , and the momentum offset  $\delta$  are unchanged after the fringe field region.

In eqn. 1 and 2,  $\rho$  is the bending radius of the dipole;  $\theta$  is the entrance or exit angle of the dipole,  $\psi$  is a parameter due to the dipole fringe field,

$$\psi = \frac{1}{\rho} K g \frac{1 + \sin \theta^2}{\cos \theta}, \quad (3)$$

where  $g$  is the full magnet gap, and

$$K = \int_{-\infty}^{+\infty} \frac{B_z(s)[B_0 - B_z(s)]}{gB_0^2} ds, \quad (4)$$

where  $B_0$  is the vertical design dipole field,  $B_z(s)$  is the magnitude of the fringing field on the magnetic middle plane at a position  $s$ . The value of  $K$  is chosen as 0.5 in Tracy3.

With the first order kick maps of the dipole fringe field in Eqn. 1 and 2, the vertical chromaticity from Tracy3 is close to the value from BETA and also the experimental value in the storage ring of the Synchrotron Soleil, which has a circumference 354 m, the beam energy 2.739 GeV, and the dipole bending radius 5.36 m. However, the natural vertical chromaticity from Tracy3 is -5.072 which has a great discrepancy from the value -11.255 calculated using BETA, for the ThomX ring which has a circumference 16.8 m, the

beam energy 50 MeV, and the dipole bending radius 0.352 m. This discrepancy is in the expectation, since it is well known that the second order term of the dipole fringe field is non-trivial to the calculation of the vertical chromaticity in a ring with a small bending radius [8].

The K. Brown's second order matrix of dipole fringe field is an accurate model, but it can't be used directly in the particle tracking codes like Tracy3, because an accurate and symplectic model is necessary to have a correct prediction to the beam dynamics using particle tracking.

### IMPORTANCE OF THE SYMPLECTICITY TO THE PARTICLE TRACKING

In the particle tracking, the particle coordinates before ( $X_i$ ) and after ( $X_f$ ) the lattice element are connected using a map  $M$ , That is  $X_f = M(X_i)$ . To have a correct prediction of the particle motion using the particle tracking,  $M$  must be both accurate and symplectic, otherwise a wrong picture of the beam dynamics will be obtained.

Figure 1 compares the phase space tracking during 1000 turns in the ThomX ring with a symplectic model (eqn. 1 and 2) and a non symplectic model (K. Brown's second order transfer matrix) of the dipole fringe field. To keep the simplicity of the tracking, the sextupoles are turned off, and only dipoles, quadrupoles are included in the lattice. With a symplectic model, the phase space is a closed ellipse (Fig. 1 a) which is in the expectation; while with a non symplectic model, the phase space spirals out (Fig. 1 b) which is the wrong physical picture, since the system is a linear dynamic system.

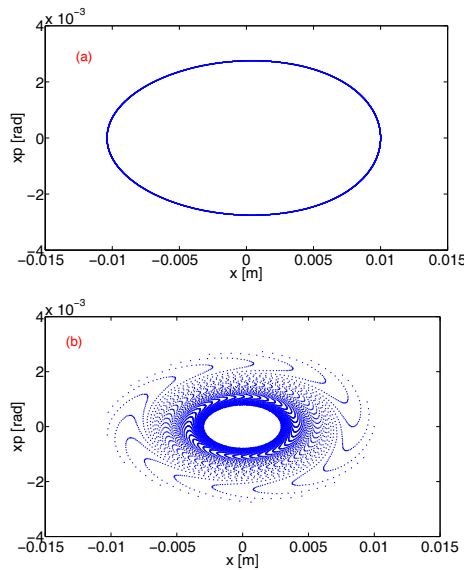


Figure 1: Phase space tracking in 1000 turns using a symplectic (a) and non symplectic model (b) in the ThomX ring without sextupoles.

### GEOMETRIC CORRECTION OF THE LINEAR DIPOLE FRINGE FIELD

The field distribution of the dipoles in the ThomX ring is shown in Fig. 2. The position  $s = 0$  is the physical edge of the dipole. The effective dipole field  $B_z$  begins to decrease from the design value  $B_0$  at the position  $-\varepsilon$  and disappears at the position  $\varepsilon$ .

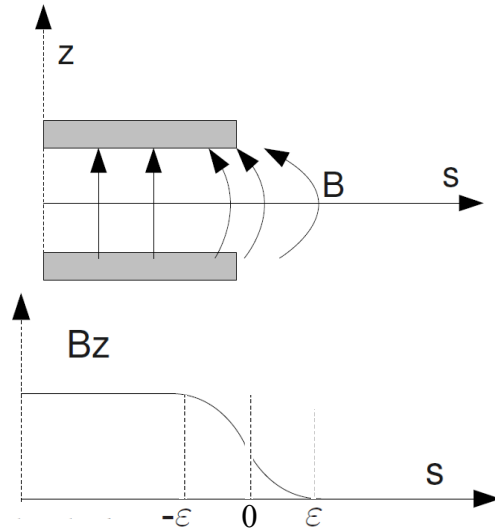


Figure 2: Sketch of the dipole field in the ThomX ring.

From the Maxwell equation  $\nabla \times \vec{B} = 0$ , it is easy to get the longitudinal field  $B_s = \frac{\partial B_z}{\partial s} z$ . From the Lorentz equation, the particle is vertically kicked by a force  $dP_z/dt = -qV_x B_s$  with  $P_z$  the vertical momentum,  $q$  the particle charge,  $V_x$  the horizontal speed. Here the horizontal dipole fringe field  $B_x$  is omitted since the magnet width is much larger than the horizontal coordinate  $x$ .

The design particle enters/exits the dipole with its closed orbit normal to the dipole face, so its  $V_x = V_s \tan \theta$  with  $V_s$  the longitudinal speed, and the vertical focusing along the fringe field at the entrance/exit of the dipole is

$$\frac{dp_z}{ds} = \frac{-\tan \theta}{B\rho} \frac{\partial B_z}{\partial s} z = K_z z \quad (5)$$

where  $K_z$  is the usual notation of the normalized quadrupole strength,  $ds = V_s dt$ ,  $p_z = P_z/P_0$  and  $B\rho = P_0/q$ . Furthermore, integrating  $K_z$  over the dipole fringe field region leads to the total focusing effect ( $1/f_z$ ) in the vertical plane:

$$\frac{1}{f_z} = \int_{-\varepsilon}^{+\varepsilon} K_z ds = \frac{-\tan \theta}{\rho}, \quad (6)$$

which is the linear kick map of eqn. 2 without the correction term  $1/(1 + \delta^i)$  and  $\psi$ .

However, the closed orbit of the off momentum particle is not normal to the dipole pole faces when it enters/exits the dipole, and its horizontal divergence  $x'$  or  $p_x$  at the entrance/exit of the dipole contributes to the absolute speed

$V_x$ , that is  $V_x = V_s \tan(\theta \pm p_x/(1 + \delta))$  with “+” at the entrance while “-” at the exit of the dipole. Consequently, the modified first order kick map of eqn. 1 and 2 with this geometric correction is

$$p_x^f = p_x^i + \frac{\tan \theta}{\rho} x^i, \quad (7)$$

$$p_z^f = p_z^i - \frac{1}{1 + \delta^i} \frac{\tan(\theta - \psi \pm \frac{p_x^i}{1 + \delta^i})}{\rho} z^i. \quad (8)$$

In Eqn. 7 and 8,  $\theta$  is due to the pole face rotation from the Cartesian coordinate system in the straight section to the curvilinear coordinate system in the dipole,  $\theta = 0$  for the dipoles in the ThomX ring;  $\psi$  is due to the dipole fringe field as shown in eqn. 3 and 4;  $p_x^i/1 + \delta^i$  is due to the closed orbit of the off momentum particle is not normal to the dipole pole face, and this term has a non-trivial contribution to the vertical chromaticity.

## SYMPLECTIC MAP OF THE DIPOLE FRINGE FIELD

In a conservative accelerator system without the energy loss (radiation damping) and the energy gain (RF cavities off), the particle motion can be described by a Hamiltonian,

$$\begin{aligned} H(x, p_x; z, p_z; -ct, \delta; s) \\ = -(1 + \frac{x}{\rho}) \sqrt{(1 + \delta)^2 - (p_x - \frac{A_x}{B\rho})^2 - (p_z - \frac{A_z}{B\rho})^2} \\ + \frac{x}{\rho} + \frac{x^2}{2\rho^2} - \frac{A_s}{B\rho} + \delta, \end{aligned}$$

where  $A_x$  and  $A_z$ , are respectively the horizontal, vertical, vector potential of the magnet. Normally  $A_z$  is omitted since the magnet width is much larger than the horizontal coordinate  $x$ ;  $A_x$  is non zero when the longitudinal fringe field is considered.  $x/\rho + x^2/2\rho^2$  is the longitudinal vector potential of the dipole, and  $A_s$  is the longitudinal vector potential of the other higher order magnets (quadrupoles, sextupoles, etc).

With the Hamiltonian  $H$  and following the procedures proposed by E. Forest [9], the symplectic map of the dipole fringe field up to second order is

$$z^f = \frac{2z^i}{1 + \sqrt{1 - 2\frac{\partial \phi}{\partial p_z^i} z^i}} \quad (9)$$

$$x^f = x + \frac{1}{2} \frac{\partial \phi}{\partial p_x} (z^f)^2 \quad (10)$$

$$p_z^f = p_z^i - \phi z^f \quad (11)$$

$$(-ct)^f = (-ct)^i - \frac{1}{2} \frac{\partial \phi}{\partial \delta} (z^f)^2 \quad (12)$$

where

$$\begin{aligned} \phi(p_x, p_z, \delta) = \\ \frac{p_x}{\rho(1 + p_z^2)} - \frac{gK}{\rho^2} \left\{ \frac{(1 + \delta)^2 - p_z^2}{p_s^3} + \frac{p_x^2}{p_s^2} \frac{(1 + \delta)^2 - p_x^2}{p_s^3} \right\} \end{aligned}$$

From Eqn. 9 to 12, it is clear that if  $z^i = 0$ , the particle coordinates are changed after the dipole fringe field region.

Repectively with the linear kick map (eqn. 1 and 2), the geometric correction of the linear kick map (eqn. 7 and 8) and the Forest's symplectic map (eqn. 9 to 12), the betatron tunes and natural chromaticities of the ThomX ring were calculated using the code Tracy3; and then compared to the values from BEATA, MADX and ELEGANT which use the K. Brown's second order transfer matrix of the dipole fringe field. The results are shown in Table 2. From this table, the second order term of the dipole fringe field has no obvious effects on the betatron tunes and horizontal chromaticity, while has a strong effect on the vertical chromaticity of the small machine with a small bending radius. The geometric correction to the modified linear kick map of the dipole fringe field is a simple and appropriate model and it gives the values close to the Forest's symplectic model.

Table 2: Tunes and Natural Chromaticities of ThomX Ring

	$\nu_x$	$\nu_z$	$\xi_x$	$\xi_z$
Tracy 3 (Corr.)	3.175	1.64	-10.663	-10.860
Tracy 3 (Forest)	3.175	1.64	-10.663	-10.860
Tracy 3 (no corr.)	3.175	1.64	-10.663	-5.072
BETA	3.175	1.64	-10.522	-11.255
MADX	3.175	1.64	-10.522	-11.255
ELEGANT	3.175	1.64	-10.523	-10.735

## CONCLUSION

The importance of the symplecticity in the particle tracking is shown in this paper. A simple geometric correction to the K. Brown's modified first order transfer matrix of the dipole fringe field is proposed, and compared to the symplectic model and also K. Brown's second order transfer matrix. The results show that this geometric correction can give the correct prediction of the vertical chromaticity of the small ring like the ThomX ring.

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