EIGENMODE COMPUTATION FOR THE GSI SIS18 FERRITE CAVITY *

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Abstract

At the GSI Helmholtzzentrum für Schwerionenforschung in Darmstadt the heavy-ion synchrotron SIS18 is operated to further accelerate stable nuclei of elements with different atomic numbers. Two ferrite-loaded cavity resonators are installed within this ring. During the acceleration phase their resonance frequency has to be adjusted to the revolution frequency of the heavy ions to reflect their increasing speed. To this end, dedicated biased ferrite-ring cores are installed inside the cavities for a broad frequency tuning. By properly choosing a suited bias current, the differential permeability of the ferrite material is modified, which finally enables to adjust the eigenfrequency of the resonator system. Consequently, the actual resonance frequency strongly depends on the magnetic properties of the ferrites. The goal of the current study is to numerically determine the lowest eigensolutions of the GSI SIS18 ferrite-loaded cavity. For this purpose, a new solver based on the Finite Integration Technique has been developed.

INTRODUCTION

Two ferrite-loaded cavity resonators are installed in the GSI SIS18 synchrotron for the further acceleration of heavy ions. Due to saturation effects the differential permeability of the ferrite material decreases when exposed to an increasing static magnetic field. Dedicated current loops are located around the ferrite ring cores for biasing parallel to the superimposed radio-frequency (RF). As a result, by applying a bias current of up to several hundred amperes, the eigenfrequency of the whole resonator system can be modified in the range from about $0.6\,\mathrm{MHz}$ to $5.0\,\mathrm{MHz}$. Obviously, the properties of the ferrite material have a large impact on the resulting resonance frequency. Hence, an appropriate choice of a material which best meets the requirements of the accelerator has to be made. Accurate numerical simulations of the whole cavity resonator provide one option to facilitate such decisions.

In this paper, a numerical solver, which has been newly developed for the computation of the lowest eigenmodes of biased ferrite-loaded cavity resonators, is presented. After briefly summarizing the applied computational approach, two numerical examples are discussed. Whereas the first one aims at the verification of the solver, the second one serves as an illustration of its supported features and particularly focuses on the implementation of the material properties.

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COMPUTATIONAL APPROACH

The need for a dedicated solver for the calculation of eigenvectors of ferrite-filled resonators arises from the challenging material properties of ferrites subject to a static magnetic field. Their permeability does not only exhibit a tensor structure but is also dependent both on the bias magnetic field and on the frequency of the superimposed RF. As a consequence, firstly the field generated by the bias current has to be computed by a magnetostatic solver with support for a nonlinear constitutive relation. This specifies a working point at which the subsequent nonlinear eigenproblem is solved. The solver is capable of handling complex eigenvalues such that magnetic losses can be taken into account. Moreover, the implementation aims at efficient parallel computation on distributed memory machines. More details about relevant fundamental relations as well as the computational model can be found in [1].

NUMERICAL EXAMPLES

Biased Cylinder Resonator

For the verification of the eigenvalue solver, a longitudinally biased ferrite cylinder similar to the setup described in [1] is considered. However, whereas in [1] no magnetic losses are taken into account, here the computation is carried out with a loss parameter of $\alpha = 0.1$. Since an equation determining the eigenfrequencies of radially symmetric TM-modes can be formulated analytically [2] also for the lossy case, the eigenfrequencies obtained with the numerical computation using the solver described before are compared with these reference values. Good agreement is indicated by convergence studies both for the real and the imaginary part of the eigenfrequencies, which is demonstrated for the two lowest radially symmetric TM-modes in Fig. 1.

Biased Ferrite-Loaded Cavity Resonator

The second example is a simplified model of the GSI SIS18 cavity (cf. Fig. 2). The bias field is generated by six figure-of-eight current windings. Additionally, four gap capacitors with a capacitance of $250\,\mathrm{pF}$ each are included as lumped elements in the model. Both the cavity housing and the hollow beam pipe are modeled as perfectly electric conducting material. Though the overall dimensions of the cavity are adopted from the real resonator, the numerical model is not yet complete. Instead of 32 ferrite ring cores with copper plates in between on each side of the accelerating gap only one ferrite ring each is modeled. Also

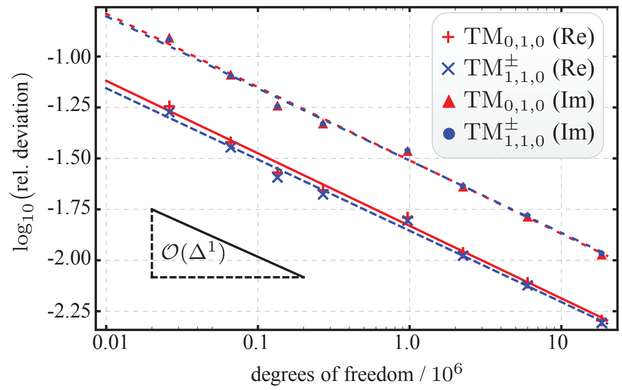

Figure 1: Relative deviation $|\omega - \omega_0|/\omega_0$ of the numerically obtained value ω to the (semi-)analytical reference value ω_0 as a function of the degrees of freedom for the two lowest radially symmetric TM-modes for a lossy, ferrite-filled cylindrical cavity resonator.

regarding the ferrites, the quantitative properties of the material, which is actually installed in the cavity, are not fully taken into account, yet. Nevertheless, for a proof of concept a model describing a ferrite in the fully saturated state including the anisotropic character and the dependence on the bias magnetic field and frequency according to the theory by Polder [3] is implemented. Thus, the permeability tensor for fully magnetized ferrite materials is given by

$$\overleftrightarrow{\mu}_d = \mu_0 \begin{pmatrix} \mu_{\mathrm{diag}} & \mathrm{i}\kappa & 0 \\ -\mathrm{i}\kappa & \mu_{\mathrm{diag}} & 0 \\ 0 & 0 & \mu_z \end{pmatrix} \quad \left(\text{for } \vec{H}_0 \parallel \vec{e}_z\right), \quad (1)$$

with

$$\mu_{\mathrm{diag}} = 1 + \chi, \quad (2a)$$

$$\chi = \frac{(\omega_0 + \mathrm{i}\omega\alpha)\,\omega_{\mathrm{M}}}{(\omega_0 + \mathrm{i}\omega\alpha)^2 - \omega^2}, \quad (2b)$$

$$\kappa = \frac{-\omega\omega_{\mathrm{M}}}{(\omega_0 + \mathrm{i}\omega\alpha)^2 - \omega^2}, \quad (2c)$$

$$\omega_0 = -\gamma_e \mu_0 H_0, \quad (2d)$$

$$\omega_{\mathrm{M}} = -\gamma_e \mu_0 M_{\mathrm{sat}}, \quad (2e)$$

$$\mu_z = 1. \quad (2f)$$

Here, M_{sat} is the saturation magnetization, whereas H_0 is the strength of the total effective static magnetic field composed of the applied DC field H_{DC} and the crystalline anisotropy field H_a; throughout this analysis we assume that $H_0 = H_{\mathrm{DC}} + H_a$. Moreover, the gyromagnetic ratio is given by

$$\gamma_e = -\frac{g\,e}{2m_e} \quad (3)$$

with g being the Landé g-factor, e the positive elementary charge and m_e the electron mass. The free parameters H_a, M_{sat}, α and g have to be determined from experimental data. At this point it is worth noting that relevant material parameters given in data sheets are usually subject to large experimental errors. In the following, the parameters implemented in this model are discussed.

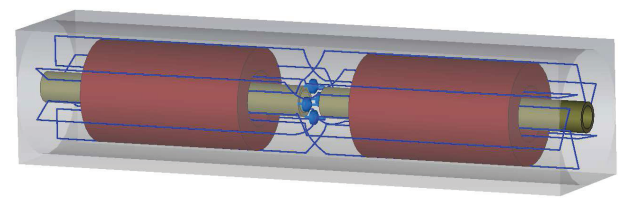

Figure 2: Simplified model of the SIS18 cavity. The six current windings are indicated by solid blue lines. Additional lumped element capacitors are installed around the gap.

In practice, the SIS18 cavity is operated in a cycle in which the bias current is only modified in a range between zero and a maximum current, i.e. without inverting its polarity. Consequently, it is reasonable to model only the upper branch of the nonlinear constitutive relation and to neglect hysteresis effects. Taking the values from the Ferroxcube data sheet [4] for the material 8C12 as a reference, a constitutive relation of the form

$$B(H) = (a \cdot \arctan(H \cdot b) + H + M_0) \cdot \mu_0, \quad (4)$$

is taken as a basis, which is fitted with parameters $a = 76600\,\mathrm{A/m}$, $b = 0.0078\,\mathrm{m/A}$ and $M_0 = 90000\,\mathrm{A/m}$. The latter quantity M_0 can be interpreted as the remanence magnetization. The associated reversible permeability at a given static magnetic bias field H_{DC}, which is defined as [4]

$$\mu_{\mathrm{rev}} := \lim_{\Delta H \to 0} \mu_\Delta := \lim_{\Delta H \to 0} \frac{1}{\mu_0} \left.\frac{\Delta B}{\Delta H}\right|_{H = H_{\mathrm{DC}}}, \quad (5)$$

then takes the form

$$\mu_{\mathrm{rev}} = 1 + \frac{ab}{1 + b^2 H^2}. \quad (6)$$

Using the same parameters as stated before, equation (6) approximates the data available for the incremental permeability μ_Δ [4] reasonably well. Another parameter which has to be checked for consistency in this context is the saturation magnetization M_{sat}. Considering Eq. (4) for the fully saturated region gives

$$M_{\mathrm{sat}} = \lim_{H \to \infty} a \cdot \arctan(H \cdot b) + M_0 \approx 2.1 \cdot 10^5\,\mathrm{A/m}, \quad (7)$$

which is just in the middle of the range specified in the data sheet with $M_{\mathrm{sat}} = 2.0 \cdot 10^5\,\mathrm{A/m}$ at 25°C and $M_{\mathrm{sat}} = 2.2 \cdot 10^5\,\mathrm{A/m}$ at 40°C. The remaining three free parameters are determined by the frequency dependence of the permeability. For lack of appropriate information, the data available in [4] are scaled such that the real part of the frequency dependent permeability in the static limit coincides with the value of μ_{rev} for $H_{\mathrm{DC}} = 0\,\mathrm{A/m}$. Picking up the

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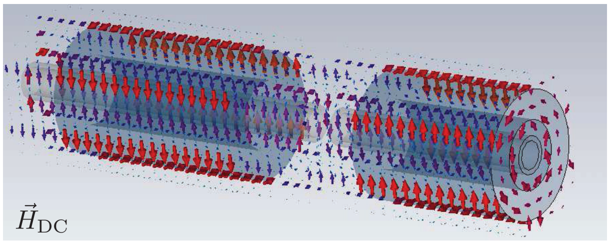

Figure 3: Bias magnetic field computed with the magnetostatic solver for a bias current of 600 A.

idea of [5], the resulting curves for the real and imaginary part are then fitted with an expression of the form

$$\mu' = \frac{1}{3}\left(1 + 2\sqrt{\mu_{\rm diag}^2 - \kappa^2}\right), \quad (8)$$

where $\mu_{\rm diag}$ and κ are defined as in Eq. (2). From this, a value of $H_a = 235\,{\rm A/m}$ is found for the anisotropy field, whereas the loss parameter α and the g-factor are obtained as 0.2 and 2.1, respectively. Besides, the relative permittivity is approximated with a value of 25 [4] independent of frequency. Of course, a frequency dependence can easily be incorporated in future material models, which will be handled by the solver in a similar manner as for the permeability.

For the given geometry and parameters, the lowest eigenmodes are computed with the newly developed solver for a bias current of 600 A. An analysis of the calculated static bias magnetic field (cf. Fig. 3) demonstrates that a symmetric and sufficiently homogeneous field distribution is obtained. In the elaborated setup, the fundamental eigenmode is observed to be a monopole mode, which features a longitudinal electric field inside the gap and is hence suited for acceleration (cf. Fig. 4(a)). Accordingly, the magnetic field is concentrated in the ferrite ring cores (cf. Fig. 4(b)). Its associated frequency of about 5.5 MHz strongly depends on the capacitance of the gap capacitors. This value is close to the frequency range covered by the SIS18 cavity. Yet, due to the mentioned simplifications made in the numerical modeling, in particular with respect to the material properties, a quantitative comparison is omitted at this point.

In order to estimate the effect of the anisotropic character of the permeability, the computation is repeated with the tensor replaced by an effective scalar permeability as given in Eq. (8). The lowest mode is still found to be an accelerating mode. Yet, besides of that the obtained eigenvalue spectrum is drastically changed. Not only is the frequency of the fundamental eigenmode lower by a factor of about 3 but also the sequence of the higher order modes is modified. Hence, it is not reasonable to replace the permeability tensor with a scalar value as given by Eq. (8).

SUMMARY AND OUTLOOK

The computation of the lowest eigenmodes of biased ferrite-loaded cavity resonators has been demonstrated.

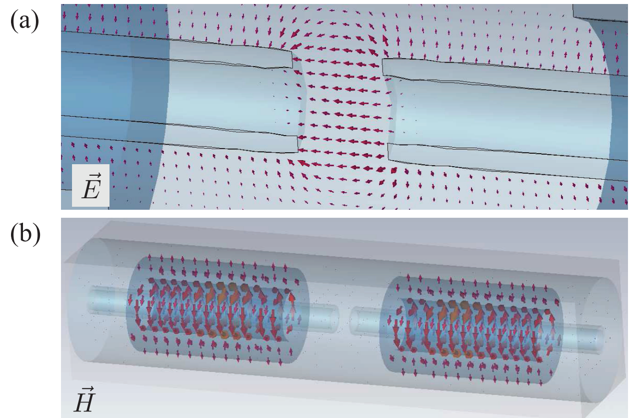

Figure 4: Distribution of the electric field (a) and magnetic field (b) of the fundamental mode. For the electric field a zoomed in, cutaway view of the gap region is shown.

The solver specifically developed for this purpose provides the following features: The anisotropic character of the ferrite material is taken into account by supporting a non-diagonal permeability tensor. This tensor is evaluated according to the strength and direction of the local DC magnetic field, which is obtained by the solution of the magnetostatic field problem in a first step. Moreover, in order to consider the frequency dependence of the permeability as well as to incorporate magnetic losses, the solver is able to handle both nonlinear and complex eigenvalue problems. Furthermore, the implementation is designed for an efficient distributed parallel computing.

Regarding the properties of the ferrite material, models for the fully magnetized as well as partially magnetized state are implemented. It is clear that for an accurate simulation of ferrite cavities the reliable and proper incorporation of relevant material properties plays a crucial role. For this reason, experimental measurements will have to be carried out and available results thoroughly implemented. To sum up, the developed solver is ready for use provided that reliable material data is on hand.

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