

# OPTIMIZATION OF TRIPLET QUADRUPOLES FIELD QUALITY FOR THE LHC HIGH LUMINOSITY LATTICE AT COLLISION ENERGY \* †

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## Abstract

Beta functions at two interaction points (IP) in the high luminosity LHC upgrade lattice (HL-LHC) at collision energy will be significantly reduced compared to the nominal LHC lattice. This will result in much higher beta functions in the inner triplet (IT) quadrupoles adjacent to these IPs. The consequences are a larger beam size in these quadrupoles, higher IT chromaticity, and stronger effects of the IT field errors on dynamic aperture (DA). The IT chromaticity will be compensated using the Achromatic Telescopic Squeezing scheme [1]. The increased IT beam size will be accommodated by installing large aperture Nb3Sn superconducting quadrupoles with 150 mm coil diameter. The stronger effects of the IT field errors can be remedied by optimizing the IT field error specifications. The latter must satisfy two conditions: provide an acceptable DA and be compatible with realistically achievable field quality. Optimization of the IT field errors was performed for the LHC upgrade layout version SLHCV3.01 with IT gradient of 123 T/m and IP beta functions of 15 cm. Dynamic aperture calculations were performed using SixTrack. Details of the optimization are presented along with recommendation for improving the field error correction.

## INTRODUCTION

In order to increase the LHC luminosity, beta functions at collision energy at the interaction points IP1 and IP5 in the high luminosity upgrade lattice (HL-LHC) will be significantly reduced compared to the nominal lattice. This will result in much higher beta functions in the inner triplet (IT) quadrupoles adjacent to these IPs. The consequences are a larger beam size in these quadrupoles, higher IT linear and nonlinear chromaticity, and stronger impact of the IT non-linear field errors on dynamic aperture (DA). New large aperture Nb3Sn superconducting IT quadrupoles with 150 mm coil diameter will be installed to accommodate the increased beam size as well as improve the field quality. The IT chromatic effects will be compensated using the Achromatic Telescopic Squeezing scheme [1]. The remaining issue is the stronger effects of the IT non-linear field errors which create tune shift and excite high-order resonances, thus limiting the DA. These effects may be significant to high order because the corresponding non-linear field is not negligible at large particle trajectories in the IT

due to the high beta functions. The most critical low order field errors will be compensated by the included IT non-linear field correctors up to the 6th order [2]. The effects of uncorrected high order errors require evaluation and optimization, leading to field quality specification for the 150 mm aperture IT quadrupoles.

The desired IT field quality should satisfy two conflicting conditions: 1) the field errors must be sufficiently small for achieving an acceptable DA, however 2) they must be sufficiently large to be compatible with realistically achievable field quality. Meeting these conditions requires optimization of the field errors based on their impact on the DA, including the effects of the IT correction.

In this study, the optimization was performed for the LHC upgrade layout version SLHCV3.01, where the IT quadrupole gradient is 123 T/m and  $\beta_{x,y}^* = 15$  cm at the IP1 and IP5. The acceptable minimum DA was set to  $10.5\sigma$ , where  $\sigma$  is the rms beam size. The DA calculation was done using long-term tracking in SixTrack [3]. The typical tracking conditions were:  $10^5$  turns, 11 angles, 30 particle pairs per  $2\sigma$  amplitude step, 60 random error seeds, 7 TeV beam energy with initial energy offset  $\Delta p/p = 2.7 \times 10^{-4}$ , normalized beam emittance of  $3.75 \mu\text{m}\cdot\text{rad}$ , and betatron tune of 62.31, 60.32. Note that the number of turns and random seeds affects the accuracy of the DA calculation which is at least  $0.1\sigma$  in this case. Besides the IT errors, the tracking included arc errors and their correction, and the low order IT non-linear field correctors [2]. The latter compensate the effects of IT  $a_3, b_3, a_4, b_4, b_6$  field terms (see definition below). No field errors were included in the interaction region D1 and D2 separation dipoles and Q4 quadrupoles. The error study for these magnets is presented separately [4]. Finally, no beam-beam effects are included in this study.

## EXPECTED IT FIELD QUALITY

Magnetic field in a quadrupole can be expressed as [5]

$$B_y + iB_x = 10^{-4} B_{2r_0} \sum_{n=2}^{\infty} (b_n + ia_n) \left( \frac{x + iy}{r_0} \right)^{n-1}, \quad (1)$$

where  $a_n, b_n$  are skew and normal coefficients in units of  $10^{-4}$  at a reference radius  $r_0$ , and  $B_{2r_0}$  is the main quadrupole field at  $r_0$ . In LHC design, each of  $a_n$  and  $b_n$  is split in the mean (m), uncertainty (u) and random (r) terms related to systematic and random type errors defined in terms of Gaussian sigmas of the error distribution (see detailed description e.g. in [2]).

The expected to be achieved field quality can be evaluated in magnetic field calculations. Table 1 shows the latest estimate of the achievable field quality in a 150 mm aperture quadrupole [6, 7]. This table will be referred to

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Table 1: Expected to achieve field errors in 150 mm aperture quadrupoles at  $r_0 = 50$  mm (error table “target6”).

$n$	$a_{nm}$	$a_{nu}$	$a_{nr}$	$b_{nm}$	$b_{nu}$	$b_{nr}$
3	0	0.800	0.800	0	0.820	0.820
4	0	0.650	0.650	0	0.570	0.570
5	0	0.430	0.430	0	0.420	0.420
6	0	0.310	0.310	0.800	1.100	1.100
7	0	0.190	0.190	0	0.190	0.190
8	0	0.110	0.110	0	0.130	0.130
9	0	0.080	0.080	0	0.070	0.070
10	0	0.040	0.040	0.150	0.200	0.200
11	0	0.026	0.026	0	0.026	0.026
12	0	0.014	0.014	0	0.018	0.018
13	0	0.010	0.010	0	0.009	0.009
14	0	0.005	0.005	-0.040	0.023	0.023

as error table “target6”. The goal of the IT error optimization is to maximize the  $a_n, b_n$  towards the values in the table “target6” while providing sufficient DA. To avoid extremely tight tolerances, we set an additional constraint that the  $a_n, b_n$  are not smaller than 50% of the values in the table “target6”.

### OPTIMIZATION

Previously, the DA sensitivity to the IT field errors was studied in detail [8], and an optimized error table “target39” was constructed providing a minimum DA of  $12.3\sigma$ . This solution, however, did not take into account if the optimized  $a_n, b_n$  terms are realistically achievable. As a result, the coefficients in tables “target39” and “target6” do not match well as seen in Fig. 1. For example, the high order skew uncertainty terms  $a_{nu}$  in the table “target39” are too loose, while many other terms are too tight. Nonetheless, the results of this study are useful since they determine the general sensitivities to the IT errors.

As a next step in the optimization process, we take into account the expected to achieve errors in table “target6” and use a lower minimum DA value of  $10.5\sigma$ . The latter helps reducing the impact of the high order terms since their field is proportional to high power of  $x$  and  $y$ . It is desirable that the high order terms are not too tight since it may be more difficult to achieve their specifications. The

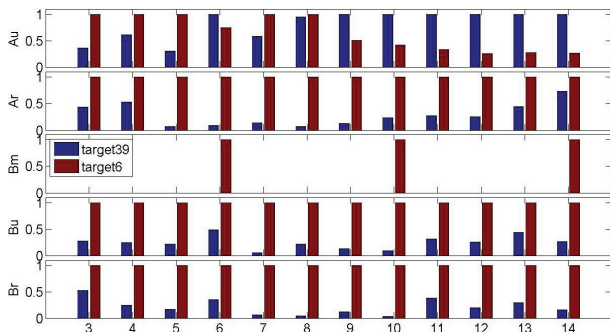


Figure 1: Comparison of  $a_n, b_n$  values in the previously optimized error table “target39” and the expected to achieve values in table “target6” (normalized units).

table “target6” will be used as a new basis for the study of the DA sensitivities and optimization. The results of the previous study [8] will help to guide the new optimization. For an easy comparison with the table “target6”, the  $a_n, b_n$  terms will be presented in relative units normalized to “target6” values. It is worth noting that the terms  $b_6, b_{10}, b_{14}$  in the table “target6” are relatively large since these are the so called “allowed” terms due to the symmetry of the quadrupole coil. In addition, the expected average values of these terms ( $b_{6m}, b_{10m}, b_{14m}$ ) are not zero.

To test the impact of the “target6” errors, we verify the DA for two cases where in the first case all the  $a_n, b_n$  are set to “target6” values and in the second case to 50% values. The resultant minimum DA over 60 random seeds and 11  $x-y$  angles is  $6.35\sigma$  and  $8.33\sigma$ , respectively, which is well below the desired  $10.5\sigma$ . Since we restricted the specifications to be at least 50% of the “target6” values, the only way to increase the DA is to improve the IT error correction. As mentioned earlier, this lattice layout SLHCV3.01 includes IT correctors for compensation of  $a_3, b_3, a_4, b_4, b_6$  terms. Since the low order terms typically have stronger impact on the DA, it is reasonable to study additional IT correctors for  $a_5, b_5, a_6$  terms. In fact, these correctors have been already implemented in the other LHC lattice SLHCV3.1b [2]. Since these correctors are not included in the SLHCV3.01, we simulate their effect by allowing smaller  $a_5, b_5, a_6$  values assuming they are residual errors after the correction. To determine their acceptable settings, these terms are scanned from 0 to 0.5 (in units relative to “target6” values) when all other  $a_n, b_n$  terms are set to 0.5. The DA sensitivity in this combined scan is shown in Fig. 2. One can see that the acceptable value for the residual  $a_5, b_5, a_6$  terms (after correction) is 0.2 providing minimum DA of about  $10.5\sigma$ . It seems reasonable to expect that the IT correctors should be able to provide such correction assuming the uncorrected values of  $a_5, b_5, a_6$  are at least 0.5. The combination of IT errors where  $a_5, b_5, a_6$  are set to 0.2 and all other terms to 0.5 (relative to “target6”) will be further used as a reference point for studying the other terms. Scans of the uncertainty and random terms of the  $a_5, b_5, a_6$  show that the DA is least sensitive to the  $a_{5u}, a_{6u}$  components which can be somewhat relaxed.

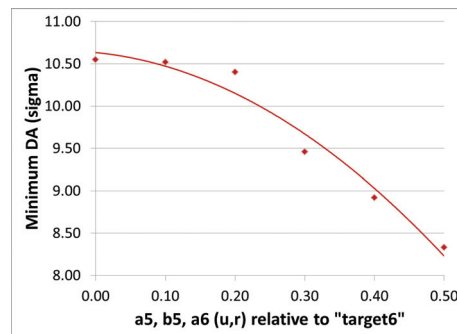


Figure 2: Combined scan of uncertainty and random  $a_5, b_5, a_6$  terms when all the other  $a_n, b_n$  are set to 0.5 (in normalized to “target6” units).

Below, we focus on the terms having potentially a stronger impact on the DA. The previous studies determined that the IT correctors provide a reasonably good compensation for the errors up to the 6th order. Therefore, as a next step we verify the lowest order uncorrected terms  $n = 7, 8$ . Fig. 3 shows that the terms  $b_7, b_8$  (u,r) and  $a_{8r}$  have a rather strong impact on the DA. Consequently, these terms must stay at the minimal 0.5 setting.

Since the expected “allowed” terms  $b_{10}, b_{14}$  are relatively large and the average terms  $b_{6m}, b_{10m}, b_{14m}$  are not zero, it is logical to verify their effect. The tracking confirmed that these terms, except the  $b_{6m}$ , decrease the DA at a higher than 0.5 settings. Therefore, they cannot be relaxed. Examples of  $b_{10}, b_{14}$  scans are shown in Fig. 4. Other tracking scans confirmed that the high order terms, especially the  $a_{nu}$ , and the corrected lowest order terms at  $n = 3, 4$  have a weak impact on the DA. Their specifications can be set close to the values in the table “target6”.

### OPTIMIZED FIELD ERROR TABLE

The performed scans of the individual error terms and their combinations help to determine their optimal settings. However, when combined together the accumulated effect of all the terms on the DA is increased. Therefore, the final optimization of the IT error table satisfying the minimum DA requires additional adjustment to some of the terms. Table 2 shows the present best optimized IT error specification table “target65”. Here, many of the terms are set to 0.8-1.0 level relative to values in the table “target6”. The

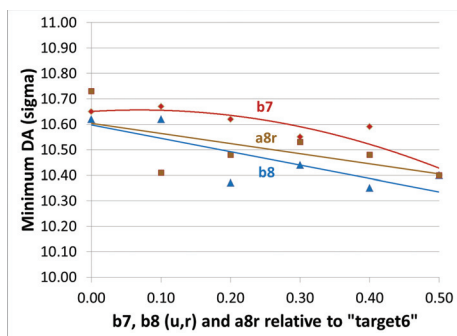


Figure 3: DA sensitivity to  $b_7, b_8$  and  $a_{8r}$ , where  $b_7, b_8$  include the uncertainty and random parts.

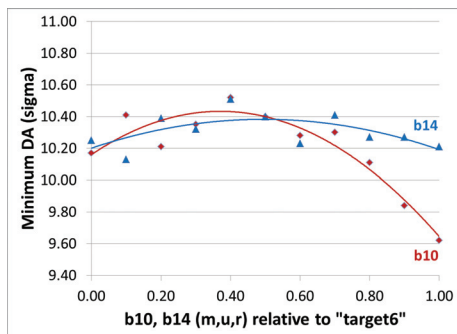


Figure 4: DA sensitivity to  $b_{10}$  and  $b_{14}$  including the mean, uncertainty and random parts.

Table 2: Optimized error table “target65” at  $r_0 = 50$  mm.

$n$	$a_{nm}$	$a_{nu}$	$a_{nr}$	$b_{nm}$	$b_{nu}$	$b_{nr}$
3	0	0.800	0.800	0	0.820	0.820
4	0	0.650	0.650	0	0.570	0.570
5	0	0.086	0.086	0	0.084	0.084
6	0	0.155	0.062	0.800	0.550	0.550
7	0	0.152	0.095	0	0.095	0.095
8	0	0.088	0.055	0	0.065	0.065
9	0	0.064	0.040	0	0.035	0.035
10	0	0.040	0.032	0.075	0.100	0.100
11	0	0.026	0.0208	0	0.0208	0.0208
12	0	0.014	0.014	0	0.0144	0.0144
13	0	0.010	0.010	0	0.0072	0.0072
14	0	0.005	0.005	-0.020	0.0115	0.0115

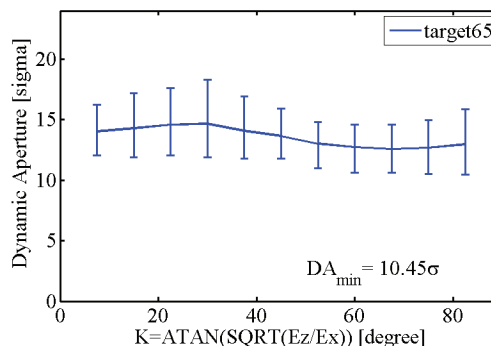


Figure 5: Dynamic aperture for the error table “target65”.

more sensitive  $b_6$ - $b_{10}$  (u,r),  $b_{10m}, b_{14}$  (m,u,r) and  $a_{7r}$ - $a_{9r}$  terms are set to 0.5 level. The lower settings for  $a_5, b_5, a_6$  assume that these are residual errors after the IT correction. Their actual values before correction will be higher. The corresponding dynamic aperture for the table “target65” is shown in Fig. 5, where the line is the average DA as a function of  $x$ - $y$  angle, and the bars show the DA spread for 60 seeds. The minimum DA is  $10.45\sigma$ .

Finally, we would like to mention that the DA is a function of the working tune. Our test calculation of the tune dependence showed that the aperture can be increased by more than  $1\sigma$  by slightly reducing the  $x$  and  $y$  tunes. Such tune optimization would give operational flexibility for improving the DA. However, this calculation did not include the beam-beam effects. Therefore, a more realistic simulation with beam-beam effects should be done.

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