Focusing Charged Particle Beams Using Multipole Magnets in a Beam Transport Line

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Introduction

- Use of multipole magnets in a beam line:
  - Modulation of the transverse phase-space distribution
  - Transformation of the real-space intensity distribution

- A “uniform beam*” can be formed using octupole magnets.
  - For low-rep-rate beam irradiation (BNL/NSRL)
  - For suppression of local target heating due to a high-intensity beam (GSI/UNILAC, IUCF, J-PARC, CSNS, ESS, IFMIF, etc.)
  - For low-fluence/short-time irradiation (JAEO Takasaki)

*Nonlinear focusing method is more capable for advanced irradiation as compared to beam scanning method.*
The transformation of the transverse distribution is studied using sextupole and octupole magnets.

\[ x'' + K_{\text{QUAD}} x + \frac{K_{\text{SXT}}}{2!} x^2 = 0 \]

\[ x'' + K_{\text{QUAD}} x + \frac{K_{\text{OCT}}}{3!} x^3 = 0 \]

General equation of motion

\[ \begin{cases} x'' + K_{\text{QUAD}} x + \frac{K_{\text{SXT}}}{2!} (x^2 - y^2) + \frac{K_{\text{OCT}}}{3!} (x^3 - 3xy^2) + \cdots = 0 \\ y'' - K_{\text{QUAD}} y - \frac{K_{\text{SXT}}}{2!} (2xy) + \frac{K_{\text{OCT}}}{3!} (y^3 - 3x^2y) + \cdots = 0 \end{cases} \]

Agenda

- Tracking simulation, and its comparison with theory
- Experiment at JAEA Takasaki Cyclotron (uniform-beam formation)
AVF cyclotron at JAEA Takasaki

Multipole magnets

- Sextupole
- Quadrupole doublet
- Octupole
- Sextupole

Typical beam optics for x-y coupling suppression

- Octupole
- QF QD
- Sextupole
- Diagnostic station
- Target

The beam cross-section is made flat at multipole magnet locations.

Cyclotron

Ion sources

Beam transport

Target (present study)
Single-particle tracking
- From Cyclotron Exit to Target
- Initial distribution: Gaussian
- Rms emittance=$10\pi$ mm.mrad
- 1D (horizontal only)

Simulation (1: Sextupole focusing)

\[ x'' + K_{\text{QUAD}} x + \frac{K_{\text{SXT}}}{2!} x^2 = 0 \]
Theoretical analysis (1: Single-particle motion)

- **Model of the beam line**
  - **Sextupole magnet**
    - Location $s_0$
    - Twiss $\alpha_0, \beta_0$
  - **Transfer matrix $M$**
    - Phase advance $\phi$
  - **Target**
    - Location $s_t$
    - Twiss $\beta_t$

- **Single-particle motion**
  1. Initial position $x_0$, initial momentum $p_0$
  2. Single-particle equation:
    \[
    \begin{pmatrix} x_t \\ p_t \end{pmatrix} = M \begin{pmatrix} x_1 \\ p_1 \end{pmatrix}
    \]
  3. Approximation at (Thin-lens approx.):
    \[
    x_t = x_0, \quad p_t = p_0 - \frac{K_{SXT} L_{SXT}}{2} x_0^2
    \]

- Approximation at (Thin-lens approx.):
  - $p_0 \approx -\left(\frac{\alpha_0}{\beta_0}\right)x_0$
  - "Linear" phase-space profile.
  - Beam size is sufficiently large.
  - (Nonlinear force can be enhanced.)

Y. Yuri et al., PRSTAB2007
Theoretical analysis (2: real-space distribution)

- **Real-space distribution function**
  \[
  dN = \rho_0 dx_0 = \rho_t dx_t \\
  \rho_t = \rho_0 \left( \frac{dx_t}{dx_0} \right)^{-1}
  \]
  (Particle number is preserved.)

- **Real-space distribution on the target:**
  \[
  \rho_t = \rho_0 \left( \frac{dx_t}{dx_0} \right)^{-1} \\
  = \rho_0 \sqrt{\frac{\beta_t}{\beta_0}} \cos \phi - \sqrt{\beta_0 \beta_t} \sin \phi \left( K_{SXT} L_{SXT} \right) x_0
  \]

**Diagram**

- Sextupole magnet
- Transfer matrix **M**
- Target
- Phase advance \(\phi\)
- Location \(s_0\)
- Twiss \(\alpha_0, \beta_0\)
- Real-space distribution \(\rho_0\)
- \((x_0, p_0)\) to \((x_1, p_1)\)
- Location \(s_t\)
- Twiss \(\beta_t\)
- Real-space distribution \(\rho_t\)

Y. Yuri et al., PRSTAB2007
Theoretical analysis (3: Moment)

Statistical information of the beam can be obtained from moments.

**1st-order moment: Beam centroid displacement**

\[
X \equiv \langle x_t \rangle = \int x_t \rho_t \, dx_t \\
= -\frac{1}{2} \varepsilon \beta_0 \sqrt{\beta_0 \beta_t} (K_{SXT} L_{SXT}) \sin \phi
\]

Displaced due to sextupole force

**2nd-order moment: RMS beam radius (envelope)**

\[
\sigma = \sqrt{\langle (x_t - X)^2 \rangle} = \sqrt{\int (x_t - X)^2 \rho_t \, dx_t} \\
= \sqrt{\varepsilon \beta_t \sqrt{1 + \frac{1}{2} \varepsilon \beta_0^3 (K_{SXT} L_{SXT})^2} \tan^2 \phi |\cos \phi|}
\]

Always increase due to sextupole force

A Gaussian distribution has been assumed as an initial distribution \( \rho_0 \).
Comparing simulation results with the theoretical predictions.

Centroid displacement on the target

Rms envelope on the target

\[ X = -\frac{1}{2} \varepsilon \beta_0 \sqrt{\beta_0 \beta_t} \left( K_{\text{SXT}} L_{\text{SXT}} \right) \sin \phi \]

\[ \sigma = \sqrt{\varepsilon \beta_t} \sqrt{1 + \frac{1}{2} \varepsilon \beta_0^3 \left( K_{\text{SXT}} L_{\text{SXT}} \right)^2} \tan^2 \phi |\cos \phi| \]
Simulation (2: Octupole focusing)

- Single-particle tracking
  - From Cyclotron Exit to Target
  - Initial distribution: Gaussian
  - Rms emittance = 10π mm.mrad
  - 1D (horizontal only)

\[ x'' + K_{\text{QUAD}} x + \frac{K_{\text{OCT}}}{3!} x^3 = 0 \]
Beam experiment at JAEA Takasaki:
- 10MeV protons from Cyclotron
- Focused by octupole magnets
- Measured on-target 2D profile using radiochromic films

Beam optics for a large-area uniform beam

The beam cross-section is made flat at multipole magnet locations

Y. Yuri et al., NIMA2011
Experiment @ JAEA (2: Sextupole focusing)

- Beam experiment at JAEA Takasaki:
  - 10MeV protons from Cyclotron
  - Focused by sextupole magnets
  - Measured on-target 2D profile using radiochromic films

The beam cross-section is made flat at multipole magnet locations
We investigated the transformation of the transverse distribution by multipole magnets theoretically, numerically, and experimentally.

The centroid displacement and rms envelope change of the beam focused by a sextupole or octupole magnet were shown.

Furthermore, the intensity distribution can be transformed from a Gaussian one to a uniform one by octupole focusing or by combined sextupole focusing.

Such uniform beams tailored by means of the nonlinear focusing method are used for applications in materials sciences (ion-track membranes, space-use device test, etc.) at JAEA Takasaki.