EQUILIBRIUM BUNCH DENSITY DISTRIBUTION WITH PASSIVE HARMONIC CAVITIES IN THE MAX IV 3 GeV STORAGE RING

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Abstract

The MAX IV storage rings will use third harmonic cavities operated passively to lengthen the bunches and alleviate collective instabilities. These cavities are an essential ingredient in the MAX IV design concept and are required for achieving the final design goals in terms of stored current, beam emittance and beam lifetime. This paper reports on fully self-consistent calculations of the longitudinal bunch density distribution in the MAX IV 3 GeV storage ring, which indicate that up to a factor 5 increase in RMS bunch length is achievable with a purely passive system.

INTRODUCTION

The MAX IV facility [1], currently under construction in Lund, Sweden, includes a 3 GeV storage ring featuring ultra-low emittance (down to 0.2 nmrad) optimized for hard X-rays and a 1.5 GeV storage ring optimized for soft X-rays and UV radiation production. A 3 GeV linear accelerator plays the role of full-energy injector into both rings as well as delivers beam to a short pulse facility designed to produce spontaneous radiation from undulators with pulse lengths down to 100 fs.

A key ingredient in achieving stable operation of the MAX IV rings at high beam current (500 mA nominal stored beam current) is the use of a low frequency (100 MHz) RF system and third harmonic RF cavities which together lead to RMS bunch lengths on the order of 5-6 cm. The long bunches are essential to achieve the ultimate design performance parameters of the MAX IV rings. In fact, it is only with lengthened bunches that the low emittance can be preserved under the action of intra-beam scattering and the design intensity can be guaranteed against coherent collective effects – in particular, the long bunches permit to keep the heat load due to induced fields in vacuum chamber components at an acceptable level and avoid excitation of high frequency trapped (high Q) modes in the chamber structures and RF cavity high order modes. Additionally, the long bunches allow us to cope with coupled bunch resistive wall instabilities [2] that are enhanced by the very compact design of the storage ring vacuum chambers [3], which is in turn a consequence of the compact magnet design [4] required to reach a very low emittance in a relatively short machine circumference through the multi-bend achromat lattice concept [1]. Moreover, the harmonic cavities increase the synchrotron frequency spread within the bunches, enhancing Landau damping.

In this paper, we focus on the 3 GeV ring and describe calculations of the equilibrium longitudinal bunch density distributions in the double RF system (main and harmonic cavities), having in mind that the harmonic cavities will be operated passively, i.e., the fields in those cavities will be excited by the beam itself. Passive operation implies therefore that the fields excited in the harmonic cavities depend on the bunch density distribution, which, in turn, is determined by the sum of the fields in the main cavities and those in the harmonic cavities. Clearly a self-consistent solution for the density distribution needs to be found.

This problem has been treated by various authors before (e.g.[5]). A self-consistency equation is established for the determination of the equilibrium density distribution, in which the excitation of fields in the harmonic cavities is described by means a form factor, which is essentially the overlap between the harmonic cavity frequency response and the beam frequency spectrum. The beam frequency spectrum depends on the bunch shape and the frequency response of the harmonic cavity depends on the cavity properties (shunt impedance, quality factor, tuning angle). As long as the field in the harmonic cavities are such that the resulting equilibrium bunch density distribution is still symmetric, this form factor may be treated as a real parameter, i.e., one is justified in assuming that the phase of the fields in the harmonic cavity is independent of the bunch shape.

The treatment described above (which we call a "scalar self-consistent treatment") works well at relatively low harmonic cavity shunt impedances. However, passive operation of the harmonic cavities also implies operation on the Robinson unstable slope of those cavities, generating a Robinson growth rate that needs to be counteracted by the Robinson damping in the main cavities. This is made easier if the harmonic cavities are tuned far away from resonance, which in turn implies the need for high shunt impedance to reach the necessary field amplitudes that provide enough lengthening. This situation leads to a significant deformation of the bunch and it cannot be treated by the scalar self-consistent approach. Instead, both the amplitude and phase of the fields in the harmonic cavities must be taken as variables in writing up the self-consistent equations, which then become two dimensional – in other words, the bunch form factor that describes the excitation of fields in the harmonic cavities is now a complex number with an amplitude and a phase and we may define a "fully" self-consistent solution.

In the following sections we quickly review the calculations of the equilibrium bunch density distribution and analyse two cases: a low shunt impedance small detuning case and a high shunt impedance and large detuning case, which highlights the difference between the scalar and fully self-consistent calculations.

An overview of collective effects in the MAX IV storage rings has been given earlier [2] and a more detailed account based on time domain tracking calculations is given in this conference [6].

EQUILIBRIUM BUNCH DENSITY DISTRIBUTION IN DOUBLE RF SYSTEMS

Double RF systems have been analysed by many authors [5,7] to which we refer the reader for a detailed discussion. Below we list the relevant results and establish our notation and conventions. We assume an RF system composed of main and harmonic cavities so that the total voltage seen by the beam on every turn is given by:

$$V(\varphi) = V_{MC}(\varphi)+V_{HC}(\varphi) \quad (1)$$

where $V_{MC}(\varphi) = V_{rf}\sin(\varphi+\varphi_s)$ is the main cavity voltage, $V_{HC}(\varphi) = kV_{rf}\sin(n\varphi+n\phi_h)$. The harmonic cavity is assumed to resonate at a frequency close to the n-th harmonic of the RF frequency. The parameters k and ϕ_h define the amplitude and phase of the fields in the harmonic cavity. The equations of motion for the phase φ and energy deviation ϵ are:

$$\frac{d\varphi}{dt} = \alpha_c \frac{2\pi h}{T_0}\epsilon$$

$$\frac{d\epsilon}{dt} = \frac{1}{E_0 T_0}[e_0 V_T(\varphi) - U_0]$$

where $\alpha_c = 3.07\times10^{-4}$ is the momentum compaction factor, $h = 176$ is the harmonic number, $E_0 = 3$ GeV is the nominal energy, $T_0 = 1.76$ µs is the revolution period and U_0 is the energy loss per turn. The synchronous phase is given by:

$$e_0 V_T(0) = e_0 V_{rf}(\sin(\varphi_s) + k\sin(n\phi_h)) = U_0$$

The equations of motion above can be derived from a Hamiltonian:

$$H(\varphi,\epsilon) = \frac{1}{2}\epsilon^2 - \frac{1}{\alpha_c^2}\frac{\alpha_c}{2\pi h E_0}\int_0^{\varphi}[e_0 V_T(\varphi') - U_0]d\varphi'$$

and the corresponding equilibrium bunch density distribution is given by:

$$\rho(\varphi,\epsilon) = \rho_0 \exp\left(-\frac{H(\varphi,\epsilon)}{\sigma_p^2}\right)$$

where $\sigma_p = 7.8\times10^{-4}$ is the equilibrium energy spread (determined from the interplay of quantum excitation and radiation damping). This distribution in the (φ,ϵ) phase space can be projected onto the φ axis to yield the longitudinal equilibrium bunch density distribution:

$$\rho(\varphi) = \rho_0 \exp\left(-\frac{1}{\alpha_C^2\sigma_p^2}\Phi(\varphi)\right) \quad \textbf{(2)}$$

where

$$\Phi(\varphi) = \frac{-\alpha_C e_0 V_{rf}}{2\pi h E_0}\Big\{\cos(\varphi_s) - \cos(\varphi+\varphi_s) + \frac{k}{n}\big((\cos(n\phi_h) - \cos(n\varphi+n\phi_h)\big) - (\sin(\varphi_s) + ksin(n\phi_h))\varphi\Big\}$$

The equations above are general in the sense that they apply to both actively and passively operated cavities. In the active case, however, both amplitude and phase can be chosen independently, whereas in the passive case, once the cavity shunt impedance is fixed (by its construction) only one parameter is available for optimization, namely the cavity tuning angle (or equivalently the cavity resonant frequency). In particular, we may choose to operate with "optimized" conditions [7], i.e. adjust the harmonic cavity voltage and phase so that both the first and second derivatives of the voltage at the synchronous phase are zero, so that an approximately quartic potential well is formed. This is possible for both passive and active operation, but in the passive case, optimized conditions are only reached at a given beam current for a given harmonic cavity detuning.

SCALAR SELF-CONSISTENCY

The response of the harmonic cavity to the excitation by the beam can be described by the cavity impedance

$$Z_{HC} = R_s\left(1 + iQ\frac{\omega_r^2-\omega^2}{\omega\omega_r}\right)^{-1} \cong R_s\left(1+iQ\frac{2\Delta f}{f_r}\right)$$

where R_s is the cavity shunt impedance, Q the quality factor and $\omega_r = 2\pi f_r$ the resonant frequency. In terms of the harmonic cavity detuning Δf and tuning angle ψ_h

$$\Delta f = nf_{rf} - f_r$$

$$\tan\psi_h = 2Q\frac{\Delta f}{f_r}$$

we may write the voltage induced in the cavity as

$$V_{HC}(\varphi) = -2I_0 R_s F\cos\psi_h\cos(n\varphi - \psi_h) \quad \textbf{(3)}$$

where I_0 is the stored beam current and we have introduced the bunch form factor

$$F = \left|FT(\rho(t))_{\omega=n\omega_{rf}}\right| \quad \textbf{(4)}$$

given by the Fourier transform of the bunch density distribution at the nth harmonic of the RF frequency and we identify:

$$k = \frac{2I_0 F R_s|\cos\psi_h|}{V_{rf}}$$

$$n\phi_h = \frac{\pi}{2} - \psi_h$$

The equations above give us the recipe for finding $\rho(\varphi)$, namely, given the beam current, harmonic cavity shunt impedance, harmonic cavity tuning angle and a bunch form factor, we may calculate the harmonic cavity voltage from eq. (3), determine the total voltage from eq. (1) and finally calculate the bunch density distribution from eq.(2). From this, the bunch form factor can be calculated back from eq.(4), which leads to a self-consistency equation in one variable (F) of the form

$$F = f(I, R_s, \psi_h, F)$$

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The equilibrium bunch form factor, which is a solution of the equation above, can be easily determined numerically. Figure 1 shows an example of such a calculation, in which the parameters have been chosen to correspond to the optimized case discussed in the previous section. A relatively low shunt impedance of 2.02 MΩ is enough to generate a 54 mm RMS bunch length for 500 mA stored beam current. Note also that bunch shape is quite symmetric and centred around the synchronous phase. In this case, a full self-consistent analysis as described in the following section leads to nearly the same results.

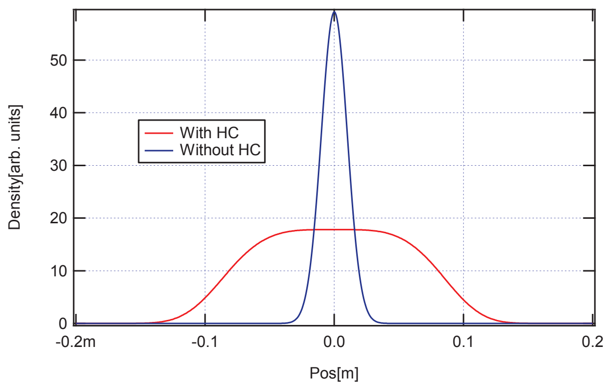

Figure 1: Equilibrium (scalar self-consistency) bunch density distribution for R_s = 2.017 MΩ, HC detuning = -28.4 kHz (Q=21600). The RMS bunch length is 54 mm. Energy loss per turn = 856 keV (machine loaded with 19 IDs). Main cavity voltage = 1.63 MV. For comparison, the distribution with no HC present is also shown, corresponding to an RMS bunch length of 10.12 mm.

FULL SELF-CONSISTENCY

The full self-consistency is implemented by writing the harmonic cavity fields as

$$V_{HC}(\varphi) = kV_{rf}\sin(n\varphi + n\phi_h + \varphi_{FF})$$

where we have added the form factor phase φ_{FF}. The same self-consistent equation used above can be used, but the form factor is now a complex quantity $\mathcal{F} = |\mathcal{F}|e^{i\varphi_{FF}}$ and the numerical root finding algorithm is replaced by a two dimensional minimization procedure.

Figure 2 shows an example of fully self-consistent analysis for shunt impedance of 4.2 MΩ. Here a much larger cavity detuning is used, reducing the Robinson growth rate from the harmonic cavity fundamental mode. We note that a larger bunch lengthening can be obtained at the expense of a non-symmetric bunch shape. Finally, Figure 3 shows the resulting RMS bunch length and peak bunch density as a function of HC shunt impedance for a fixed HC tuning angle.

CONCLUSIONS

We have described a fully self-consistent analysis for the determination of the equilibrium bunch density distribution in a double RF system with passively operated harmonic cavities. The analysis has been applied to the case of the MAX IV 3 GeV ring and indicates that significant bunch lengthening of up to about a factor 5 can be obtained, even at large detuning angles for the harmonic cavities.

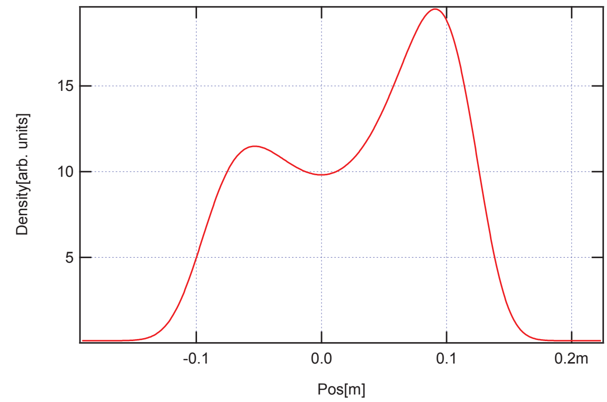

Figure 2: Fully self-consistent equilibrium density distribution for R_s = 4.2 MΩ, HC detuning = -56.5 kHz.. The RMS bunch length is 69 mm. Other parameters are the same as in Figure 1.

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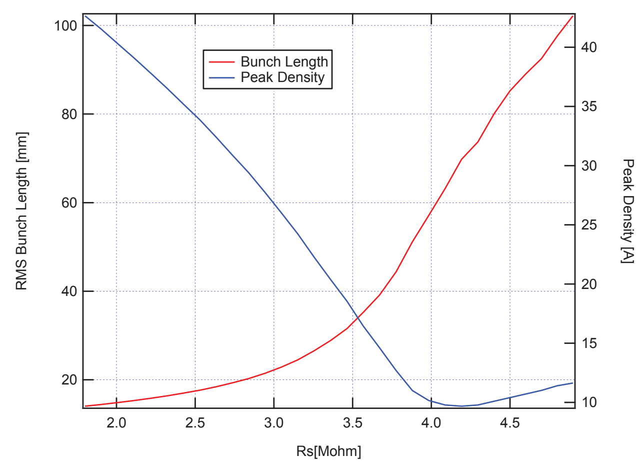

Figure 3: RMS bunch length and peak bunch density as a function of HC shunt impedance for a fixed HC detuning of -56.5 kHz.

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