THEORETICAL STUDY ON THE TWO-STAGE COLLIMATION SYSTEM DESIGN*

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Abstract

Two-stage collimation system is widely used in high intensity machines to localize the beam losses in a restricted area. In the well-known theory, the optical constrains are expressed by the betatronic phase advances between the primary and secondary collimators, which minimize the size of the secondary halo. In this paper, the physical model is developed considering the characteristic of the space charge dominated beams. Numerical studies are performed to verify the theoretical model.

INTRODUCTION

The concept of two stage collimation system was first presented by Trenkler and Jeanneret [1, 2]. The basic law is to minimize the amplitude of the secondary beam halo. The phase advances between the primary and secondary collimators are considered as only quantity defining the efficiency of the collimation system. The efficiency of the collimation system is defined by the geometrical properties of the system rather than by the true scattering mechanism in the jaws.

However, in actual machines, the performance of a collimation system is usually constrained by the limited physical aperture of the vacuum chambers. Particles scattered by the primary collimator with increased amplitude may be lost in an aperture restriction before being absorbed by the secondary collimators. Besides, for space charge dominated beams, the emmitance growth is evident, and the impact parameter can not be neglected.

In the new theory developed, the coulomb scattering effect is included. The dependence of the collimation efficiency on the physical aperture and the impact parameter are investigated. Numerical results are given and compared with the theoretical estimations.

PHYSICAL MODEL

In this paper, one dimensional betatron collimation system is considered. The primary collimator has an aperture of n_1 in the normalized phase space (Y, Y'), which are expressed as

$$\begin{pmatrix} Y \\ Y' \end{pmatrix} = \frac{1}{\sigma_x}\begin{pmatrix} 1 & 0 \\ \alpha_x & \beta_x \end{pmatrix}\begin{pmatrix} y \\ y' \end{pmatrix}. \quad (1)$$

Two secondary collimators of aperture n_2 locate at phase advances of μ and $\pi-\mu$ downstream of the primary jaw act as absorbers.

The scattering angles of the particles received from the primary collimator are described by [3, 4]

$$g(\theta)d\theta = \frac{1}{\sqrt{2\pi}\theta_0}\exp(-\frac{\theta^2}{2\theta_0^2})d\theta, \quad (2)$$

where

$$\theta_0 = \frac{13.6MeV}{\beta cp} Z\sqrt{\frac{X}{X_0}}(1+0.038\ln(\frac{X}{X_0})), \quad (3)$$

and X/X_0 is the thickness of the primary jaw in unit of radiation length.

According to Ref. [1], the minimum elastic kick needed for the particle to reach the secondary collimator is

$$K_c = \frac{n_2 - n_1\cos\mu}{\sin\mu}, \quad (4)$$

which has a minimum value at $\mu_c = \mathrm{acos}(n_1/n_2)$.

Assume that the particles hit the secondary collimator are absorbed without forming tertiary particles. Then the ratio of the halo particles absorbed by the collimators is

$$f = \int_{-\infty}^{-k_c} g(\theta)d\theta + \int_{k_c}^{\infty} g(\theta)d\theta = Erfc(\frac{K_c}{\sqrt{2}\theta_0}), \quad (5)$$

where $Erfc(z) = 1 - Erf(z)$, and $Erf(z)$ is the error function.

PHYSICAL APERTURE RESTRICTION

Limited physical apertures will restrict the performance of the collimation system. In this case, the classical theory of evaluating the minimum extension of the halo particles can not well define the efficiency of the system. Instead, we use the collimation efficiency, which is the ratio of the beam lost in the collimators and the total beam loss, to evaluate the efficiency of the collimation system.

The vacuum chambers have physical aperture of n_r in phase advance of 0° ~ 90° and $-n_r$ in 90° ~ 180°. Similar to the case of the collimators, the minimum kick needed for the particle to be caught by the physical aperture is

$$K_r = \frac{n_r - n_1\cos\mu}{\sin\mu}. \quad (6)$$

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The expression has minimum value of $K_{ri} = \sqrt{n_r^2 - n_1^2}$ at phase advance of $\mu_r = \mathrm{acos}(n_1/n_r)$. As $n_r > n_2$, we have $k_r > k_c$, and $\mu_r > \mu_c$.

The expression of the collimation efficiency depends on the relation between K_c and K_{ri}. In the case of $K_c \geq K_{ri}$, the total beam loss is expressed as

$$L_{total} = \int_{k_{ri}}^{\infty} g(\theta)d\theta + \int_{-\infty}^{-k_{ri}} g(\theta)d\theta, \qquad (7)$$

and the loss on the collimators is

$$L_{coll} = \int_{k_c}^{k_r} g(\theta)d\theta. \qquad (8)$$

As $g(\theta)$ is an odd function of θ, we get the expression of the collimation efficiency as

$$f = \frac{Erf(\frac{K_r}{\sqrt{2}\theta_0}) - Erf(\frac{K_c}{\sqrt{2}\theta_0})}{2 \cdot Erfc(\frac{K_{ri}}{\sqrt{2}\theta_0})}. \qquad (9)$$

In the case of $K_c < K_{ri}$, the total beam lost and the loss on the collimators are

$$\begin{aligned} L_{total} &= \int_{k_c}^{\infty} g(\theta)d\theta + \int_{-\infty}^{-k_c} g(\theta)d\theta, \\ L_{coll} &= \int_{k_c}^{k_r} g(\theta)d\theta + \int_{-k_{ri}}^{-k_c} g(\theta)d\theta. \end{aligned} \qquad (10)$$

Therefore, the collimation efficiency is

$$f = \frac{Erf(\frac{K_r}{\sqrt{2}\theta_0}) + Erf(\frac{K_{ri}}{\sqrt{2}\theta_0}) - 2Erf(\frac{K_c}{\sqrt{2}\theta_0})}{2 \cdot Erfc(\frac{K_c}{\sqrt{2}\theta_0})}. \qquad (11)$$

We can see that the collimation efficiency depends not only on the phase advance between the primary and secondary collimators, but also the physical aperture and the scattering angle of the primary jaws.

Particle tracking simulations are proceeded to verify the theoretical results. In the simulation, we choose $n_1 = 6$, $n_2 = 7$, $n_r = 8$. The dependence of the collimation efficiency on the phase advance is estimated. As shown in Fig. 1, the efficiency of the collimation system has a peak value of 63% at phase advance of 0.35. The result optimum phase advance is much lower than $\mathrm{acos}(n_1/n_2) = 0.54$. So what is optimum in the well-known theory is not optimum at all when limit physical apertures are considered.

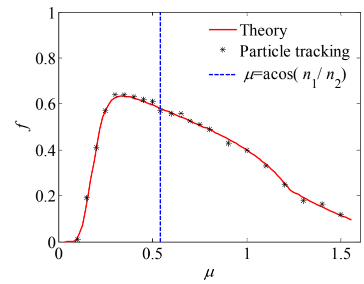

Figure 1: Collimation efficiency with phase advance μ.

The dependences of the optimum phase advance and collimation efficiency on the physical aperture n_r are also estimated. As shown in Fig. 2, the optimum phase advance varies with physical aperture, and the collimation efficiency increases with the increase of the physical aperture.

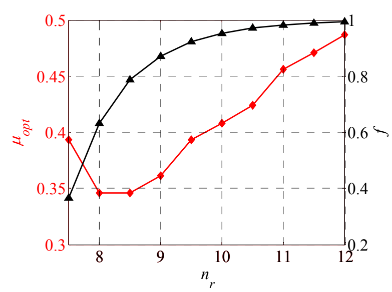

Figure 2: The optimum phase advance and collimation efficiency with different physical aperture n_r.

The influence of the scattering angle of the primary jaw is shown in Fig.3. We can see that both the optimum phase advance and collimation efficiency decrease with the increase of the scattering angle. That's because when θ_0 is small, the amplitude of the scattered particle is considerably small to be intercepted by the physical aperture.

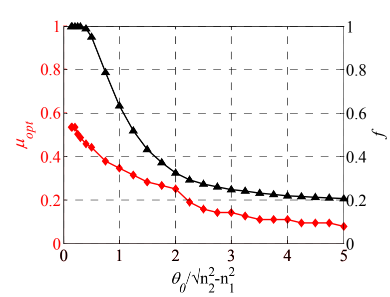

Figure 3: The optimum phase advance and collimation efficiency with the scattering angle of the primary jaw.

NONZERO IMPACT PARAMETER

In this section, nonzero point of impact on the primary collimator with $Y_0 \geq n_1$ and $Y_0' \neq 0$ is investigated. For convenience, we use the transverse azimuthal angle α to define the coordinate of the particle. The coordinates of the particle before scattering are

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$$\vec{A}_0 = (Y_0, Y_0') = (A\cos\alpha, A\sin\alpha)\,, \qquad (12)$$

where $n_1 \le A \le n_2$ and $-\mu_c \le \alpha \le \mu_c$. The halo particles are assumed to have uniform distributions in the region. The coordinates of the particle after scattering are

$$\vec{A}_1 = (A\cos\alpha, A\sin\alpha + K)\,. \qquad (13)$$

Then the particle is transformed by a phase advance of μ, and the coordinates become

$$\vec{A}_2 = \begin{pmatrix} A\cos\alpha\cos\mu + (A\sin\alpha + K)\sin\mu \\ (A\sin\alpha + K)\cos\mu - A\cos\alpha\sin\mu \end{pmatrix}. \qquad (14)$$

We repeat the analysis as in the former section, and get the collimation efficiency for a single particle as

$$f = \begin{cases} \left(Erf(\frac{K_{c2}}{\sqrt{2}\theta_0}) - Erf(\frac{K_{r2}}{\sqrt{2}\theta_0})\right) / \left(Erfc(-\frac{K_{r2i}}{\sqrt{2}\theta_0}) + Erfc(\frac{K_{r1i}}{\sqrt{2}\theta_0})\right), & (K_{c1} \ge K_{1ri}, \mu \ge \mu_0) \\ \left(Erf(\frac{K_{r1}}{\sqrt{2}\theta_0}) - Erf(\frac{K_{c1}}{\sqrt{2}\theta_0})\right) / \left(Erfc(-\frac{K_{r2i}}{\sqrt{2}\theta_0}) + Erfc(\frac{K_{r1i}}{\sqrt{2}\theta_0})\right), & (K_{c1} \ge K_{1ri}, \mu < \mu_0) \\ \dfrac{-Erf(\frac{K_{c1}}{\sqrt{2}\theta_0}) + Erf(\frac{K_{c2}}{\sqrt{2}\theta_0}) + Erf(\frac{K_{r1i}}{\sqrt{2}\theta_0}) - Erf(\frac{K_{r2}}{\sqrt{2}\theta_0})}{Erfc(-\frac{K_{c2}}{\sqrt{2}\theta_0}) + Erfc(\frac{K_{c1}}{\sqrt{2}\theta_0})}, & (K_{c1} < K_{1ri}, \mu \ge \mu_0) \\ \dfrac{-Erf(\frac{K_{c1}}{\sqrt{2}\theta_0}) + Erf(\frac{K_{c2}}{\sqrt{2}\theta_0}) + Erf(\frac{K_{r1}}{\sqrt{2}\theta_0}) - Erf(\frac{K_{r2i}}{\sqrt{2}\theta_0})}{Erfc(-\frac{K_{c2}}{\sqrt{2}\theta_0}) + Erfc(\frac{K_{c1}}{\sqrt{2}\theta_0})}, & (K_{c1} < K_{1ri}, \mu < \mu_0) \end{cases} , \qquad (15)$$

where K_{c1}, K_{c2}, K_{r1} and K_{r2} are given by

$$\begin{aligned} K_{c1} &= \frac{n_2 - A\cos\alpha\cos\mu}{\sin\mu} - A\sin\alpha, \\ K_{c2} &= -\frac{n_2 - A\cos\alpha\cos\mu}{\sin\mu} - A\sin\alpha, \\ K_{r1} &= \frac{n_r - A\cos\alpha\cos\mu}{\sin\mu} - A\sin\alpha, \\ K_{r2} &= -\frac{n_r - A\cos\alpha\cos\mu}{\sin\mu} - A\sin\alpha, \end{aligned} \qquad (16)$$

K_{r1i} and K_{r2i} are extreme values of K_{r1} and K_{r2} at $\mu_r = \text{acos}(A\cos\alpha/n_r)$ and $\pi-\mu_r$, respectively.

Accordingly, the normalized collimation efficiency is

$$\bar{f} = \frac{\int_{-\theta_1}^{\theta_1} A d\theta \int_{\frac{n_1}{\cos\theta}}^{n_2} f dA}{\int_{-\theta_1}^{\theta_1} A d\theta \int_{\frac{n_1}{\cos\theta}}^{n_2} dA} = \frac{\int_{-\theta_1}^{\theta_1} A d\theta \int_{\frac{n_1}{\cos\theta}}^{n_2} f dA}{n_2^2\theta_1 - n_1\sqrt{n_2^2 - n_1^2}} . \qquad (17)$$

The dependence of the collimation efficiency on the phase advance is shown in Fig.4. With nonzero impact parameter, the optimum phase advance is smaller than the case of zero impact parameter.

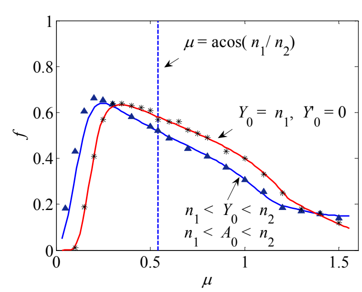

Figure 4: The collimation efficiency with different phase advance μ for nonzero impact parameter.

CONCLUSIONS

A new theory is presented to describe a two stage collimation system. Collimation efficiency is used to evaluate the performance of a collimation system, instead of the extent of the beam halo. The influences of the physical aperture and impact parameter on the collimation efficiency are investigated.

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