

SUPPRESSING TRANSVERSE BEAM HALO WITH NONLINEAR MAGNETIC FIELDS

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Abstract

Traditional space charge driven resonances, such as beam halo, arise due to the underlying linear nature of accelerator lattices. In this talk, we present initial results on a new class of intrinsically nonlinear lattices, which introduce a large tune spread naturally. The resulting nonlinear decoherence suppresses the onset of beam halo.

INTRODUCTION

High intensity beams have broad applications for high energy physics, neutron sources, and in the nuclear industry. For example, at Fermilab the Project X machine will deliver MW proton beams in the range of 3 to 120 GeV for the purposes of generating neutrinos for the proposed Long Baseline Neutrino Experiment, muons and kaons for the measurement of rare decays. It will also be used for the creation of exotic nuclei, and as a testbed for the transmutation of nuclear waste. For the intense beams desired for these purposes it is necessary to keep beam loss to a minimum to prevent activating the surrounding equipment. For example, in the 1.4 MW CW beam at SNS, it is necessary to keep beam loss to the pipe below 1 W/m. It is therefore crucial for these future applications that methods of mitigating intensity-dependent effects be developed.

Traditional beams gain stability from Landau damping, where a spread in the ensemble frequency response to an external perturbation prevents the ensemble as a whole from gaining energy. However, various other ensemble effects can result in a loss of Landau damping. Traditional linear lattice storage rings must still be designed to avoid various orders of rational tune resonances to maintain stability.

To bypass resonance-based restrictions on beam current, Danilov and Nagaitsev [1] proposed the use of controlled nonlinear lattices which still preserve one or two constants of the transverse motion. This assures bounded trajectories, while benefiting from a very large spread in frequencies for each trajectory. This *nonlinear decoherence* is the primary subject of this proceeding, and we hope to illustrate its properties with a simple example.

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NONLINEAR DECOHERENCE

A key distinction between nonlinear decoherence and Landau damping is that Landau damping is a property of the particle ensemble, while nonlinear decoherence is a property of the lattice. As such, nonlinear decoherence will occur regardless of the beam intensity or the specifics of the underlying beam distribution.

Consider an example Hamiltonian which has a purely quartic potential:

$$H = \frac{p_x^2}{2} + \frac{1}{4}\lambda^4 x^4 \tag{1}$$

This Hamiltonian is completely integrable, and the Hamiltonian in terms of the action variable is

$$\bar{H} = \left(\frac{\lambda J_x}{2\alpha} \right)^{4/3} \tag{2}$$

where $\alpha = \sqrt{\pi} \Gamma(5/4)/\Gamma(7/4)$ is a constant. The tune for an oscillation with an action J is therefore

$$\nu = \frac{1}{2\pi} \frac{\partial \bar{H}}{\partial J} = \frac{1}{2\pi} \frac{4}{3} \left(\frac{\lambda}{2\alpha} \right)^{4/3} J^{1/3} \tag{3}$$

Because this Hamiltonian has an amplitude-dependent tune, any forcing that is initially resonant will eventually be detuned out of resonance by the single particle dynamics. This is a purely single particle effect, and in no way depends upon the ensemble of particles proper. For a harmonic oscillator Hamiltonian, which is the basis of linear accelerator lattices, the Hamiltonian in the action-angle variables is

$$\bar{H} = J\omega_0 \tag{4}$$

for a frequency ω_0 , and hence the tune is independent of the amplitude. In this case, a periodic forcing would drive the particle to arbitrarily large amplitudes. This amplitude-dependent tune is the heart of nonlinear decoherence.

Despite originating from different physical effects, Landau damping and nonlinear decoherence have many similar traits. To see this, consider the two plots in Fig. 2 depicting the total single particle energy for a two-dimensional ensemble of particles initially populating a fixed Hamiltonian value H_0 . The left plot is the total energy for an ensemble of harmonic oscillators with a small frequency spread,

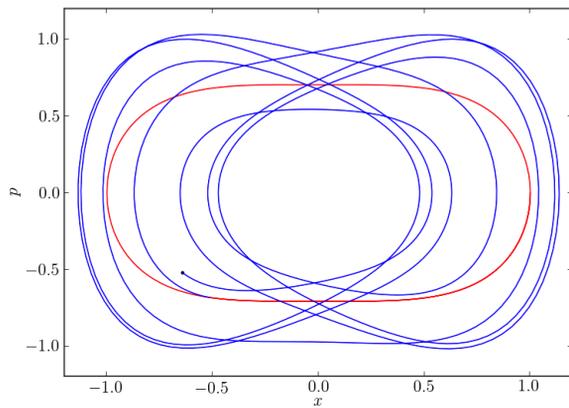


Figure 1: The $p - x$ phase space trajectory of a quartic oscillator without resonant forcing (red) and with a forcing resonant with the initial frequency (blue).

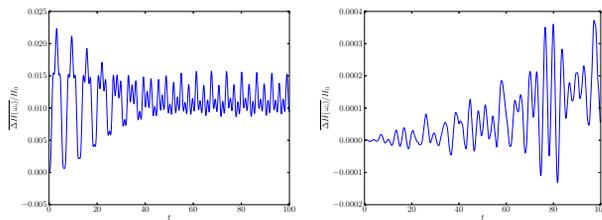


Figure 2: The average particle energy for the harmonic oscillator (left) and quartic oscillator (right) for many resonant forcing periods.

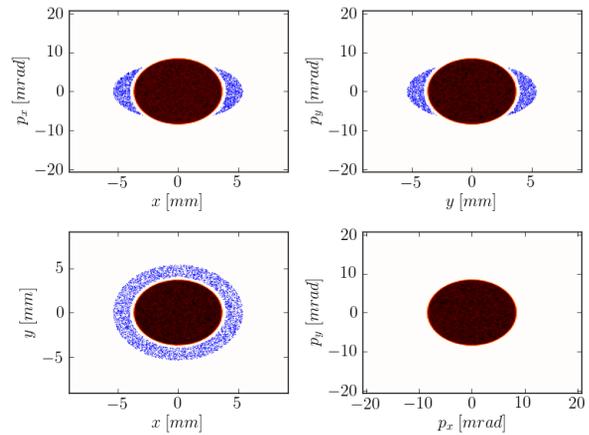
resonantly forced at the central frequency. The right is an ensemble of quartic oscillators. Because there is a curve of $J_x - J_y$ space which yields a frequency spread at fixed energy, the ensemble has a natural frequency spread.

In practice both prevent the growth of energy from resonant forcing – if tracked for long enough the nonlinear decoherence shows some super-periodic oscillations in the energy but no resonant growth, while the Landau damping has saturated out. The key feature that distinguishes the nonlinear decoherence is that it does not require many particles, or an interaction with the environment, to initiate its effects. There is no straightforward way to achieve a loss of nonlinear decoherence.

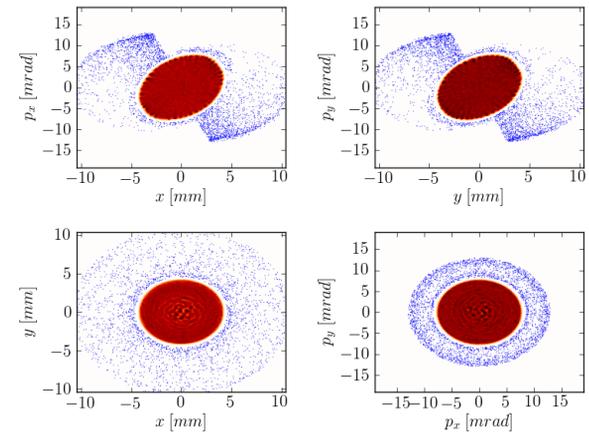
BEAM HALO IN LINEAR LATTICES

To test these results in a more realistic setting, we consider the beam core-halo model described by Gluckstern [2]. In this model a mismatched “core” experiences transverse breathing at the betatron tunes. This can resonantly force particles not in this core to large amplitudes, before the nonlinear space charge forces outside the beam core proper detune the resonant forcing. As demonstrated by Bruhwiler [3], this can be rigged to rapidly form if a “pre-halo” of matched particles is co-propagating with the mismatched core.

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(a) Blue dots indicate pre-halo particles outside of 2 RMS beam radius



(b) Particles in the pre-halo begin making large amplitude oscillations driven by the resonant space charge forcing of the beam core

Figure 3: Coherent linear oscillations of the beam envelope drive halo formation.

The first test case is the conventional linear lattice, comprised of a 2 m drift space with equal beta functions, matching a single element of the IOTA test ring design. We use an effective double-focusing element to mirror the more complicated optics of the IOTA lattice. We use a 100 A CW beam with $\gamma_0 = 2$ and zero longitudinal energy spread. The beam core is matched to a lattice with a beta function 30% larger than the actual lattice. As can be seen in Fig. 3, particles in the pre-halo are rapidly forced to twice the core radius, withing 500 passes through this trial lattice.

Linear lattices are commonplace because of their regular dynamics. However, they are highly susceptible to resonances such as the beam halo described here. This is because the frequency of oscillations is independent of the amplitude of transverse motion – this allows conventional resonant forcing by any perturbation. Traditional mitigation schemes, such as the addition of octupoles to an otherwise linear lattice, introduce some amplitude-dependent

tune content to detune a particle trajectory from resonance. However these schemes can reduce the dynamic aperture. We instead consider an alternative approach.

INTEGRABLE ELLIPTIC LATTICE

By comparison, we examine the integrable elliptic lattice (IEL), which yields completely integrable two-dimensional motion for its transverse particle trajectories. The effective single-turn Hamiltonian, in the normalized coordinates, is given by

$$H(p_x, p_y, x, y) = \frac{1}{2} (p_x^2 + p_y^2) + \frac{1}{2} (x^2 + y^2) + \frac{f_2(\xi) + g_2(\eta)}{\xi^2 - \eta^2} \quad (5)$$

Here

$$\xi = \frac{\sqrt{(x+c)^2 + y^2} + \sqrt{(x-c)^2 + y^2}}{2c} \quad (6)$$

$$\eta = \frac{\sqrt{(x+c)^2 + y^2} - \sqrt{(x-c)^2 + y^2}}{2c}$$

are hyperbolic coordinates with foci at $x = \pm c$. The two potential functions are

$$f_2(\xi) = \xi \sqrt{\xi^2 - 1} [d + t \cosh^{-1}(\xi)] \quad (7)$$

$$g_2(\eta) = \eta \sqrt{1 - \eta^2} [b + t \cos^{-1}(\eta)]$$

where d, b , and t are free parameters. We have taken $d = 0$, $b = \pi/2$, and $t = -0.5$. We show the equipotential surfaces for this Hamiltonian in Fig. 4. The issue of beam matching is intimately related to the equipotential surfaces depicted.

The Hamiltonian here is a constant of the motion, and there exists a second constant of the motion for the IEL lattice. This guarantees bounded regular motion that remains almost regular under perturbations, as described by the KAM theorem. However, because the Hamiltonian is strongly nonlinear, there is not a single “tune” in the way a conventional linear lattice has a tune. In this way, tune diagrams are irrelevant for IEL type lattices.

Any function of the invariants of motion form a stationary solution to the Vlasov equation [7, 8]. For strong focusing lattices, these are the Courant-Snyder invariants [6]. The IEL similarly has two invariants, and we have chosen to match to the total Hamiltonian. As a result of this, a beam with a phase space distribution that is a delta function in the Hamiltonian

$$f(\vec{p}_N, \vec{q}_N) = \delta(\mathcal{H}(\vec{p}_N, \vec{q}_N) - \varepsilon_0) \quad (8)$$

will be a fixed point which uniformly fills all two-dimensional projections in phase space, with the $x - y$ projection determined by the contour $V(\vec{q}_N = \varepsilon_0)$. This ε_0 takes the role of the transverse emittance in a conventional linear lattice. Indeed, this is equivalent exactly to the Courant-Snyder invariant in the limit that all the nonlinear components of the Hamiltonian vanish.

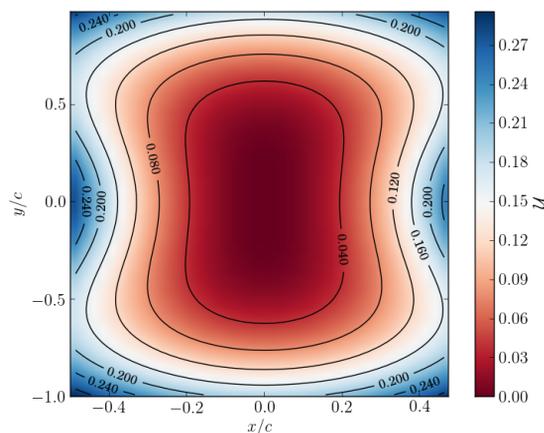


Figure 4: Equipotential surfaces for the IEL potential.

This is the value of the isoenergetic contours in Fig. 4 – they mark the transverse beam profile for a properly matched beam in an IEL. There is likely a beam matching that separates out two individual invariants [4], equivalent to the vertical and horizontal emittances in linear lattices, and this remains open to exploration.

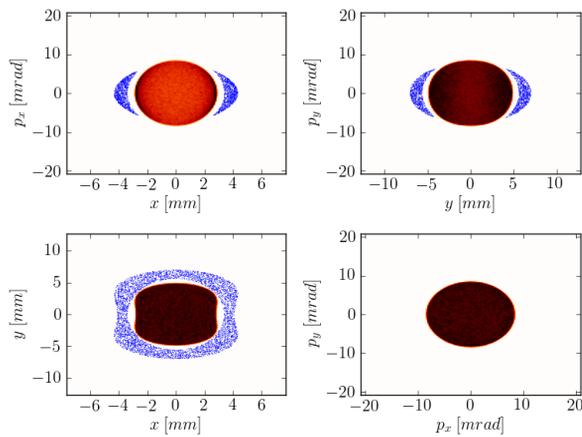
By matching a core with a 30% larger beta function as with the linear lattice, and then a properly matched pre-halo is added. As can be seen in Fig. 5, the beam rapidly equilibrates to a stationary solution. There are no periodic core oscillations to drive a resonance, and the beam is totally stable.

Interestingly, these beam configurations closely resemble the results of a mismatch without space charge, or a proper matching with space charge. Generically, these nonlinear lattices rapidly filament in phase space, reaching a quasi-static equilibrium distribution so long as the initial distribution is sufficiently close in form to a properly matched distribution.

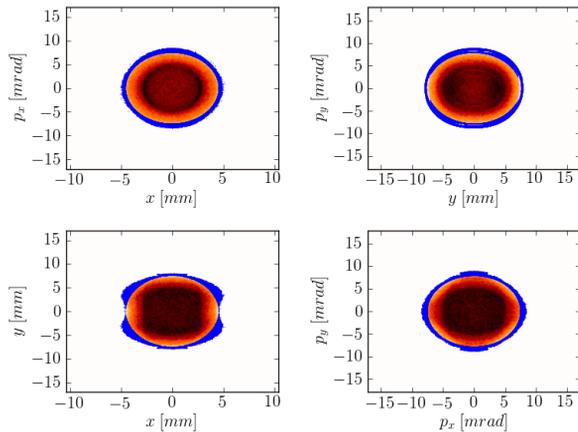
Danilov and Nagaitsev also discuss a chaotic bounded octupole lattice, to be discussed in a later publication [5], and which when tested yielded similar results. This would seem to suggest that a second invariant of the motion is not a necessary condition for bounded nonlinear trajectories. The exact interplay between integrability and space charge forces for a general integrable Hamiltonian remains an open question.

CONCLUSION

This demonstrates a very promising first step for the proposed nonlinear lattices with two invariants. In a situation rigged for failure, the nonlinear decoherence of the lattice prevents completely the onset of beam halo in a situation where a linear lattice cannot cope. Future work at the Integrable Optics Test Accelerator will elucidate the single particle properties in the presence of realistic lattice issues such as fringe fields, dispersion, and other properties which may break the integrability of the pure two-dimensional system.



(a) Blue dots indicate pre-halo particles outside of 2 RMS beam radius



(b) The IEL beam rapidly equilibrates to the properly matched physical dimensions with no halo formation

Figure 5: Nonlinear decoherence prevents the rapid formation of beam halo in the IEL.

We have presented preliminary results for the efficacy of a novel lattice design which uses controlled nonlinearities to maintain the dynamic aperture while introducing nonlinear decoherence to the single-particle dynamics. It is this effect which efficiently prevents a variety of resonances – here we have discussed the resonant interaction of beam mismatch oscillations and space charge to produce a beam halo. We have shown that these lattices are capable of completely preventing the formation of beam halo where a linear lattice would see the halo instability immediately. This is a promising initial result for future advances in the intensity frontier.

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