

# PRECISION TUNE, PHASE AND BETA FUNCTION MEASUREMENT BY FREQUENCY ANALYSIS IN RHIC\*

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## Abstract

A new program of analyzing turn-by-turn (TbT) beam position data to retrieve linear optics information has been developed for RHIC. High precision measurements of tune ( $\sim 2 \times 10^{-5}$ ) and phase (0.1 ~ 0.2) deg have been achieved with this program. The algorithm and its applications are presented in this report.

## INTRODUCTION

Turn-by-turn measurements of the beam position with an applied excitation to the beam have been used at many accelerators to infer fundamental optical parameters such as the tune, the phase advance between BPMs, and with input from the accelerator model, the beta functions. Many different algorithms for data analysis have been successfully applied such as fitting in time domain [1], interpolated FFT technique in frequency domain [2, 3] and statistical techniques (PCA, ICA) [4, 5] finding beam motions in a high dimension data.

At RHIC we have recently applied two different algorithms and have demonstrated higher precision in determination of the beam's optical parameters compared to those algorithms applied at RHIC previously. One is based on fitting of the data in the time domain [1] while the other is based on analysis in the frequency domain.

In this report we describe the new frequency-domain based algorithm and present experimental data acquired using two different techniques, multiple kicks by ARTUS kicker [6] and sustained excitation by AC dipole [7]. Turn-by-turn BPM data of both types have been used to extract linear optics parameters [8, 9] at RHIC in the past. The goal of repeating the effort is two-fold: one is to verify the improvement of BPM data quality, the other is to demonstrate the improved measurement precision.

Tune and phase measurements are considered more precise compared with beta function measurement intrinsically because of their independence of BPM gains, calibration and so on. The criteria for a good measurement of the tune is a narrow distribution of tunes as measured by all BPMs. For phase measurements, the standard deviation of the measured phases for all BPMs from multiple data sets must be small.

## INTERPOLATED FOURIER TRANSFORM

A Gaussian window is applied on the TbT data first, the resulting data is then Fourier transformed using FFT tech-

nique [10]. A set of data points on the spectrum peak are selected and fitted as Gaussian distribution. The frequency corresponding to the fitted peak position is the interpolated betatron tune. The measurement of tune and phase are uncoupled. The parameters of Gaussian window are adjusted for higher tune measurement precision. The tunes from all BPMs are averaged. The phases of BPMs are calculated by continuous FFT,

$$X(f) = \frac{1}{N} \sum_{n=1}^N x_n \exp(-2\pi f n i), \quad (1)$$

$$\phi = \text{angle}(X), \quad (2)$$

Where N is the number of data points of the TbT orbit. The beta functions at BPMs can be calculated using [11]

$$\beta_{1,exp} = \beta_{1,model} \frac{\cot(\phi_{12,exp}) - \cot(\phi_{13,exp})}{\cot(\phi_{12,model}) - \cot(\phi_{13,model})} \quad (3)$$

here,  $\beta_{1,model}$  is the model beta function,  $\phi_{12,exp}$  is the measured phase advance between BPM 1 and 2,  $\phi_{12,model}$  is the model phase advance between BPM 1 and 2.

## TUNE MEASUREMENT

For linear optics measurement, all BPMs take TbT data simultaneously. Interpolated FFT is applied on TbT data from all BPMs. As an example, the tune values from all BPMs are shown as a histogram in Fig. 1. The rms of the tune distribution is  $1.8 \times 10^{-5}$ .

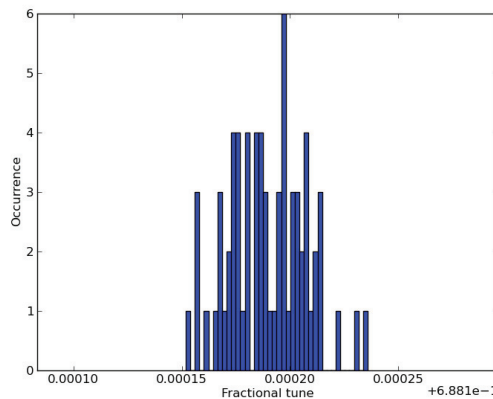


Figure 1: Histogram of the measured tunes at all BPMs in Blue ring.

To further study the tune measurement precision, multiple data sets were taken during 1 minute duration, the rms of the tunes is  $1.6 \times 10^{-5}$ .

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## PHASE MEASUREMENT

The tune applied in measurements of the phases is the average of the tunes from all BPMs TbT data. The raw phase from the continuous FFT calculation reflects the phase advance between ARTUS kicker and BPM of interest. However, for comparing with model phase advances, it is convenient to show phase advances from a defined origin. In RHIC, the origin is IP6, one of the two interaction points for experiments. The phases are shifted by an offset which is determined by the average difference between the measured and model phases of all BPMs.

As a tradition, the phase at RHIC advances in the clockwise direction in both rings as defined in the model. The measured phases agree with this definition in Blue ring because the beam direction is clockwise as well. However, measured phase increases count-clockwise (beam direction) in Yellow. Therefore, the phase difference between consecutive BPMs in Yellow is reversed to be consistent with the model.

The phase advance between BPMs are calculated. Both the measurement and model are shown in Fig. 2.

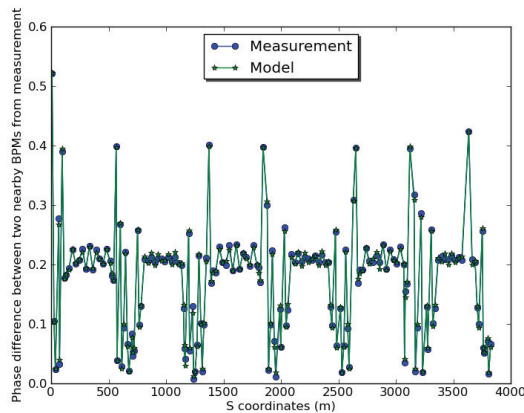


Figure 2: Measured and model phase advances between BPMs in Blue ring.

The integrated phase advance is of interest for projects like e-lens, which requires a phase advance of  $k\pi$  ( $k$  is an integer) between one of the IPs and e-lens location. The integrated phase advance from the same measurement as in Fig. 2 is shown in Fig. 3.

For impedance localization study and phase beat based optics correction, a high precision phase measurement is critical. The precision of phase measurement is to first order dominated by the BPM signal-to-noise ratio. A high precision tune measurement is another key factor for good phase precision. The phase measurement precision in Blue ring from 8 data sets is shown in Fig. 4.

The phase measurement precision has also been studied with sustained oscillation data produced by an AC dipole [7]. Fig. 5 is based on 4 data sets which were not taken the same time as in Fig. 4. The improvement of phase mea-

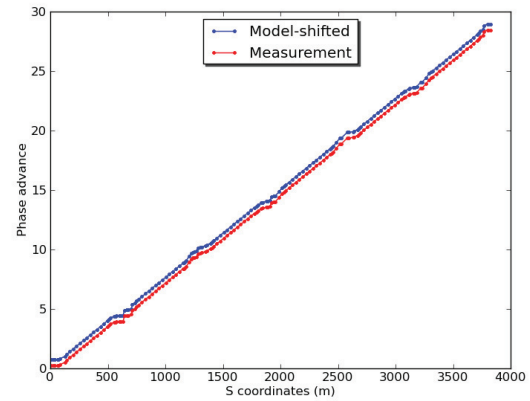


Figure 3: Measured integrated phase advance compared with model integrated phase advance (shifted by  $\pi$ ) in Blue ring.

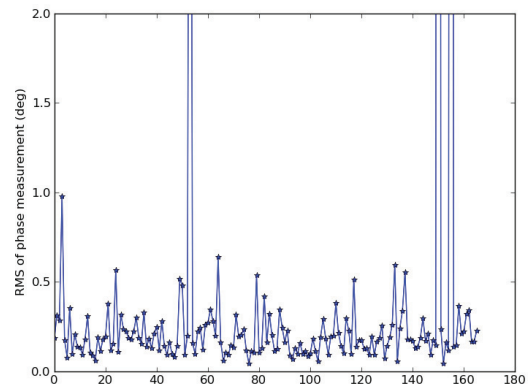


Figure 4: Standard deviations of phase measurements at all BPMs from 8 sets of ARTUS data.

surement precision in Fig. 5 is apparently due to the higher signal-to-noise ratio.

## BETA FUNCTION MEASUREMENT

In addition to beta functions calculated based on Eq. 3, beta functions can also be inferred from the amplitude of the interpolated FFT peak. The model beta functions at arc BPMs are used to calculate a scale factor, which is then applied on the amplitude of all BPMs to compute the beta functions. Neither of these two methods are model independent. The result from Eq. 3 has a relative large error when the phase advance between BPMs is close to  $\pi/4$ . The beta functions calculated using Eq. 3 is shown in Fig. 6. The beta functions inferred from the interpolated FFT amplitude of the same data set is shown in Fig. 7. The outliers in Fig. 6 are due to phase advances being close to  $\pi/4$  (see Fig. 2).

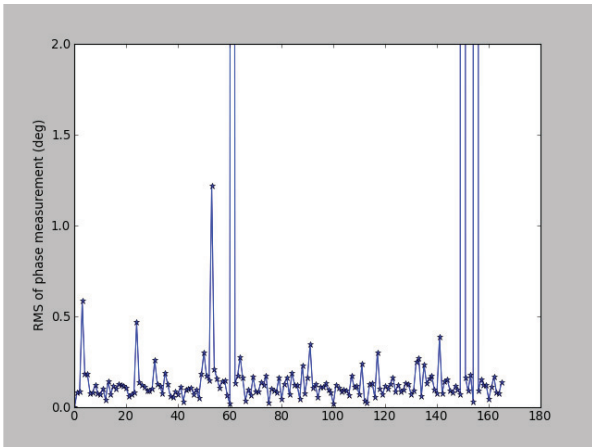


Figure 5: Standard deviations of phase measurements at all BPMs from 4 sets of AC dipole data.

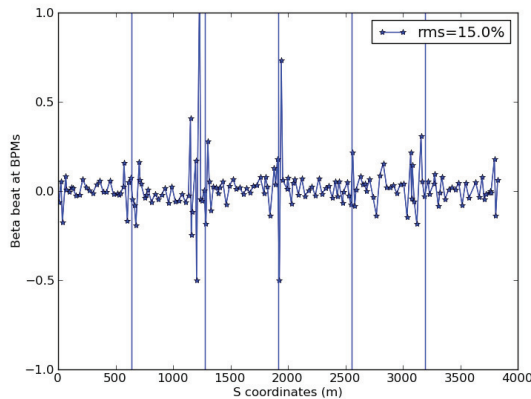


Figure 6: Relative difference between the measured and model beta functions at all BPMs by Eq. 3.

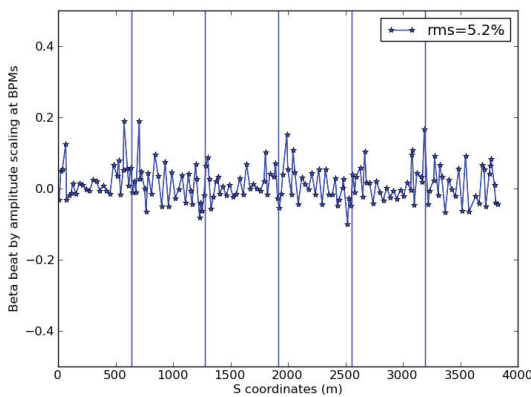


Figure 7: Relative difference between the measured and model beta functions at all BPMs by scaling amplitude, the data file being used is the same as that of Fig. 6.

## SUMMARY

The algorithm is optimized to measure tunes precisely. The average tune is then used in continuous FFT for phase determination. With ARTUS data, the standard deviation of phase measurements is  $\sim 0.2$  deg. The statistical error is reduced to  $\sim 0.1$  deg when applying the algorithm to AC dipole data. The high precision measurements are proof of high quality beam position measurements in RHIC. The new algorithm of analyzing TbT BPM data achieved unprecedented high precision and revealed the good performance of BPM system. Application of the program is being extended to allow measurement and correction of the accelerator optics during energy ramp.

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