

# EMITTANCE AND BETA FUNCTION MEASUREMENTS FOR THE MAX IV LINAC

N. Čutić\*, E. Mansten,

Lund University, MAX IV Laboratory, P.O.B. 118, Lund, Sweden

## Abstract

We plan to determine transverse beam emittance and Twiss parameters for the MAX IV linac using multiple-quadrupole scans. We investigate the possibility to perform such scans using matching section quadrupoles combined with beam profile measurements on fluorescent YAG screens. Beam pipe size, resolution and screen saturation limits and strengths of quadrupoles are taken into consideration. Our approach to this problem using a Kalman filter is presented.

## INTRODUCTION

The MAX IV linear accelerator will be a full energy injection linac for two storage rings and at its full energy (3 GeV) it will also provide compressed electron bunches for a linac based short pulse facility [1]. The linac contains two bunch compressors: BC1 operating at 264 MeV and BC2 at full energy. Both of these compressors have matching sections before and after (MS1 to MS4) in which knowledge of the beam parameters is crucial for operation of the compressors. We aim to perform measurements of Twiss pa-

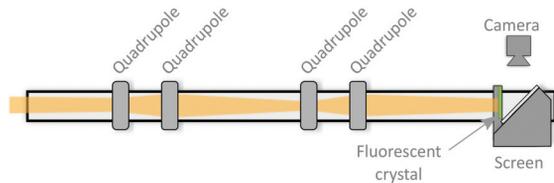


Figure 1: Illustration of the problem. Determine the beam parameters (Twiss parameters and emittance) by changing the strengths of several quadrupoles in the matching section only by measuring the beam size on the fluorescent screen near the end of the section. The beam size must stay within a certain range (in horizontal and vertical plane).

rameters and emittances in each of these matching sections using the available quadrupoles and a fluorescent screen at the end of each matching section. Performing this measurement using a simple quadrupole scan is inhibited by saturation effects for tightly focused beams on the screen or poor signal to noise ratios for large beam sizes. One can elegantly circumvent these problems by keeping the beam size nearly constant while changing the strengths of several quadrupoles and performing a least-squares fit (LS) as presented by Tenenbaum [2]. In this paper we report on

our approach based on a Kalman filter [3, 4, 5]. We briefly present the formalism behind LS and Kalman filter (KF), we describe the way quadrupole values for scanning are found and in the last section of this paper we show how the Kalman filter performs in a simulation of MS1 to retrieve emittance and Twiss parameters.

## METHODS

A beam comprised of  $N$  electrons with transverse coordinates  $\vec{v}_i = (x_i, x'_i)$  can be (to second order) described by central moments  $\Sigma_{20}, \Sigma_{11}, \Sigma_{02}$  where  $\Sigma_{mn} = \frac{1}{N} \sum (x_i - \langle x \rangle)^m (x'_i - \langle x' \rangle)^n$ . If the beam enters a matching section described by a transport *abcd-matrix*  $M$  then the new coordinates for each particle are  $v_{i,2} = Mv_i$ . The covariance-variance matrix given by

$$\Sigma = \begin{bmatrix} \Sigma_{20} & \Sigma_{11} \\ \Sigma_{11} & \Sigma_{02} \end{bmatrix} = \begin{bmatrix} \epsilon\beta & -\epsilon\alpha \\ -\epsilon\alpha & \epsilon\gamma \end{bmatrix} \quad (1)$$

will then transform during passage through the transport as:  $\Sigma_2 = M\Sigma_1M^T$ . The  $\alpha, \beta$  and  $\gamma = \frac{1+\alpha^2}{\beta}$  are the Twiss parameters of the beam and  $\epsilon$  the transverse RMS emittance. Using  $\Sigma_{20} = \sigma_x^2$  we gather that the beam size transforms as

$$\Sigma_{20,2} = a^2\Sigma_{20,1} + 2ab\Sigma_{11,1} + b^2\Sigma_{02,1} \quad (2)$$

## Least Squares Method

LS method is based on finding the triplet of sigmas ( $\Sigma_{20}, \Sigma_{11}, \Sigma_{02}$ ) that minimizes functional  $\chi^2 = \sum_{m=1}^k \frac{[\Sigma_{20,2,m}^2 - (a_m^2\Sigma_{20,1} + 2b_m a_m \Sigma_{11,1} + b_m^2 \Sigma_{02,1})]^2}{\delta_m^2}$  for  $k$  measurements. The  $\delta_m$  are measurement errors as defined in [2].

## Kalman Filter

With a (discrete) Kalman filter it is not necessary to collect all the measurements and then perform a fit of the data. As an iterative method it allows to update while the information is collected. A KF always keeps the same amount of data, i.e. the collected information does not grow. For each shot we measure the beam size and estimate its precision and then invoke the filter to update itself. The update consists of a prediction (where KF estimates current state and its uncertainties) and then a correction of the prediction using the newly available measurements. In general, a KF is for systems that can be modeled as:  $x = Ax + Bu + w$  where  $x$  is a state vector estimate,  $A$  is a state transition

\* nino.cutic@maxlab.lu.se

matrix,  $u$  is a control vector that directly and linearly influences the state through the matrix  $B$  and  $w$  is centered Gaussian noise with covariance  $Q$ . We observe a quantity  $z$  which is linearly connected with the state  $x$  through an observation matrix  $H$  by  $z = Hx + v$  where  $v$  is again centered Gaussian noise with covariance  $R$ . The KF then operates in the following way [6]:

- Predict the state vector and its covariance  $P$ :  
 $x \mapsto Ax + Bu$   
 $P \mapsto APA^T + Q$
- Calculate a Kalman gain factor:  
 $K = PH^T(HPH^T + R)^{-1}$
- Correct based on the observed  $z$ :  
 $x \mapsto x + K(z - Hx)$   
 $P \mapsto P - KHP$

The state vector and its covariance can be initialized in the first step (if they are not known) as  $x = H^{-1}z$  and  $P = H^{-1}R(H^T)^{-1}$  or they can be provided as a guess (our case).

For emittance scans we apply the KF using the following:

- The unknown state  $x$  is a column vector of sigmas at the entrance to the matching section  $x = (\Sigma_{20}, \Sigma_{11}, \Sigma_{02})$ .
- The matrix  $A$  is the identity matrix since we assume that the input beam parameters don't change with time.
- $B$  is zero since we do not control the state. In fact, we assume the state to be constant since we do not have control of input Twiss parameters and emittance. We assume that we have no shot to shot noise ( $Q = 0$ ) at the entrance to the matching section.
- We observe the beam size  $z_m = \Sigma_{20,2,m}$ . The observation matrix  $H_m$  is given by a row vector  $H_m = (a_m^2, 2a_m b_m, b_m^2)$ . This can be seen by comparing  $z = Hx$  and equation (2).  $H$  changes every time we change the optics of the matching section.
- For a certain range of measured beam sizes we assume covariance of measurement  $R$  to be a certain percentage of the beam size (about 5%). Outside of that range we increase the covariance continuously but drastically. Therefore, the KF will take into consideration measurements of saturated or large beam sizes but uncertainty of such measurements will be provided to the filter as being huge. In practice it was noticed that such points help to reach the correct solution faster (in fewer shots) but should be sparse enough not to bias the result.

The reader is referred to [3] for a very nice introduction into terminology and functioning of the KF.

### Preparations

Although it is possible to perform scans by blindly testing random quadrupole values, such a scan can be outperformed by a scan where quadrupole values are selected a priori using the knowledge that is available about the matching section. Trying to keep the beam size constant, or within a certain range is obviously beneficial. Alas, we do not know which pairs of  $a_m$  and  $b_m$  will result in a certain beam size since we do not know the input Twiss parameters. If we investigate a certain constant beam size ( $\beta_2$  or  $\Sigma_{20,2,m}$  constant), we know from equation (2) that matrix elements  $a_m$  and  $b_m$  will form an ellipse (non-rotated if  $\alpha_1 = 0$ ) as in Fig. 2. Dividing the equation (2) by the  $\Sigma_{02,2}$  we get a form that does not contain the emittance and which is a common form of an ellipse:

$$Aa^2 + Bab + Cb^2 = 1 \tag{3}$$

where  $A = \frac{\beta_1}{\beta_2}$ ,  $B/2 = \frac{-\alpha_1}{\beta_2}$  and  $C = \frac{\gamma_1}{\beta_2}$ . This ellipse will have semi-major and semi-minor axes  $u$  and  $v$  in direction of the eigenvectors of a matrix  $W$  and the lengths of these axes are  $\lambda^{-1/2}$  where  $\lambda$  is the corresponding eigenvalue. The area of the ellipse will be  $\pi\beta_2$ .

$$W = \begin{bmatrix} A & B/2 \\ B/2 & C \end{bmatrix} \tag{4}$$

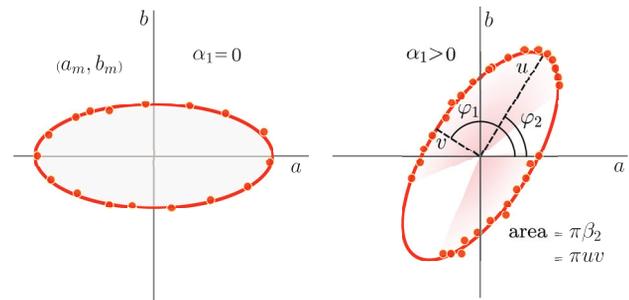


Figure 2: If the input  $\alpha_1$  function is zero the values of  $a$  and  $b$  matrix elements of the transport matrix  $M$  that keep the beam size constant on the screen following the transport section will lie on the non-rotated ellipse (left). If  $\alpha_1$  is greater than zero the semi-major axis will be in the first quadrant and the ellipse will be rotated. The area of the ellipse is always  $\pi\beta_2$ . Not necessarily all pairs of  $a$  and  $b$  can be reached (as indicated on right with some regions of the ellipse missing the dots). This limitation comes from limits on magnet strengths, the apertures in the transport and available phase advance as well as the final beam size in the orthogonal transverse plane. The orientation and eccentricity of the ellipse depends only on  $\alpha_1$  and  $\beta_1$ .

We can now choose for our scan the values of quadrupoles that are close to the ellipse that corresponds to our initial guess of  $\alpha_1$  and  $\beta_1$  and desired beam size (we initialize the script with quadrupoles set so they give a certain size; this speeds up the process but is not required).

We would like to get the scan points to surround the origin as much as possible to increase the quality of ellipse's reconstruction. Only segments of this ellipse might be available for the scan depending on the available phase advance we can achieve in the transport. As the scan progresses we can update our list of "interesting" quadrupole values with new sets that would keep the beam size more constant based on the estimates of Twiss parameters we have so far. This speeds up the measurement since we do not measure oversized or undersized beams. To measure how close a certain point is to the ellipse we use the Mahalanobis distance. As the scan progresses the measured Twiss parameters approach the real values and we also reduce the allowed Mahalanobis distance (we start requiring that the  $a_m$  and  $b_m$  deviate very little from the ellipse that is expected to produce the desired beam size). The list of quadrupole values also becomes increasingly more accurate and the beam size on the screen stabilizes. Ideally, once the scan is complete the beam changes in size only little although the quadrupole values are changing significantly (going through different phase advances, or  $\alpha_2$ ). This confirms the retrieved values of Twiss parameters since we know them well enough to produce any available desired behavior.

## SIMULATIONS

In order to test the KF we wrote a simulation script in Matlab. The simulation contains a tracking part where particles are tracked through a matching section to the screen. The tracking part also adds noise to the set values of quadrupoles and takes into account DAC values available for the magnets as well as their apertures and available strengths. Effects of the screen and camera such as screen saturation, salt and pepper noise, radiation damaged/dead pixels, background Poisson noise, camera resolution, dynamic range, and point spread function are included. The analysis part of the script generates lists of quadrupoles values to test, grabs the image from the tracking part of the simulation (as if it was getting it from the screen in the real measurement) and then filters all the noise (without being provided its characteristics) and measures the beam width. One run with 50 shots is shown in Fig 3. Every 10 shots a new list of quadrupole values that should approximately keep the beam size within the desired range is generated.

The value of  $\beta$  in the vertical plane is slightly underestimated and is also accompanied by a bias in the emittance estimate. The most likely reason for this is a bias in the beam size measurement. We hope to address this issue with further improvements on noise filtering and beam size measurements. The script could be improved with a smarter search for quadrupole values, that give certain  $a$  and  $b$ , although this mostly concerns the speed at which the script can run and not its accuracy. A typical run for 100 shots including simulation of the matching section presently takes a couple of minutes on average laptop.

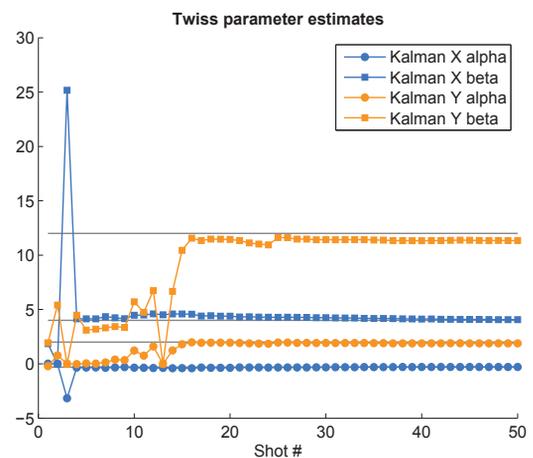


Figure 3: Two Kalman filters estimate Twiss parameters in matching section 1. Design values serve as initial guess. On purpose the "true" values have been set significantly different  $(\alpha_1, \beta_1)_x = (-0.3, 4)$  and  $(\alpha_1, \beta_1)_y = (2, 12)$ . Determined values are:  $(\alpha_1, \beta_1)_x = (-0.28, 4.06)$  and  $(\alpha_1, \beta_1)_y = (1.9, 11.3)$ . The "true" emittance values:  $\epsilon_x = 2.5 \cdot 10^{-9}$ ,  $\epsilon_y = 2.3 \cdot 10^{-9}$  and determined are:  $\epsilon_x = 2.78 \cdot 10^{-9}$ ,  $\epsilon_y = 2.56 \cdot 10^{-9}$  m rad.

## REFERENCES

- [1] M. Eriksson, J. Ahlbäck, Å. Andersson, M. Johansson et al., "The MAX IV Synchrotron Light Source", 3026 - 3028. IPAC'11 (2nd International Particle Accelerator Conference). San Sebastián, Spain, THPC058, 2011.
- [2] P. Tenenbaum, "Emittance measurements in CTF 2 Drive Beam", CLIC Note 326, CERN Libraries, Geneva, 1997.
- [3] R. M. du Plessis, "Poor Man's Explanation of Kalman Filtering or How I Stopped Worrying And Learned To Love Matrix Inversion", North American Rockwell Electronics Group, 1967.
- [4] G. Welch, G. Bishop, "An Introduction to the Kalman Filter", TR 95-041, Department of Computer Science, University of North Carolina at Chapel Hill, US, 2006.
- [5] R. E. Kalman, "A New Approach to Linear Filtering and Prediction Problems", Transaction of the ASME Journal of Basic Engineering, pp. 35-45, 1960.
- [6] M. C. Kleder, Matlab script "Learning the Kalman Filter" available from authors pages on [www.mathworks.com](http://www.mathworks.com), 2013.