GENERAL BEAM LOADING COMPENSATION IN A TRAVELING WAVE STRUCTURE

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Abstract

The well-known beam loading in a traveling wave structure is in fact a resistive beam loading which bunches travel on the crest. This type of beam loading could be compensated by increasing RF feed power. But generally, bunches could travel on each phase. General beam loading compensation is well-known for a single cell cavity and it is done by changing the RF feed power and detuning the structure together [1,2]. In this paper, the concept of detuning for a TW structure will be shown and the evolution of fundamental mode beam-induced field will be derived and finally, it will be shown how to compensate beam loading by changing the phase velocity in comparison to the beam velocity.

INTRODUCTION

Beam loading compensation in general form is well-known for a single cell and standing wave cavities and it was solved by P. B. Wilson by the equivalent circuit and phasors analysis [1, 2]. In his analysis, he compensate the beam loading by changing the RF feed power and detuning the structure together. Detuning means there is a small difference between resonant frequency of the structure and the RF source frequency. Extending the SW result to a TW structure is not so obvious and straight-forward. Also the concept of detuning for a TW structure is not so obvious. Also it will not give us the field evolution inside the structure. S. Arai [3] focused on a TW structure and L. Thorndahl [4] used his own method to solve this problem for the CTF3 SHBs as a TW structure. In this paper, another method is used that shows the concept of detuning in a TW structure and gives us the detail of fundamental mode beam-induced field inside a TW structure. This method was found during designing of the CLIC drive beam sub harmonic bunchers [5].

DETUNING CONCEPT IN A TRAVELING WAVE STRUCTURE

Detuning ($\Delta\omega$) is equal to ω_0-ω and figure 1 shows the meaning of ω and ω_0. In my definition, ω_0 is the angular frequency in the dispersion function which its related phase velocity is equal to the beam velocity (v_e). The RF source could work in a different angular frequency (ω) that produce an electro-magnetic wave with different phase velocity ($v_p \neq v_e$) inside the structure. The repetition frequency of bunches is equal to the RF source frequency. Equation 1 shows the relation between detuning and phase, beam and group velocities.

$$k_b - k = \frac{\Delta\omega}{v_g} \Rightarrow \frac{\omega_0}{v_e} - \frac{\omega}{v_p} = \frac{\Delta\omega}{v_g}$$
$$\Rightarrow \frac{\Delta\omega}{\omega} = \frac{\frac{1}{v_e} - \frac{1}{v_p}}{\frac{1}{v_g} - \frac{1}{v_e}} \tag{1}$$

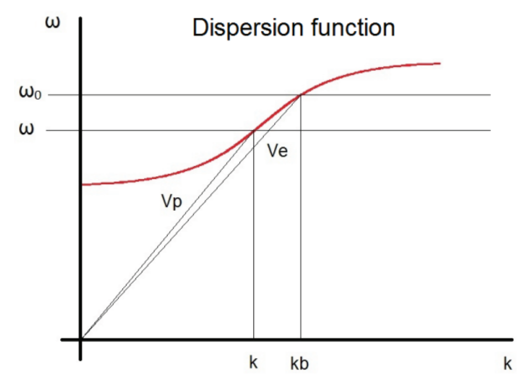

Figure 1: TW structure dispersion function and the meanings of ω and ω_0.

FORWARD TRAVELING WAVE STRUCTURE

Figure 2 shows a forward TW structure schematic layout with length L (group velocity (v_g)>0). The reference bunch (p=0) enters the structure at time t=0. The structure is divided to n sections. The length of each section is equal to the distance energy travels for a bunch time interval ($\Delta z = v_{g0} T_b$). Our goal is to find on-axis electric field in each section.

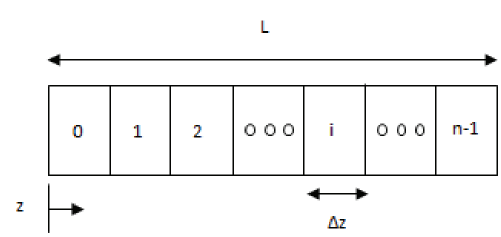

Figure 2: Schematic layout of a forward TW structure.

In first assumption, each bunch produced an electro-magnetic wave with the phase velocity equal to the beam velocity (v_e) and its angular frequency is ω_0 as described previously. This wave wants to decrease the bunch energy and transfer it to the structure or $E_{b0}/q<0$. The on-axis electric field for the reference bunch (p=0) and previous bunches (p>0) could be written as:

$$\begin{cases} E_{b,p} = E_{b0} e^{j(k_b z - \omega_0 (t - t_p))} e^{-\alpha_0 p \Delta z} \\ t_p = -pT_b = -\dfrac{p}{f} = -2\pi \dfrac{p}{\omega}, p \geq 0 \\ \Delta z = v_{g0} T_b \\ E_{b0} = -F \dfrac{\omega_0}{2} \dfrac{r}{Q_0} q = -\pi F \dfrac{r}{Q_0} I \\ k_b = \dfrac{\omega_0}{v_e} \\ \alpha_0 = \dfrac{\omega_0}{2 Q_0 v_{g0}} \end{cases} \tag{2}$$

E_{b0} is a single bunch induced field magnitude. F, T_b, r, q and I are the bunch form factor, the bunch time interval, the shunt impedance per meter, the bunch charge and current, respectively. The reference bunch sees previous bunches induced fields plus half of its induced field (fundamental theorem of beam-loading) [1,2]. Equation 3 shows the total bunch induced fields seen by the reference bunch at section i.

$$E_{i,b} = \tfrac{1}{2} E_{b0} e^{j(k_b z_i - \omega_0 t)} + \sum_{p=1}^{i} E_{b0} e^{j(k_b z_i - \omega_0 (t + pT_b))} e^{-\alpha_0 p \Delta z}, z_i = i\Delta z \tag{3}$$

This equation could be rewritten like below:

$$\begin{cases} E_{i,b} = E_{b0} e^{j(k_b z_i - \omega_0 t)} \left[\tfrac{1}{2} + A e^{i\theta} \left(1 - e^{-\left(j\frac{\Delta\omega}{v_{g0}} + \alpha_0\right) z_i}\right) \right] \\ A e^{i\theta} = \dfrac{e^{-\left(j2\pi\frac{\Delta\omega}{\omega} + \alpha_0 \Delta z\right)}}{1 - e^{-\left(j2\pi\frac{\Delta\omega}{\omega} + \alpha_0 \Delta z\right)}} \end{cases} \tag{4}$$

Beam loading compensation mechanism

Before going into more details, it is better to know how the beam loading compensation mechanism works. By assumption $\Delta\omega<<\omega$ and $\alpha_0<<1$ and knowing $t=z_i/v_e$ for the reference bunch, the equation 4 simplifies to:

$$E_{i,b} = E_{b0} \left(\tfrac{1}{2} + \frac{f z_i}{v_{g0}} \right) = E_{b0} \left(\tfrac{1}{2} + i \right) \tag{5}$$

Figure 3 shows this field evolution inside the structure. Now, if we look at RF source fundamental mode induced field inside the structure and the reference bunch related phase progress, we can understand how beam compensation mechanism works. If the reference bunch related phase is changed from point 1 to 2 in the fig. 4 when it crosses the structure with the proper rate, the beam loading could be compensated point by point and it works like it is located at the requested phase with phase velocity equals the beam velocity (see Fig. 4).

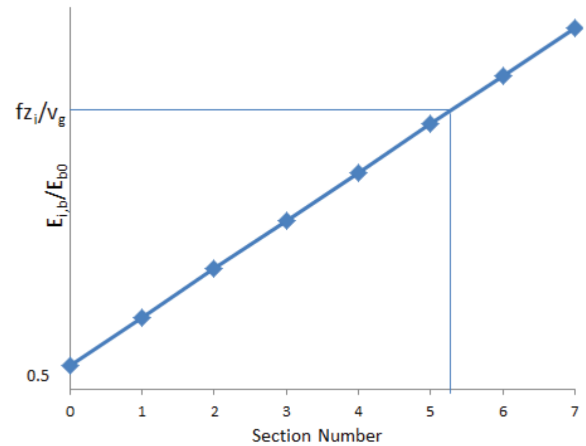

Figure 3: Field evolution inside a TW structure.

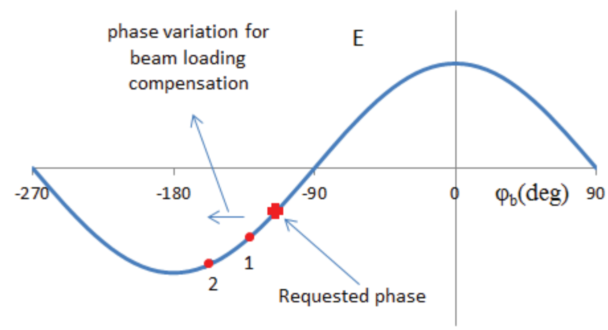

Figure 4: RF source induced field and the bunch related phase progress during crossing the structure.

Equation 6 shows the RF source fundamental mode induced field inside the structure. φ_b shows the related bunch phase as you can see in Fig. 4.

$$\begin{cases} E_g(z) = E_{g0} e^{j(kz - \omega t + \varphi_b)} e^{-\alpha z} \\ E_{g0} \geq 0, real \\ \alpha = \dfrac{\omega}{2 Q_0 v_g} \end{cases} \tag{6}$$

Therefore, the total field seen by the reference particle using equation 1, 4 and 6 by knowing $t=z_i/v_e$ is:

ISBN 978-3-95450-122-9

$$E(z) = E_{g0} e^{-jz\Delta\omega\left(\frac{1}{v_g}-\frac{1}{v_e}\right)} e^{j\varphi_b} e^{-\alpha z} + E_{b0}\left[\tfrac{1}{2} + Ae^{i\theta}\left(1 - e^{-\left(j\frac{\Delta\omega}{v_{g0}}+\alpha_0\right)z}\right)\right] \tag{7}$$

By another assumption, z_i is replaced by z to extend the result to each location for a smooth field variation inside the structure. Now we define $\Delta E(z)=E(z)-E(0)e^{-\alpha z}$. We can compensate the beam loading by keeping $|Re(\Delta E(z))|$ close to zero. Equation 8 shows the amount of $\Delta E(z)$.

$$\frac{\Delta E(z)}{e^{-\alpha z}} = -E_{g0} e^{j\varphi_b}\left(1 - e^{-jz\Delta\omega\left(\frac{1}{v_g}-\frac{1}{v_e}\right)}\right) + E_{b0} A e^{i\theta}\left(1 - e^{-\left(j\frac{\Delta\omega}{v_{g0}}+\alpha_0\right)z}\right) - \frac{E_{b0}}{2}\left(1 - e^{\alpha z}\right) \tag{8}$$

It is not possible to keep $Re(\Delta E(z))$ zero for all z but for the case that the attenuation is low ($\alpha L, \alpha_0 L \ll 1$) , $\Delta\omega \ll \omega$ and $v_g \ll v_e$, this can be reached if:

$$\frac{\frac{1}{v_e} - \frac{1}{v_p}}{\frac{1}{v_g} - \frac{1}{v_e}} = \frac{\Delta\omega}{\omega} = -\frac{1}{2\pi}\frac{E_{b0}\sin(\varphi_b)}{E_{g0}} \tag{9}$$

This amount of detuning is equal to the SW case [1,2]. You can see also, the detuning could be reached by the proper difference between phase and beam velocity. For the beam loading compensation, RF feed power and the bunch entrance phase should also be changed a little as you can see below:

$$\begin{cases} E(0) = E_{g0} e^{j\varphi_b} + \frac{1}{2} E_{b0} = E_{g0,n} e^{j\varphi_{b,n}} \\ E_{g0} = \sqrt{\left(E_{g0,n}\cos(\varphi_{b,n}) - \frac{1}{2}E_{b0}\right)^2 + \left(E_{g0,n}\sin(\varphi_{b,n})\right)^2} \\ \tan(\varphi_b) = \dfrac{E_{g0,n}\sin(\varphi_{b,n})}{E_{g0,n}\cos(\varphi_{b,n}) - \frac{1}{2}E_{b0}} \\ \dfrac{P_{g,n}}{P_g} = \left(\dfrac{E_{g0,n}}{E_{g0}}\right)^2 \end{cases} \tag{10}$$

$E_{g0,n}$ and $\varphi_{b,n}$ is the desired on-axis electric field amplitude and the bunches entrance phase (requested phase in fig.4) . Then the electric field and the entrance phase should be equal to E_{g0} and φ_b, respectively and new RF feed power ($P_{g,n}$) is found.

BACKWARD TRAVELING WAVE STRUCTURE

To analyze a backward traveling wave structure, the z axis origin in fig.1 should be located at the end of structure and its direction should be reversed. With this change, group velocity remains positive but phase and beam velocity become negative and all derived equations remain unchanged.

BUNCHING PHASE

Bunching phase is a very important special case. Bunching phase is φ_b=-90° for negative charges and φ_b=90° for positive charges. At this phase we have maximum bunching. Below you can find brief results for the bunching phase case:

$$\begin{cases} \dfrac{\frac{1}{v_e} - \frac{1}{v_p}}{\frac{1}{v_g} - \frac{1}{v_e}} = \dfrac{\Delta\omega}{\omega} = \dfrac{1}{2\pi}\left|\dfrac{E_{b0}}{E_{g0}}\right| \\ E_{g0} = \sqrt{\frac{1}{4}E_{b0}^2 + E_{g0,n}^2} \\ \varphi_b = \mp\dfrac{\pi}{2} + a\tan\left(\left|\dfrac{E_{b0}}{2E_{g0}}\right|\right) \\ \dfrac{P_{g,n}}{P_g} = \left(\dfrac{E_{g0,n}}{E_{g0}}\right)^2 \end{cases} \tag{11}$$

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