COLLISIONAL EFFECTS IN PARTICLE-IN-CELL BEAM-BEAM SIMULATIONS*

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Abstract

Self-consistent particle tracking simulations (strong-strong) can be used to investigate the deterioration of colliding beams in a storage ring. However, the use of a small number of macroparticles compared to the real number of particles magnifies the collisional effects and causes numerical noise. In particular, predictions of the emittance lifetime suffer from this numerical noise. In order to produce usable emittance predictions, the contribution of numerical noise to the simulated emittance growth has to be known. In this paper, we apply a diffusion model to strong-strong beam-beam simulations to study the numerical noise driven emittance growth. The scaling of emittance growth with numerical and physical parameters is discussed.

INTRODUCTION

The growth of the emittance and consequently the decay of the luminosity limits the storage time of beams in circular colliders and hence the overall performance. Predictions of the emittance growth are of high value for the design of new colliders like the High Luminosity LHC. Attempts to predict the emittance growth rely on simulations.

One source of emittance growth are beam-beam effects in conjunction with noise [1]. The most accurate simulation scheme of bunch collisions employs particle-in-cell (PIC) methods with each bunch being represented as a set of macroparticles (i. e. strong-strong collisions). These computations produce numerical noise, though, which causes particle diffusion and hence emittance growth. In order to obtain information about the emittance growth due to physical processes, the contribution of numerical effects must be eliminated from the total simulated emittance growth.

One way to avoid numerical noise is the computation of the beam's self-field using an analytic model that approximates the particle distribution. This approach is not self-consistent, though, and therefore not always applicable. If self-consistency is required, the numerically driven emittance growth must be quantified and subtracted from the total growth. For this reason we investigate the scaling of noise effects with various simulation parameters. The goal is to determine the contribution of numerical diffusion to the simulated emittance growth with arbitrary parameters.

In this paper we analyze numerical noise by virtue of an analytic model and simulations. First we discuss a diffusion model for numerical noise. Second we compare self-consistent simulations with simulations using a soft Gaussian model and determine the scaling with physical and numerical parameters.

NUMERICAL EMITTANCE GROWTH

Particle beams are subject to noise which gives rise to a small but finite fluctuation of the transverse position of colliding bunches. As a consequence the bunches do not collide head-on exactly but with a random offset. Due to the random offset and the non-linear shape of the beam-beam force, particles diffuse away from the center. This process is partially responsible for the emittance growth in circular colliders [1].

In PIC simulations, the bunches are represented by a set of macroparticles the number which, N_m, is several orders of magnitude smaller than the number of physical particles, N, in the real system. A scaling factor brings the average field of the macroparticle distribution in agreement with the real field. However, the macroparticle distribution is much coarser, due to the poor statistics with $N_m/N \ll 1$ and the upscaling magnifies local fluctuations. Even though a numerical mesh is used to smooth out some of those numerical fluctuations in PIC simulations, in practice, those fluctuations are not completely removed. This results in residual numerical noise in self-consistent beam-beam simulations.

Besides the numerical noise, a second important source of numerical error lies in interpolations. In each time step, the charge distribution due to the macroparticles is interpolated onto a grid. On that grid, the discrete Poisson equation is solved. The field is then interpolated to the position of the macroparticles where the corresponding kick is exerted. Both interpolations generate numerical errors.

Numerical noise can enhance the simulated emittance growth non-negligibly and even dominate over the physical growth under realistic conditions [2]. The noise described before can be avoided by calculating the beam-beam force analytically as an approximation, if an analytic expression for the self-field is known, e. g. for Gaussian beams. A self-consistent and a soft Gaussian simulation with equal beam parameters are shown in Fig. 1 for comparison. The emittance growth in the self-consistent simulation is clearly driven by noise.

However, this approach is limited because it violates self-consistency and may introduce errors, in particular if the particle distribution changes. Since self-consistency is required to acquire correct results sometimes, we try to quantify the numerical perturbations as a function of all relevant physical and numerical parameters. First we consider an analytic model to describe the emittance growth.

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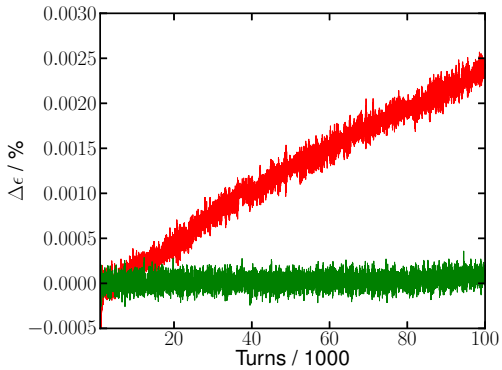

Figure 1: Emittance growth in a self-consistent (red) and a soft Gaussian simulation (green). The simulation parameters correspond to those described in section *Simulations*.

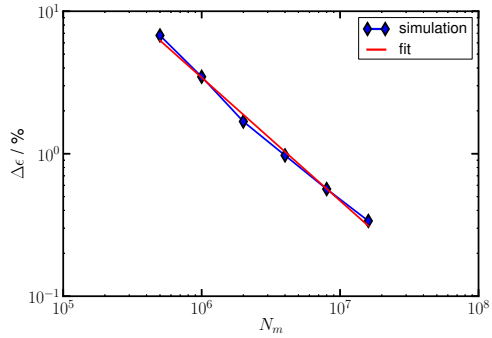

Figure 2: Numerical emittance growth versus intensity and fit.

Table 1: Expected exponent from Eq. 7 and Eq. 8, respectively, and result from fit to data.

Parameter	Theory	Simulation
N_m	-1	-0.86
N	2	2.3
ϵ	-2	-2.2

An analytic model for particle diffusion due to physical and numerical effects is described in Ref. [3]. Solving the diffusion equation with constant diffusion coefficient D,

$$\dot{\Phi}(t,x) = D\Phi''(t,x) \tag{1}$$

yields

$$\Phi(t,x) = \frac{1}{2\sqrt{Dt}} \exp\left(-\frac{x^2}{4Dt}\right). \tag{2}$$

Applying this to the horizontal size of a beam, we find

$$2Dt = \sigma^2(t) \propto \epsilon(t), \tag{3}$$

i. e. the emittance is a linear function of time and the diffusion coefficient.

D is related to a friction parameter β_f by virtue of [3]

$$D = \beta_f \frac{k_B T}{m}, \tag{4}$$

where k_B is the Boltzmann constant, T the beam temperature and m the particle mass. For physical collisions, the friction is proportional to the particle density,

$$\beta_f \propto \frac{N}{\epsilon_x \epsilon_y \epsilon_z}. \tag{5}$$

The friction in PIC simulations is given by

$$\beta_f^{sim} = \beta_f \frac{N}{N_m}. \tag{6}$$

Inserting Eq. 4 to Eq. 6 into Eq. 3 yields

$$\dot{\epsilon} \propto \frac{N^2}{N_m \epsilon_x \epsilon_y \epsilon_z}, \tag{7}$$

a scaling law for particle number, macroparticle number and the emittances.

SIMULATIONS

A series of PIC simulations was run with BeamBeam3D [4] to evaluate the emittance growth, varying one parameter at a time. The beam parameters correspond to the nominal LHC parameters [5], with exception of the vertical tune. The vertical tune was set equal to the horizontal tune ($Q = 64.31$) to avoid complications due to heterogeneous emittance evolutions. When changing the emittance, we preserved the circular shape of the beams $\epsilon_x = \epsilon_y = \epsilon$. From Eq. 7 follows

$$\dot{\epsilon} \propto \frac{1}{\epsilon^2}. \tag{8}$$

As default numerical parameters we used $N_m = 10^6$, a transverse mesh with 128^2 cells, and 4×10^5 turns. The bunches were intialized with a Gaussian distribution. For simplicity the bunches were not sliced longitudinally. Optional features like crab cavities and the transverse damper were switched off.

For comparison, for each setting a self-consistent and a soft Gaussian simulation were performed. With a single exception (for the smallest value of N considered), the emittance growth in the soft Gaussian simulations did not exceed 10 % of that in the corresponding self-consistent simulations. The results of the sweeps through N_m, N and ϵ are shown in Fig. 2, Fig. 3 and Fig. 4, respectively. The data from the soft Gaussian runs were subtracted to account for numerical effects only. The red lines in these plots correspond to linear fits to the logarithm of the the data. The scaling obtained from the fits are summarized in Table 1. The fits agree with the model within 15%.

In addition to the parameters included in the model (Eq. 7), we examined the emittance growth as a function of

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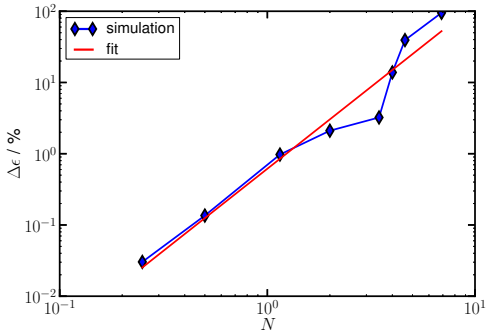

Figure 3: Numerical emittance growth versus number of macroparticles and fit.

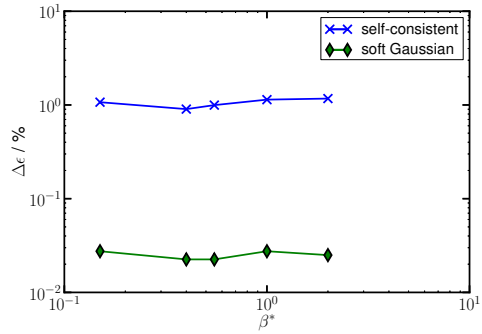

Figure 5: Emittance growth versus β^* for self-consistent and soft Gaussian simulation.

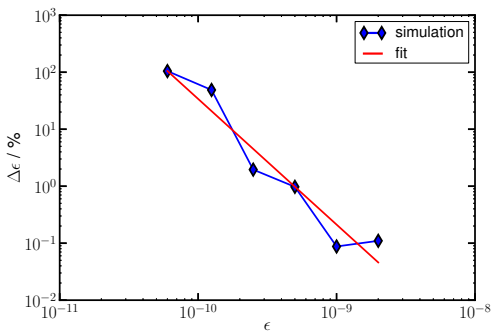

Figure 4: Numerical emittance growth versus transverse emittance.

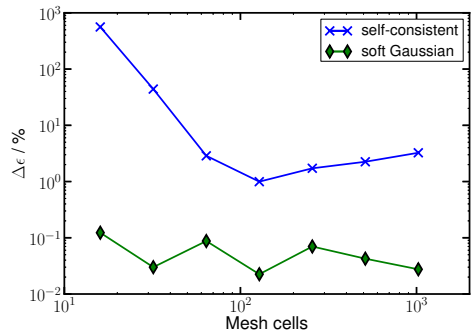

Figure 6: Emittance growth versus number of mesh cells for self-consistent and soft Gaussian simulation.

the beta function at the collision point, β^*, and the number of mesh cells. A strong dependence on β^* was not expected since the beam-beam parameter for circular beams,

$$\xi = \frac{r_p N}{4\pi\gamma\epsilon}, \tag{9}$$

where we introduced the classical proton radius r_p, and the Lorentz factor γ, does not depend on β^*. However, since the beam-beam force is non-linear, some impact on the emittance could not be excluded. As Fig. 5 reveals, that impact, if any, is not detectable in our simulations.

The number of the mesh cells, or their size, is crucial for the interpolation error. On one hand, the numerical error should decrease with increasing number of mesh cells. This expectation is confirmed for very coarse grids, as Fig. 6 shows. On the other hand, as the number of mesh cells increases, the number of macroparticles per cell decreases. This leads to larger local numerical noise. Beyond 128 cells in horizontal and vertical direction, the numerical noise effects become dominant and the emittance growth starts to accelerate.

CONCLUSION

A diffusion model for numerical emittance growth in beam-beam simulations has been compared to self-consistent strong-strong simulations. Soft Gaussian simulations showed the emittance growth in the self-consistent simulations was dominated by numerical effects. An approximate agreement was found for the scaling with the beam intensity, transverse emittance and number of macroparticles. No dependence of the growth on the beta function at the interaction point was found. The effect of the grid size is more complicated and can not be described by a single power function.

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