

# INVESTIGATION OF EMITTANCE GROWTH IN A SMALL PET CYCLOTRON CYCIAE-14

Ming Li, Shizhong An, Tianjue Zhang, CIAE, Beijing, China

## Abstract

In order to satisfy the rapidly increased domestic needs for PET in China, a small medical cyclotron named CYCIAE-14 is designed and constructed at CIAE (China Institute of Atomic Energy). As the beam intensity in CYCIAE-14 is high, the beam emittance should be controlled strictly in order to reduce the beam loss in the cyclotron. Precessional mixing and resonance crossing are the two main factors leading to emittance growth in the cyclotron with stripping extraction cyclotron. In this paper, the physical mechanism of precessional mixing in a stripping extraction cyclotron is investigated. After that, the maximum allowable field errors in CYCIAE-14 are derived using Hamiltonian and numerical simulation, which provides a reference for the cyclotron design and field shimming.

## INTRODUCTION

Compact cyclotrons to accelerate negative hydrogen ions to low energies are widely used for medical applications. To meet the rapid growth of domestic demand for PET cyclotrons in China, a 14 MeV PET cyclotron CYCIAE-14 has been designed and constructed [1] at CIAE. As CYCIAE-14 adopts a very compact structure with the designed beam intensity to higher than 400  $\mu$ A, the influence on beam emittance induced by the magnetic field error should be considered seriously.

As well known, Precessional mixing and resonance crossing are the two main factors leading to emittance growth in the cyclotron with stripping extraction cyclotron [2][3]. For the cyclotron TR30, restrictions on the magnetic field errors to take emittance growth have been estimated using analytical expressions of Hamiltonian by Kleeven and Hagedoorn [2]. This paper will introduce in detail the numerical analysis and study of the emittance growth in CYCIAE-14.

## PHYSICAL MECHANISM OF PRECESSIONAL MIXING

Magnetic field errors in a cyclotron excite coherent oscillations of the beam because they displace the central orbit or they distort the transverse phase space. The phase advance of the oscillation for a particle in the beam can be expressed as

$$\phi_p = \int^n \nu(E) dn \approx \int^n \left[ \nu(E_0) + \frac{d\nu}{dE} \Delta E \right] dn \quad (1)$$

Where  $n$  is the turn number and  $\nu$  is the tune varying with energy  $E$ . From the formula (1), the phase advance spread is brought not only by the differ of the turn

numbers which is presented by Kleeven and Hagedoorn, but also by the tune spread. In order to investigate the phenomena of precessional mixing brought by tune spread during acceleration, thousands of particles are tracked from 2.08 MeV at the centro-symmetry plane of valley region to extracted energy, supposing that the initial transverse beam emittance is 4.0pi-mm-mrad. The initial conditions of the simulation are divided into three different cases: (a) vertical beam well matched and the beam center does not shift. (b) vertical beam phase space with 50% of mismatching, which means vertical envelope is 50% of the original value and divergence angle is 2 times of the original so that the beam emittance will remain unchanged. (c) vertical beam well matched and the vertical shift of the beam center is 5 mm.

The simulation results shown in Fig. 1 indicate that the mismatching of the beam phase space or vertical shift of the beam center can lead to precessional mixing and bring an apparent increase of the emittance growth.

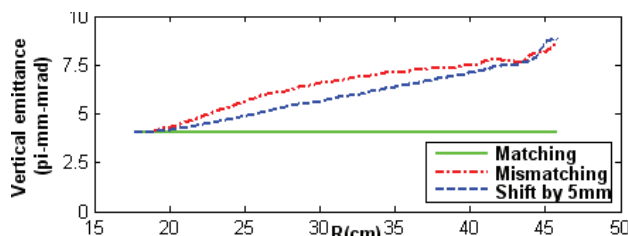


Figure 1: With three different initial distributions, (a) vertical coordinate of beam center (b) vertical beam envelope (c) vertical beam emittance as a function of radius at an azimuth ( $60^\circ$ ) near stripping foil.

## FIELD ERROR TOLERANCE WITH PRECESSIONAL MIXING

In this cyclotron, we mainly consider the resonances  $\nu_r = 1$ ,  $2\nu_r = 2$ ,  $\nu_z = k$  and  $2\nu_z = 1$ . Among these resonances whether the beam crosses or not, coherent oscillations would be excited with corresponding magnetic field errors and leading to beam emittance growth during acceleration and on the foil.

$$\nu_r = 1$$

The driving term of this resonance is first harmonic of the magnetic field error. To study this resonance, a analytical formula is derived as [2]

$$\Delta r = \frac{B_1}{B_0 \chi} \quad (2)$$

Where  $\Delta r$  is the center orbit shift,  $B_0$ ,  $B_1$  are the average field and amplitude of the first harmonic field respectively, and  $\nu_r - 1$  is assumed to be proportional with radius, i.e.  $\nu_r - 1 = \chi r$ . For the CYCIAE-14, this condition is approximatively satisfied with  $\chi = 0.15 \text{ m}^{-1}$ . Giving the average field  $B_0$  is 12 kGs, if we allow an orbit centre shift of 1 mm, the first harmonic field should be smaller than 2 Gs.

Actually, the effect does not depend only on the magnitude of the field errors but also on the radial width of the area in which they occur. A uniformly distributed first harmonic field along radial range  $R_i - R_f$  is added to the main field, results of the radial oscillation are illustrated in Fig. 2.

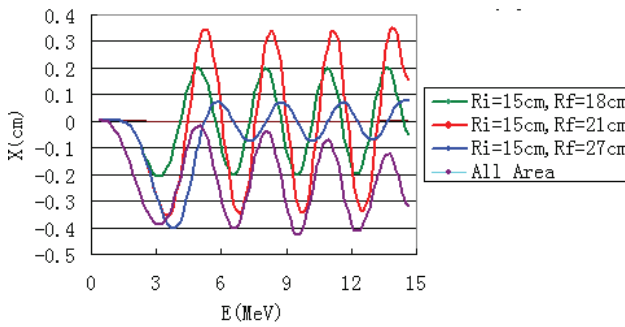


Figure 2: Radial oscillation with first harmonic field along different radial ranges (1) 15 – 18cm (2) 15 – 21 cm (3) 15 – 27 cm (4) everywhere.

$$2\nu_r = 2$$

This resonance is driven by the second harmonic field error. Assuming complete precessional mixing happened, a maximum emittance growth  $f$  could be obtained [2]

$$f = \frac{C_0 + C_2}{C_0 - C_2} \quad (3)$$

With  $C_0 = \nu_r - 1$  and  $C_2 = b_2 / 2B_0$ , where  $b_2$  is the amplitude of the second harmonic field. In the center of the CYCIAE-14,  $C_0 \approx 0.01$ , which means if the emittance growth factor  $f$  is restricted to be less than 1.5, the maximum second harmonic field should satisfy

$$b_2 < 24Gs$$

$$2\nu_r = 1$$

This resonance is driven by the first gradient of the first harmonic field error  $db_1 / dr$ . It has little influence on the radial motion but brings coherent oscillation to the axial motion. The Hamiltonian describing this resonance is given by

$$H = G \left[ \left( \nu_z - \frac{1}{2} \right) - \frac{1}{4\nu_z} \frac{b_1'}{B_0} \cos(2\psi) \right] \quad (4)$$

with  $G$  and  $\psi$  as the action-angle variables.

The maximum emittance growth  $f$  has the same form as formula (3) but with the

$$C_2 = \frac{db_1 / dr}{4\nu_z} \quad (5)$$

At the extracted energy  $E = 14 \text{ MeV}$ ,  $\nu_z = 0.56$  and  $r = 44.6 \text{ cm}$ . If the emittance growth factor  $f$  is restricted to be less than 1.5, the gradient of the first harmonic at extraction should satisfy

$$db_1 / dr < \frac{4\bar{B}}{5r} \nu_z \left( \nu_z - \frac{1}{2} \right) \approx 7Gs / cm$$

## FIELD ERROR TOLERANCE WITH RESONANCE CROSSING

As the energy is very low in this type of cyclotron, the radial tune  $\nu_r$  varies little and there is no radial resonance crossed. But in the axial direction, there are three resonances crossed as shown in Fig. 3, every of which would bring axial emittance growth and worsen the beam quality at some extent.

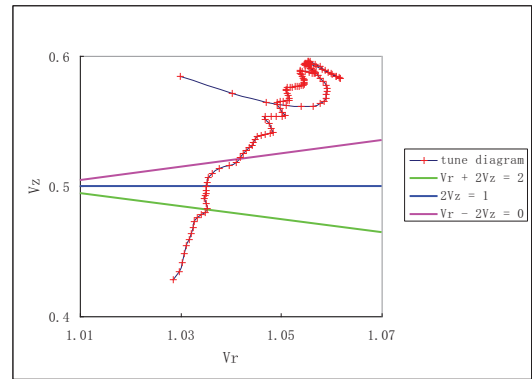


Figure 3: Tune diagram and the three main resonances crossed in CYCIAE-14.

$$2\nu_r = 1$$

According to the Hamiltonian formula (4), the motion equation is obtained

$$\frac{d\psi}{d\theta} = (\nu_z - 0.5) + \text{oscillation term} \approx \left( \frac{d\nu_z}{d\theta} \right)_{2\nu_z=1} \theta \quad (6)$$

$$\psi = \int \left( \frac{d\nu_z}{d\theta} \right)_{2\nu_z=1} \theta d\theta = \frac{1}{4\pi} \nu_z' \theta^2 \quad (7)$$

In which

$$\nu_z' = \left( \frac{d\nu_z}{dn} \right)_{2\nu_z=1} = \left( \frac{d\nu_z}{dE} \right)_{2\nu_z=1} \Delta E \quad (8)$$

with  $\Delta E$  as the energy gain every turn. The other motion equation from the Hamiltonian is given as

$$\frac{dG}{d\theta} = -\frac{\partial H}{\partial \psi} = \frac{b_1'}{B_0} G \sin(2\psi) \quad (9)$$

As the axial oscillation amplitude  $A_z \propto \sqrt{G}$ , with formula (7) and (9), we get

$$\frac{\Delta A_z}{A_z} = \frac{1}{2} \int \frac{b'_1}{B_0} \sin\left(\frac{1}{2\pi} \nu'_z \theta^2\right) d\theta = \frac{\pi}{2\sqrt{\nu'_z}} \frac{b'_1}{B_0} \quad (10)$$

According to Fig. 4, at the position of resonance crossing, it has  $dv_z/dE \approx 0.06 \text{ MeV}^{-1}$  and  $\Delta E = 0.16 \text{ MeV}$ . Given a gradient of first harmonic field of 10 Gs/cm, the relative growth of axial oscillation amplitude  $\Delta A_z/A_z$  is about 0.3. To verify the analytic result, first harmonic field gradient of 10 Gs/cm and 20 Gs/cm, as shown in Fig. 4, are added to the main field. Figure 5 illustrates the simulation results, which proves that the relative growth of axial oscillation amplitude is be proportional with first harmonic field gradient.

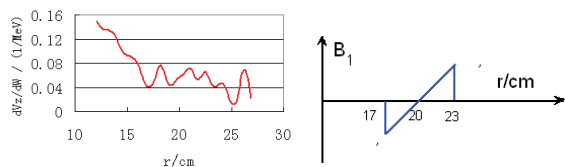


Figure 4:  $dv_z/dE$  varies radius (left); Amplitude along radius to provide a gradient of first harmonic field error (right).

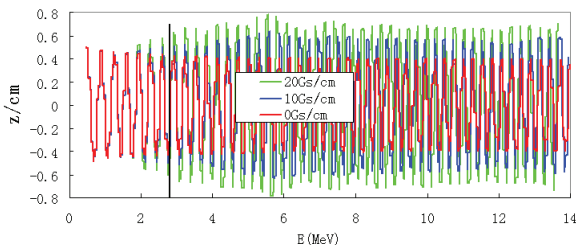


Figure 5: Axial oscillation with first harmonic field gradient of (1) 0 Gs/cm (2) 10 Gs/cm (3) 20 Gs/cm.

$\nu_r - 2\nu_z = 0$  (*Walkinshaw Resonance*)

$\nu_r - 2\nu_z = 0$  is a very dangerous resonance in this type of cyclotron as it is driven by the radial derivatives of the main field. Even there is no field errors, axial beam quality would be worsen in the case the radial beam amplitude is large. In the process of CYCIAE-14 design, the main field is optimized iteratively to avoid crossing this resonance. A centered beam and other two  $\pm 5$  mm off centered beams are accelerated in the cyclotron. Figure 6 shows the the influence of crossing this resonance on

beam emittance is tiny, demonstrating the designed main field is very effective.

$$\nu_r + 2\nu_z = 2$$

This resonance is a two dimensional nonlinear resonance driven by a second harmonic field error and its gradient. Comparing to the influence of the precessional mixing brought by the same field error, impact of this resonance could be neglectable.

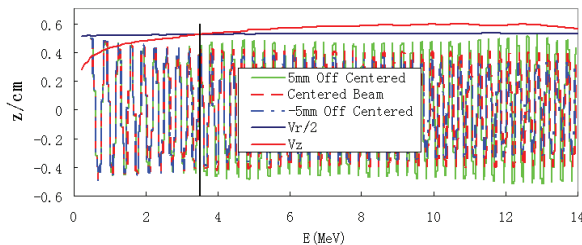


Figure 6: Axial oscillation with centered beam and  $\pm 5$  mm off centered beam.

### SUMMARY

In this paper, influence of precessional mixing and resonance crossing on beam emittance growth is investigated. Computations showed that the  $\nu_r = 1$  and  $2\nu_z = 1$  are seemed as the really important resonance in CYCIAE-14, which restrict the first harmonic field error and its gradient to be less 2 Gs and 10 Gs/cm respectively. Essentially Walkinshaw resonance could be a very dangerous, but with optimized design of main field, the impact on beam quality could be neglectable. Except that, tolerances for the second harmonic field error and its gradient other resonance are s educed by other resonances, which will provides a reference for the process of field shimming.

### REFERENCES

- [1] Tianjue Zhang, Overall Design of CYCIAE-14, a 14MeV PET Cyclotron, to be published.
- [2] W.J.G.M. Kleeven, H.L. Hagedoorn, "The Influence of Magnetic Field Imperfections on the Beam Quality in an H- Cyclotron," Proc. 13th Int. Conf. On Cyclotrons and their Applications, Canada, 1992, p. 380.
- [3] R.E. Laxdal et al., "Beam Quality Investigations For H- Extraction at Triumf", Proc. 13th Int. Conf. On Cyclotrons and their Applications , 1992, p. 415.