



TIARA
TAKASAKI ION ACCELERATORS
for
ADVANCED RADIATION APPLICATION

Focusing Charged Particle Beams Using Multipole Magnets in a Beam Transport Line

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Introduction

- Use of multipole magnets in a beam line:
 - Modulation of the transverse phase-space distribution
 - Transformation of the real-space intensity distribution
- A “*uniform beam**” can be formed using octupole magnets.
 - For low-rep-rate beam irradiation (BNL/NSRL)
 - For suppression of local target heating due to a high-intensity beam (GSI/UNILAC, IUCF, J-PARC, CSNS, ESS, IFMIF, etc.)
 - For low-fluence/short-time irradiation ([JAEA Takasaki](#))

Nonlinear focusing method is more capable
for advanced irradiation
as compared to *beam scanning method*.

*A beam with an almost uniform transverse intensity distribution

- The transformation of the transverse distribution is studied using sextupole and octupole magnets.

$$x'' + K_{\text{QUAD}}x + \frac{K_{\text{SXT}}}{2!}x^2 = 0$$

$$x'' + K_{\text{QUAD}}x + \frac{K_{\text{OCT}}}{3!}x^3 = 0$$



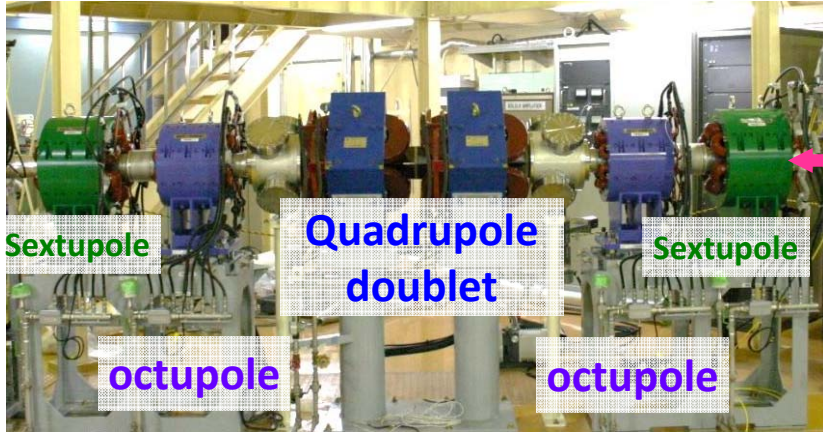
General equation of motion

$$\begin{cases} x'' + K_{\text{QUAD}}x + \frac{K_{\text{SXT}}}{2!}(x^2 - y^2) + \frac{K_{\text{OCT}}}{3!}(x^3 - 3xy^2) + \dots = 0 \\ y'' - K_{\text{QUAD}}y - \frac{K_{\text{SXT}}}{2!}(2xy) + \frac{K_{\text{OCT}}}{3!}(y^3 - 3x^2y) + \dots = 0 \end{cases}$$

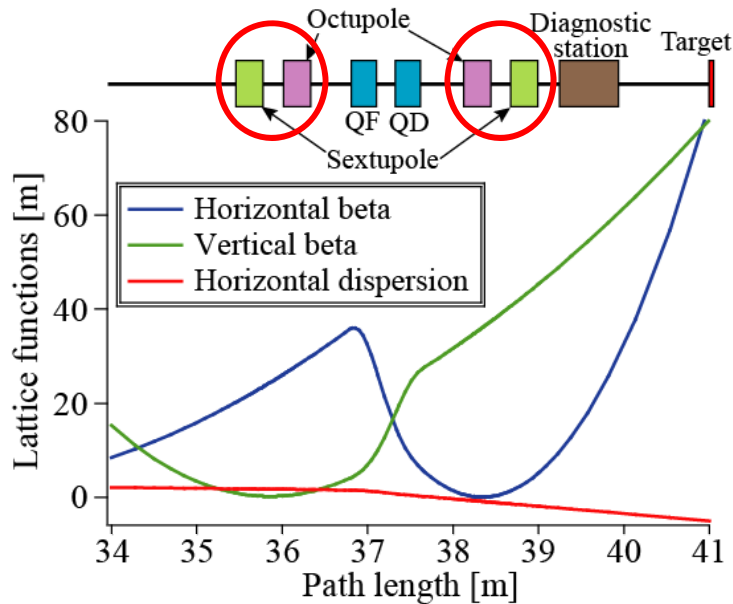
□ Agenda

- Tracking simulation, and its comparison with theory
- Experiment at JAEA Takasaki Cyclotron
(uniform-beam formation)

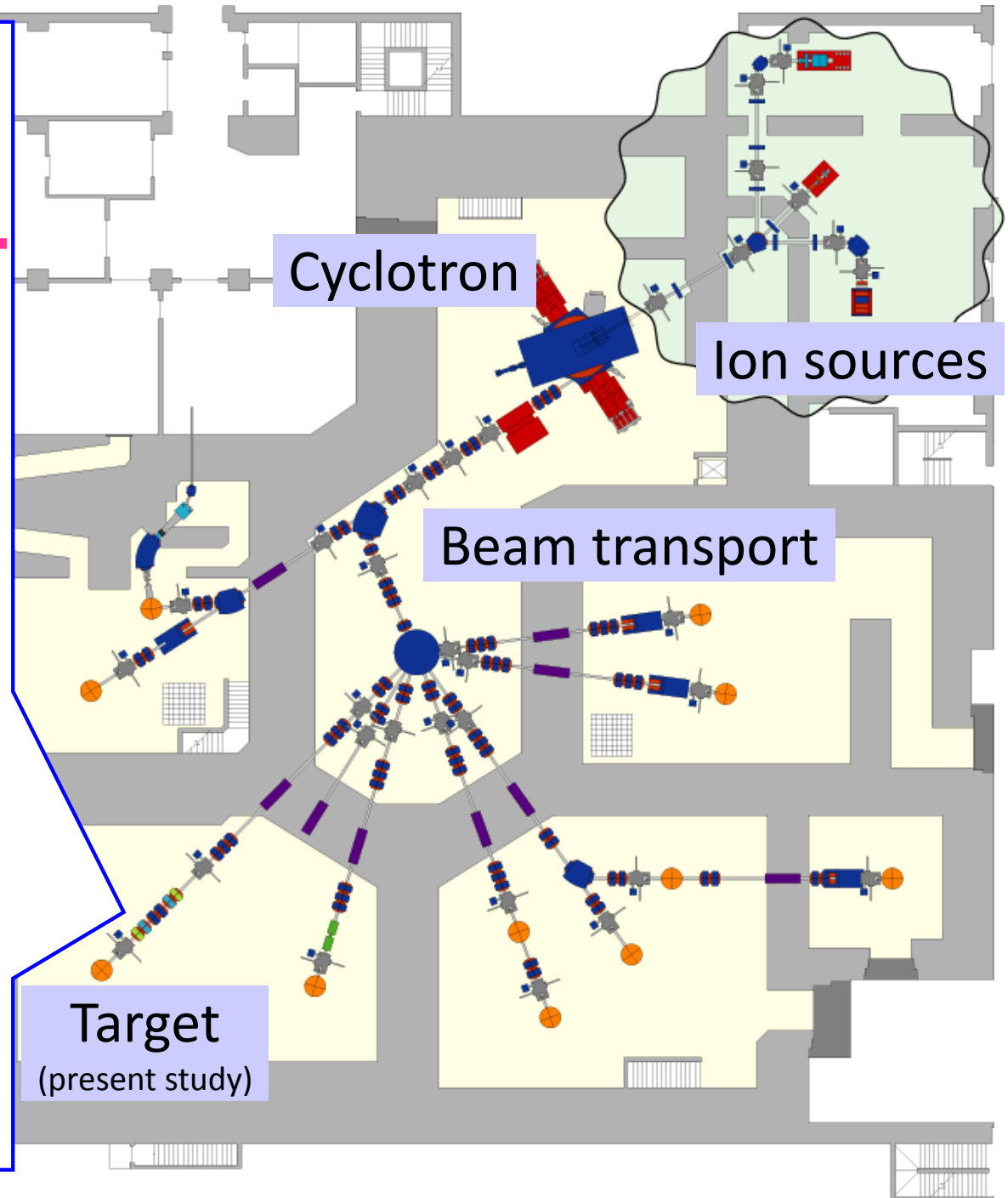
Multipole magnets



Typical beam optics for x-y coupling suppression

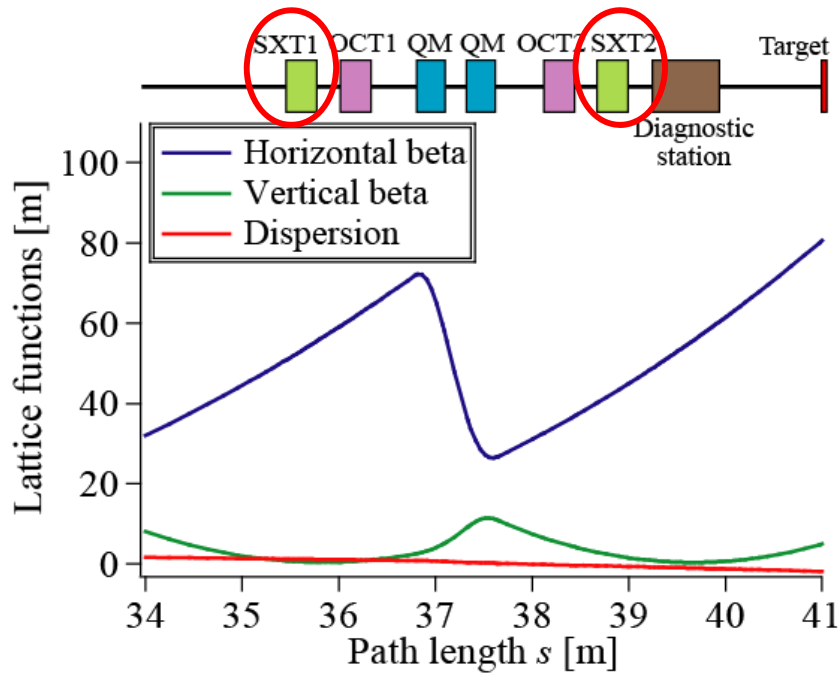


The beam cross-section is made flat at multipole magnet locations

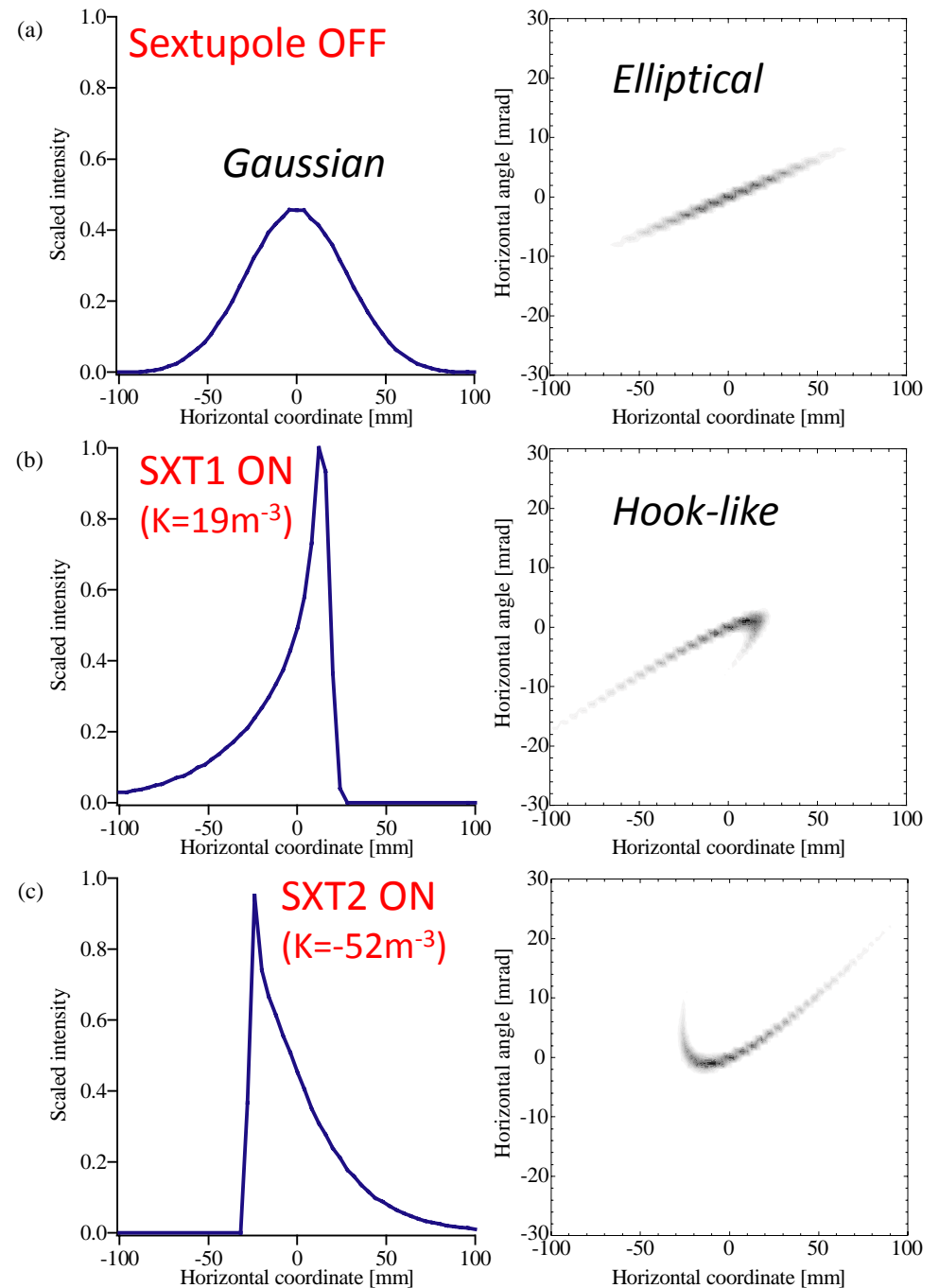


Simulation (1: Sextupole focusing)

- Single-particle tracking
 - From Cyclotron Exit to Target
 - Initial distribution: Gaussian
 - Rms emittance = 10π mm.mrad
 - 1D (horizontal only)

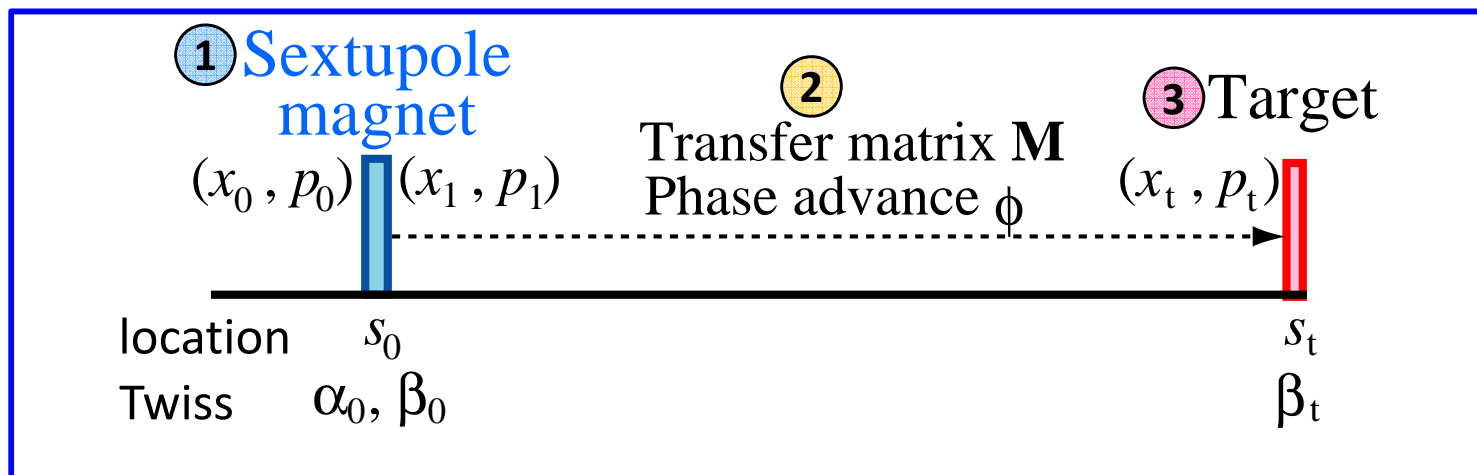


$$x'' + K_{\text{QUAD}}x + \frac{K_{\text{SXT}}}{2!}x^2 = 0$$



Theoretical analysis (1: Single-particle motion)

□ Model of the beam line



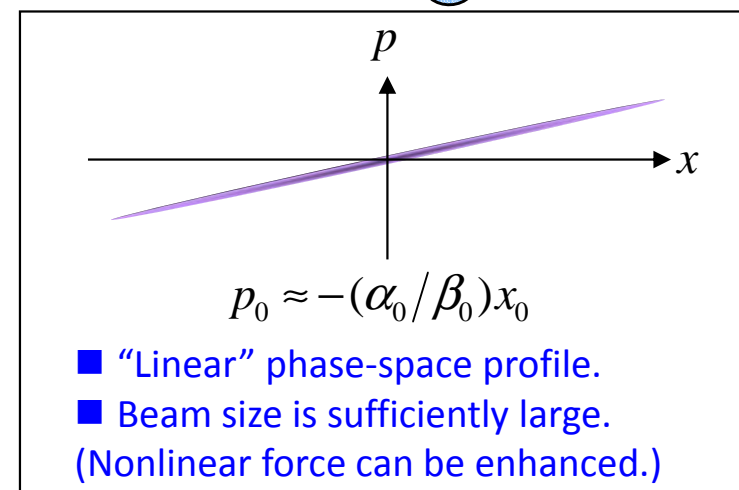
□ Single-particle motion

①
$$\begin{cases} x_1 = x_0 \\ p_1 = p_0 - \frac{K_{\text{SXT}} L_{\text{SXT}}}{2} x_0^2 \end{cases} \quad (\text{Thin-lens approx.})$$

②
$$\begin{pmatrix} x_t \\ p_t \end{pmatrix} = \mathbf{M} \begin{pmatrix} x_1 \\ p_1 \end{pmatrix}$$

③
$$x_t = \sqrt{\frac{\beta_t}{\beta_0}} \cos \phi x_0 - \sqrt{\beta_0 \beta_t} \sin \phi \frac{K_{\text{SXT}} L_{\text{SXT}}}{2} x_0^2$$

Approximation at ①



Theoretical analysis (2: real-space distribution)

Real-space distribution function

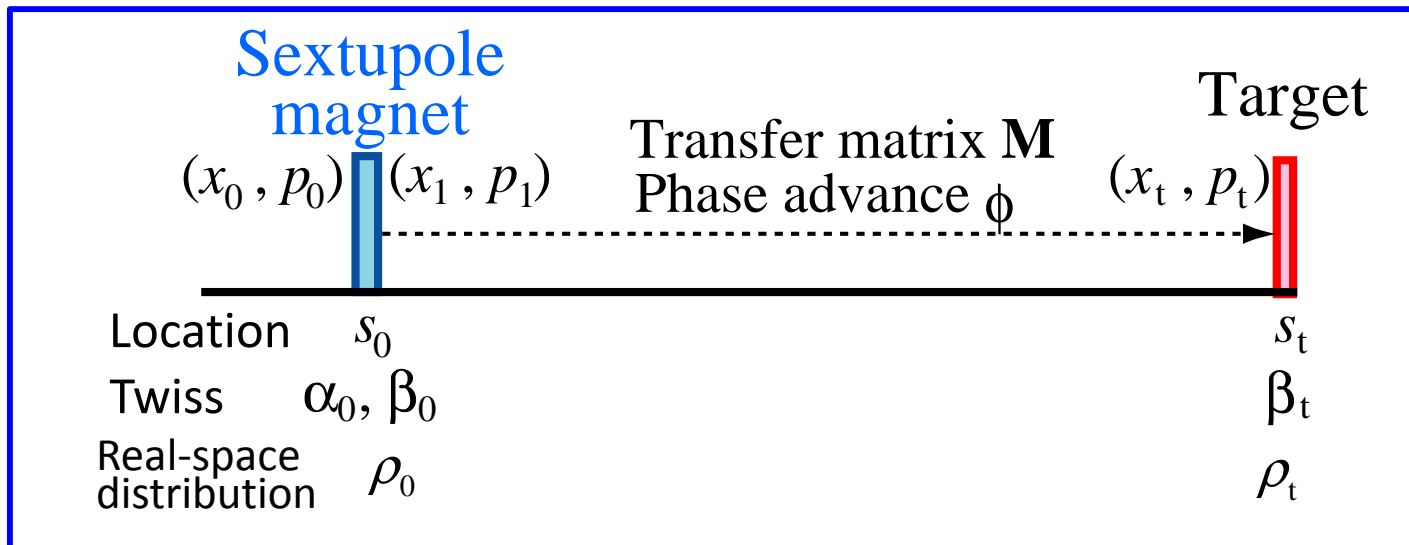
$$dN = \rho_0 dx_0 = \rho_t dx_t \quad \longrightarrow \quad \rho_t = \rho_0 \left(\frac{dx_t}{dx_0} \right)^{-1}$$

(Particle number is preserved.)

Real-space distribution on the target:

$$\rho_t = \rho_0 \left(\frac{dx_t}{dx_0} \right)^{-1}$$

$$= \rho_0 / \left[\sqrt{\frac{\beta_t}{\beta_0}} \cos \phi - \sqrt{\beta_0 \beta_t} \sin \phi (K_{SXT} L_{SXT}) x_0 \right]$$



Statistical information of the beam can be obtained from moments.

□ 1st-order moment: Beam centroid displacement

$$\begin{aligned}
 X &\equiv \langle x_t \rangle = \int x_t \rho_t dx_t \\
 &= -\frac{1}{2} \varepsilon \beta_0 \sqrt{\beta_0 \beta_t} (K_{\text{SXT}} L_{\text{SXT}}) \sin \phi
 \end{aligned}$$

Displaced
due to sextupole force

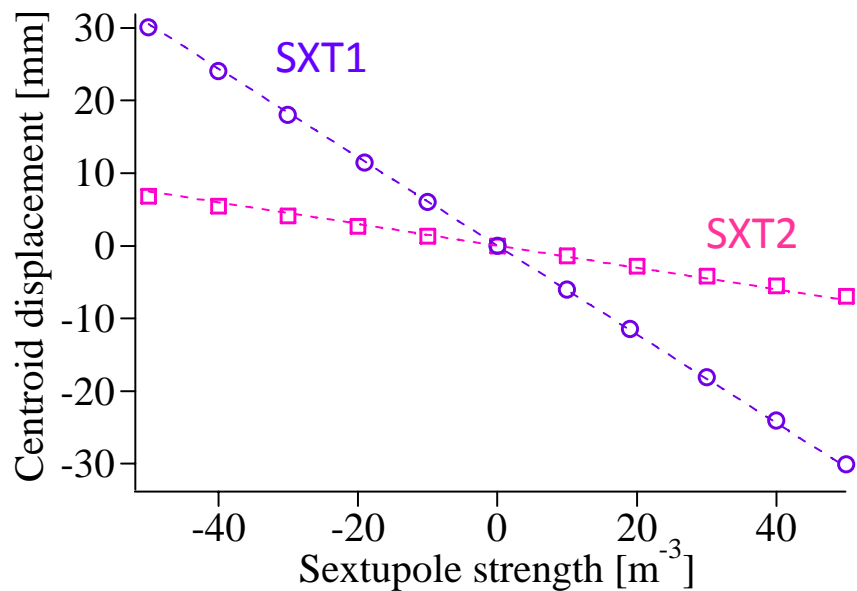
□ 2nd-order moment: RMS beam radius (envelope)

$$\begin{aligned}
 \sigma &\equiv \sqrt{\langle (x_t - X)^2 \rangle} = \sqrt{\int (x_t - X)^2 \rho_t dx_t} \\
 &= \sqrt{\varepsilon \beta_t} \sqrt{1 + \frac{1}{2} \varepsilon \beta_0^3 (K_{\text{SXT}} L_{\text{SXT}})^2 \tan^2 \phi} |\cos \phi|
 \end{aligned}$$

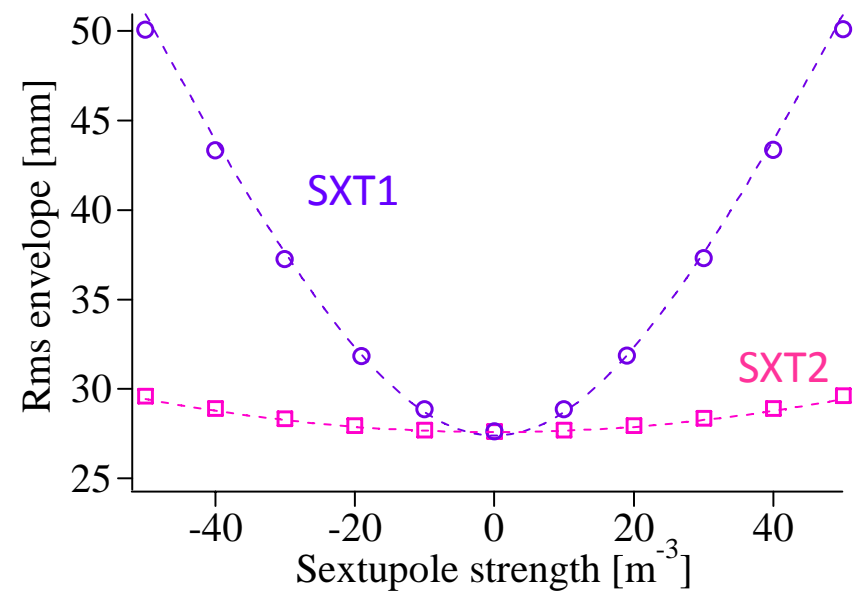
Always increase
due to sextupole force

- Comparing simulation results with the theoretical predictions.

Centroid displacement on the target



Rms envelope on the target



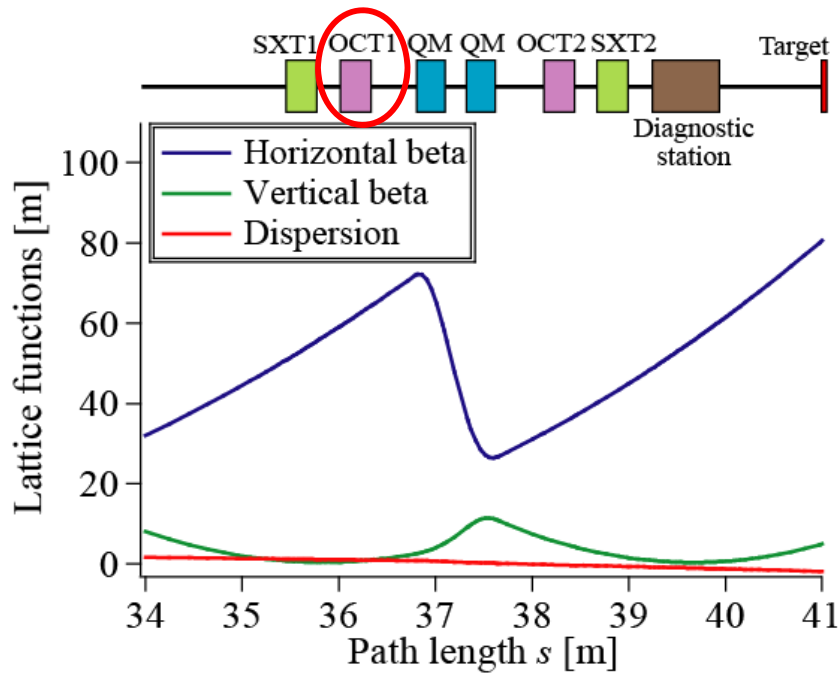
Dashed lines: Theory
 ○, □: Simulations

$$X = -\frac{1}{2} \epsilon \beta_0 \sqrt{\beta_0 \beta_t} (K_{\text{SXT}} L_{\text{SXT}}) \sin \phi$$

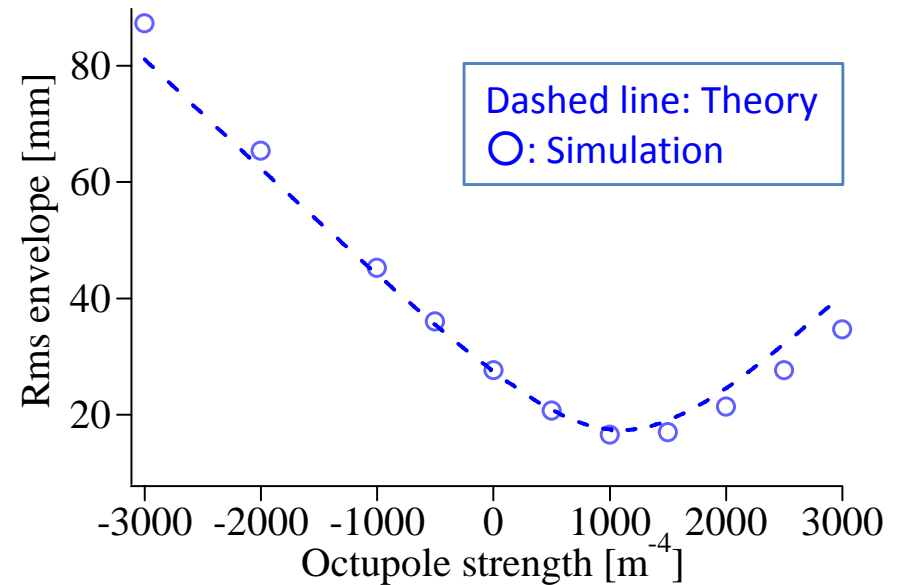
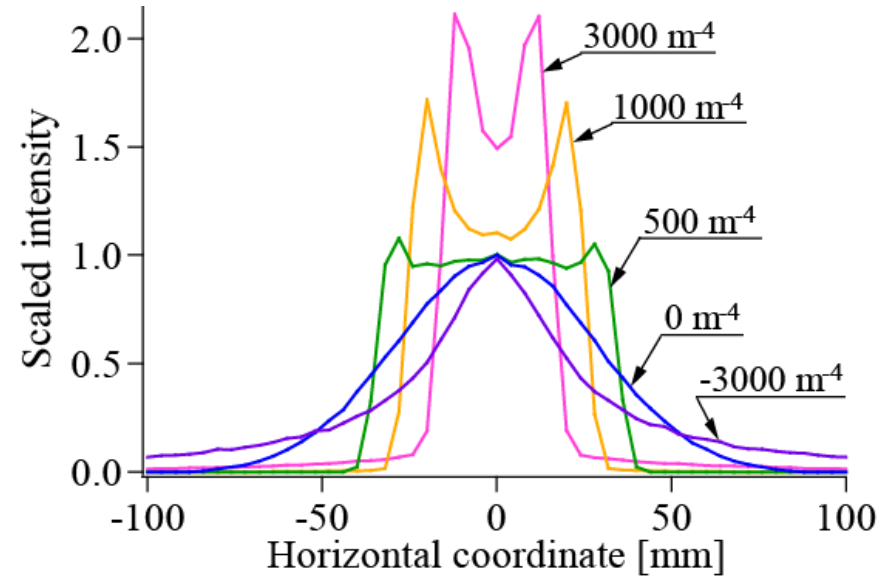
$$\sigma = \sqrt{\epsilon \beta_t} \sqrt{1 + \frac{1}{2} \epsilon \beta_0^3 (K_{\text{SXT}} L_{\text{SXT}})^2 \tan^2 \phi} |\cos \phi|$$

Simulation (2: Octupole focusing)

- Single-particle tracking
 - From Cyclotron Exit to Target
 - Initial distribution: Gaussian
 - Rms emittance = 10π mm.mrad
 - 1D (horizontal only)



$$x'' + K_{\text{QUAD}}x + \frac{K_{\text{OCT}}}{3!}x^3 = 0$$



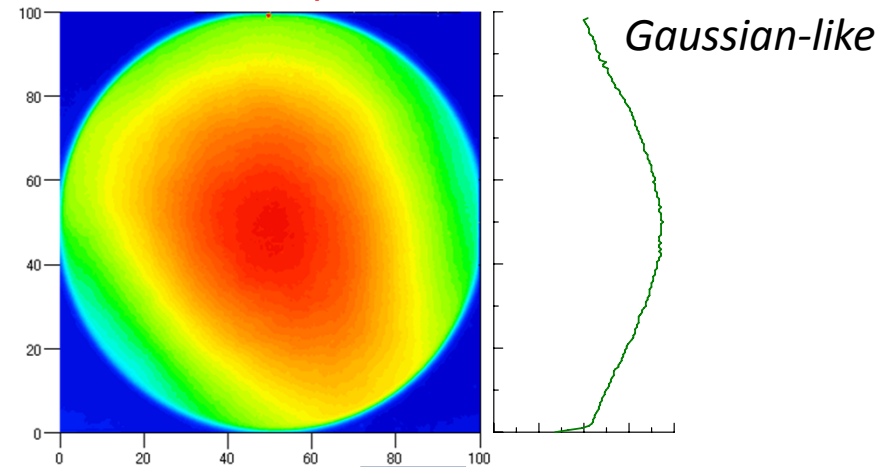
$$\sigma = \sqrt{\epsilon\beta_t} \sqrt{1 - \epsilon\beta_0^2 (K_{\text{OCT}}L_{\text{OCT}}) \tan\phi + \frac{5}{12} [\epsilon\beta_0^2 (K_{\text{OCT}}L_{\text{OCT}}) \tan\phi]^2 |\cos\phi|}$$

Experiment @ JAEA (1: Octupole focusing)

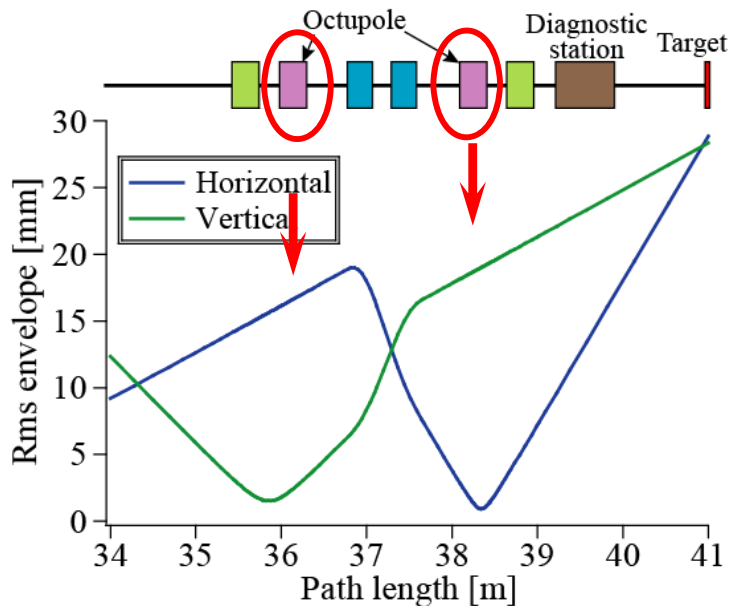
Beam experiment at JAEA Takasaki:

- 10MeV protons from Cyclotron
- Focused by octupole magnets
- Measured on-target 2D profile using radiochromic films

Octupoles: OFF



Beam optics for a large-area uniform beam

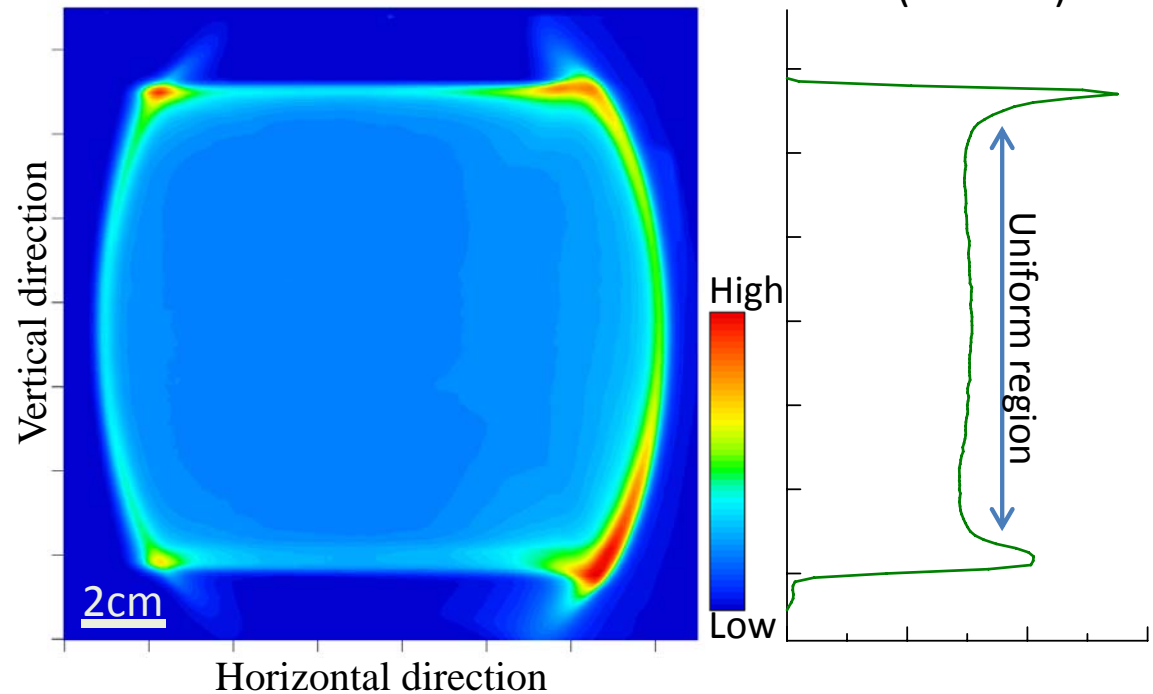


The beam cross-section is made flat at multipole magnet locations

2D relative intensity

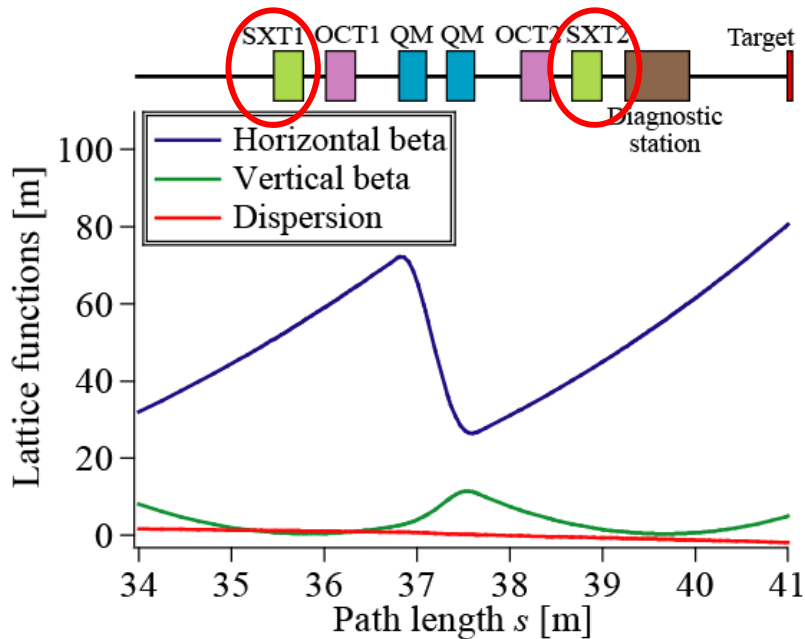


1D relative intensity (Vertical)

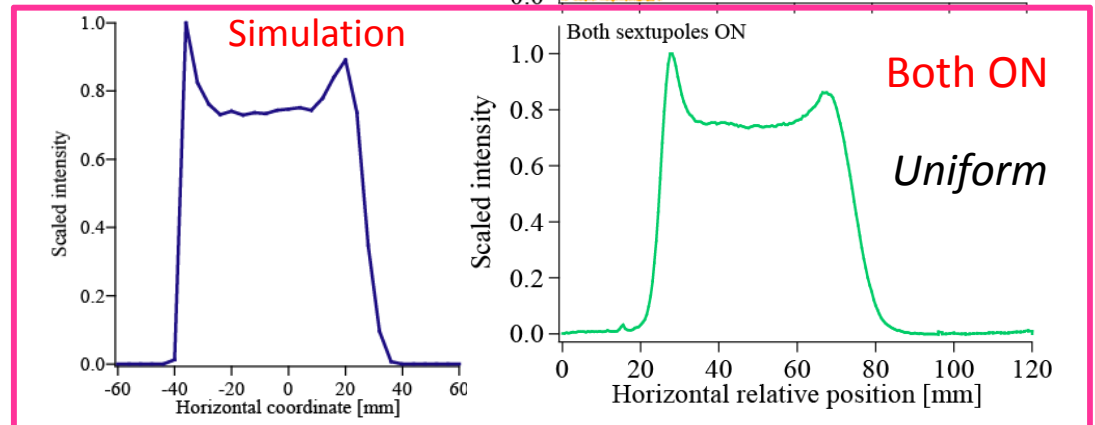
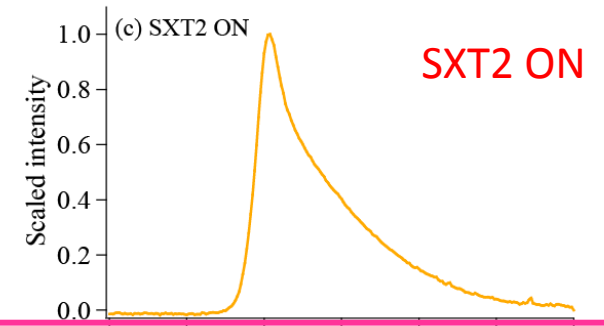
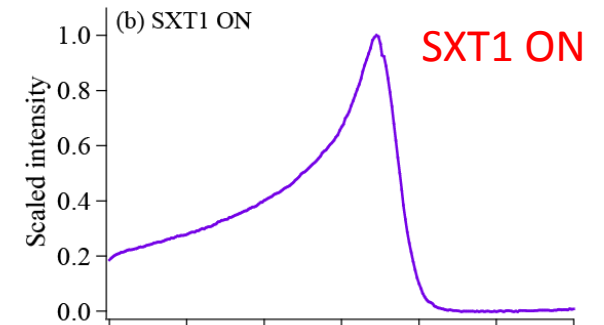
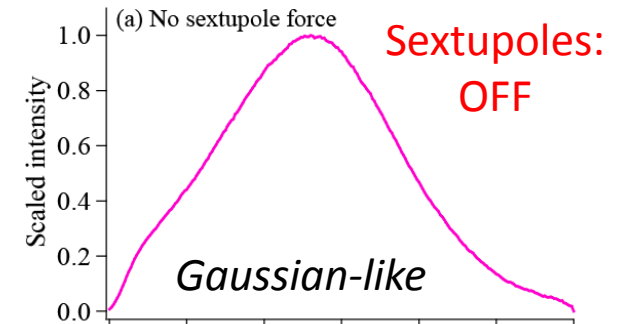


Experiment @ JAEA (2: Sextupole focusing)

- ❑ Beam experiment at JAEA Takasaki:
 - ❑ 10MeV protons from Cyclotron
 - ❑ Focused by sextupole magnets
 - ❑ Measured on-target 2D profile using radiochromic films



The beam cross-section is made flat at multipole magnet locations



Summary

- ❑ We investigated the transformation of the transverse distribution by multipole magnets theoretically, numerically, and experimentally.
- ❑ The centroid displacement and rms envelope change of the beam focused by a sextupole or octupole magnet were shown.
- ❑ Furthermore, the intensity distribution can be transformed from a Gaussian one to a uniform one by octupole focusing or by combined sextupole focusing.
- ❑ Such uniform beams tailored by means of the nonlinear focusing method are used for applications in materials sciences (ion-track membranes, space-use device test, etc.) at JAEA Takasaki.