

# WAKE-FIELD REDUCTION IN HYBRID PHOTONIC CRYSTAL ACCELERATOR CAVITIES

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## Abstract

Photonic crystals (PhCs) have attractive properties for manipulating electromagnetic radiation. In one application, PhCs are composed of a number of dielectric rods that can be arranged to make an accelerator cavity. These structures trap an accelerating mode and allow higher-order modes to propagate out. This work focuses on optimizing PhC cavities to reduce transverse wake-fields by minimizing the presence of unwanted modes that deflect bunches off axis and lead to beam instabilities. The transverse wake-fields in the optimized structure are compared with transverse wake-fields for a triangular lattice structure and the CLIC cavity with higher-order mode (HOM) damping.

## INTRODUCTION

For many applications, transverse wake-fields in accelerators are undesirable. Transverse wake-fields deflect particle bunches off-axis and lead to beam instabilities that limit luminosity. Photonic crystal accelerator cavities are attractive because they have lower wake-fields than traditional closed metal cavities.

Photonic crystals are periodic arrays of electromagnetic wave scatterers (in this case, made of dielectric rods) that have a frequency band gap — a range of frequencies where no electromagnetic modes exist. This allows one to trap an accelerating mode by removing a rod from the lattice and filling the defect with radiation of a frequency within the band-gap. Because these structures are open, they allow HOMs to propagate out, without the need for additional damping mechanisms to be built in to the structure.

Our work investigates optimizing the design of photonic crystal accelerator cavities to minimize transverse wake-fields. Lattice structures, such as the 4-layer triangular lattice structure shown in Fig. 1, have significantly lower transverse wake-fields than closed metal cavities, but have higher transverse wake-fields than the Compact Linear Collider (CLIC) with HOM-damping waveguides.

In order to minimize transverse wake-fields, we use an optimization process that adjusts the rod positions in the structure. Our results show significant lowering of transverse wake-fields in the optimized structure, compared to the 4-layer triangular lattice structure. The transverse wake-fields in the optimized structure remain comparable to the CLIC cavity with HOM-damping waveguides; however, a large number of parameters remain to be optimized in the photonic crystal cavities. We describe future efforts to reduce transverse wake-fields further.

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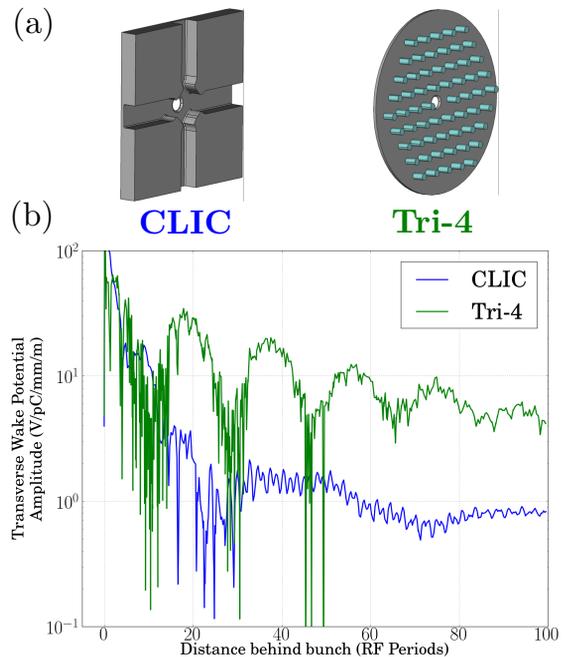


Figure 1: (a) Cross-sections of the CLIC and Tri-4 Cavities. (b) Wake-field comparison of CLIC and Tri-4 cavities. Computational calculations [7] are done for 8-cell 3D cavities.

## OPTIMIZATION PROCESS

The optimization routine uses a Nelder-Mead simplex [1, 2], where the  $x$  and  $y$  rod locations are the degrees of freedom. The optimization process works by minimizing a ‘cost’ value calculated for each step.

Each step in our optimization process corresponds to an electromagnetic simulation run with a given set of rod positions. We use VORPAL FDTD software [3] to simulate the electromagnetics, and then calculate the cost value based on the results of each simulation. After calculating the cost value, the optimization moves the rods and eventually finds a downhill slope to follow.

To address the transverse wake-fields in the cost value, we looked for correlations between 2D cavity modes and transverse wake-field calculations in 3D (3D calculations are done for 8-cell cavities, where one cell consists of dielectric rods between metal end-caps). For example, the Fourier transform of the transverse wake potential (Fig. 2) of the 4-layer triangular lattice structure shows high peaks near 15 GHz (the accelerating mode is chosen to be at 12 GHz, the same as CLIC). These peaks correspond to dif-

ferent modes, and 2D mode extractions calculated from the cavity fields [4] show dipole-like modes at these frequencies (see Fig. 2).

By extracting 2D dipole modes with frequencies that match 3D calculations, the wake-fields can be reconstructed by calculating the dipole loss factor  $k_n$ , frequency  $\omega_n$ , and decay constant  $\gamma_n$  for each mode  $n$ . The dipole loss factor is a measure of the beam-dipole mode coupling [5, 6]:

$$k_n = cL^2 a \frac{|\nabla_{\perp} E_{z,n}|^2}{4\omega_n U_n} \quad (1)$$

where  $|\nabla_{\perp} E_{z,n}|$  is calculated at the center of the cavity,  $L$  is the length of the cavity,  $a$  is the transverse offset of the exciting bunch, and  $U_n$  is the stored energy for each mode  $n$ . It is important to note that the stored energy depends on the volume (in 2D, area) chosen, and unlike the case of closed metal cavities, there is no obvious boundary to choose to calculate the stored energy. In our simulations, we have chosen to use a rectangular boundary just outside the outer-most rods to calculate  $U_n$ . This way, any radiation outside the outer-most rods is known to be propagating out of the structure, and not to be reflecting off other rods. By including the time-dependence for each mode and summing over all modes, the reconstructed transverse wake-fields can be calculated:

$$w_{\perp,dip}(t) = \frac{1}{aL} \sum_{n=1}^N k_n \sin(\omega_n t) e^{-\gamma_n t} \quad (2)$$

This quantity and its Fourier transform agree well with the wake-fields found for 8-cell 3D simulations. For example, Fourier transforms of the 3D wake-fields for the 4-layer triangular lattice and the 2D reconstructed wake-fields are compared in Fig. 2.

These results indicate that the dipole loss factor can be used in the cost function in 2D simulations. Several different cost functions were created and tried in optimization routines, the most successful being the following:

$$\text{cost} = \sum_{i=1}^2 k_i / Q_{rad,acc} \quad (3)$$

Here,  $Q_{rad,acc}$  is the radiative  $Q$ -factor of the accelerating mode, and  $k_n$  is the dipole loss factor (Eq. 1) for mode  $n$ . The two highest dipole loss-factors are taken in the sum. The accelerating mode  $Q$ -factor is included to ensure that the accelerating mode does not disappear (for example, one can imagine the optimization routine moving all rods far apart from each other to reduce wake-fields, in which case we don't have an accelerating mode at all).

## RESULTS

Starting from a 2-layer triangular lattice (18-rod) structure, the cost function of Eq. 3 was used to optimize the rod positions. In 2D, the optimization significantly lowered the  $k_n$ . In 3D, wake-fields were drastically reduced from the

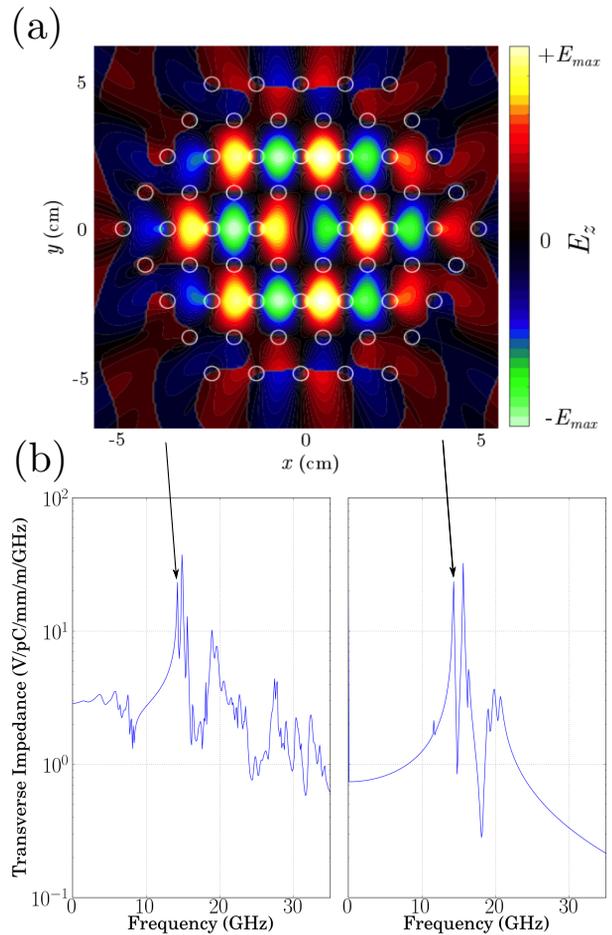


Figure 2: (a) Dipole mode extracted at the frequency shown in the plot below. (b) Fourier transform of transverse wake-fields for the 8-cell 3D cavity (left) and reconstructed wake-fields from the 2D  $k_n$  (right).

initial lattice configuration. The Fourier transform of the wake-fields for the optimized structure, the CLIC cavity, and the 4-layer triangular lattice structure are compared in Fig. 3.

Similar cost functions were tried in this process, and the final 3D calculations for these structures all showed less improvement than for the optimized structure shown in Fig. 3. For example, we have tried starting at structures other than the lattice structure, adding rods to the above structure and restarting the optimization process, and trying out cost functions that take into account the  $Q$ -factors of the dipole modes, preferring modes that decay more slowly over modes that decay quickly. None of these has proven to be drastically more effective in reducing transverse wake-fields, though none have been drastically worse. A general finding is that deviation from the lattice structure significantly lowers transverse wake-fields. For example, in prior work [2], optimization of  $Q_{rad,acc}$  resulted in structures with transverse wake-fields nearly as low as those shown in Fig. 3.

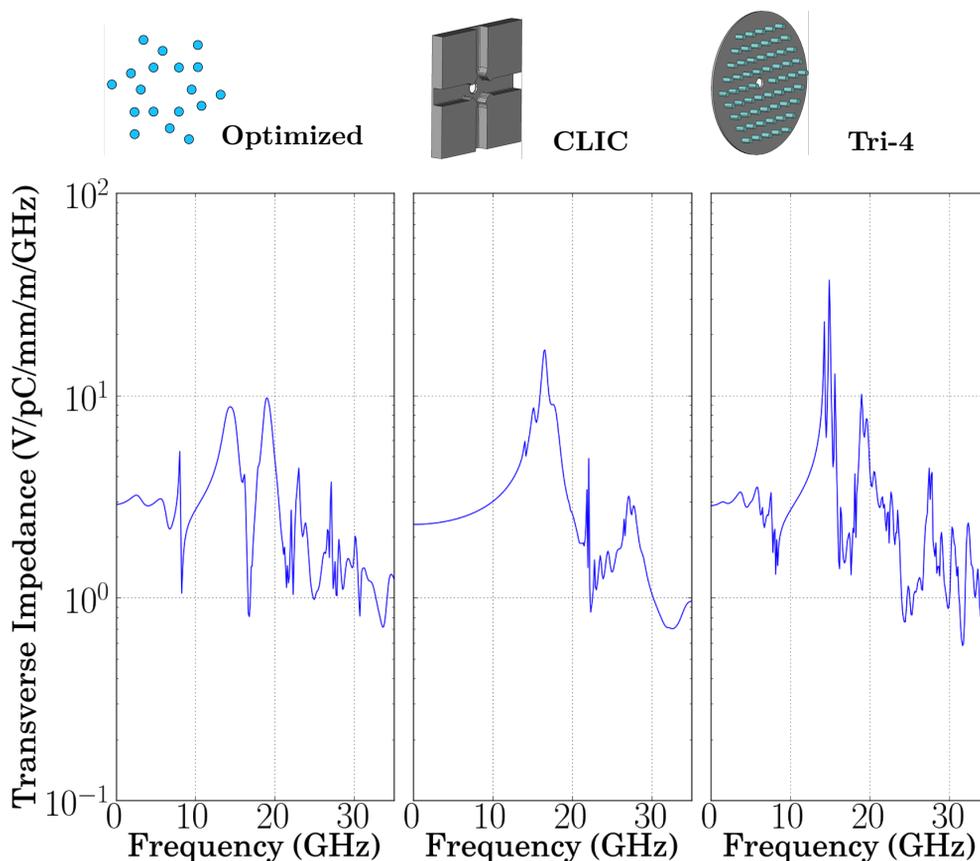


Figure 3: Fourier transform of transverse wake-fields for different structures. The wake-fields for the optimized PhC structure are significantly lower those of the lattice structure.

### SUMMARY

Transverse wake-fields are undesirable in many accelerator cavity designs, and minimizing their presence is of high priority for future linacs and other accelerators. Transverse wake-fields are significantly reduced by optimizing the rod positions to reduce the dipole loss factors ( $k_n$ ) of the most prominent and harmful dipole modes.

To reduce wake-fields further, we plan on using 2D time-domain simulations, in which the wake-fields are calculated directly from the cavity fields, rather than from extracting cavity modes and calculating the  $k_n$ . One reason for making this switch is that as the presence of the high  $k_n$  modes diminish, they become more difficult to extract with mode extraction because modes decay so quickly that the mode extraction routine can no longer find them. In a 2D time-domain simulation, this is of no concern, since we no longer rely on mode extraction to calculate the cost value.

A large number of other parameters remain to be changed and optimized, such as the radii of the rods, the use of metal rods, and the use of elliptic rods, instead of circular ones. Another step forward will be to optimize a 60-rod structure, in order to better compare the Tri-4 lattice with an optimized structure. Additionally, moving to 3D optimizations may be feasible, given that we can reduce the simulation time to run on large-scale supercomputers.

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