# INFLUENCE OF ELECTRON BEAM PARAMETERS ON COHERENT ELECTRON COOLING

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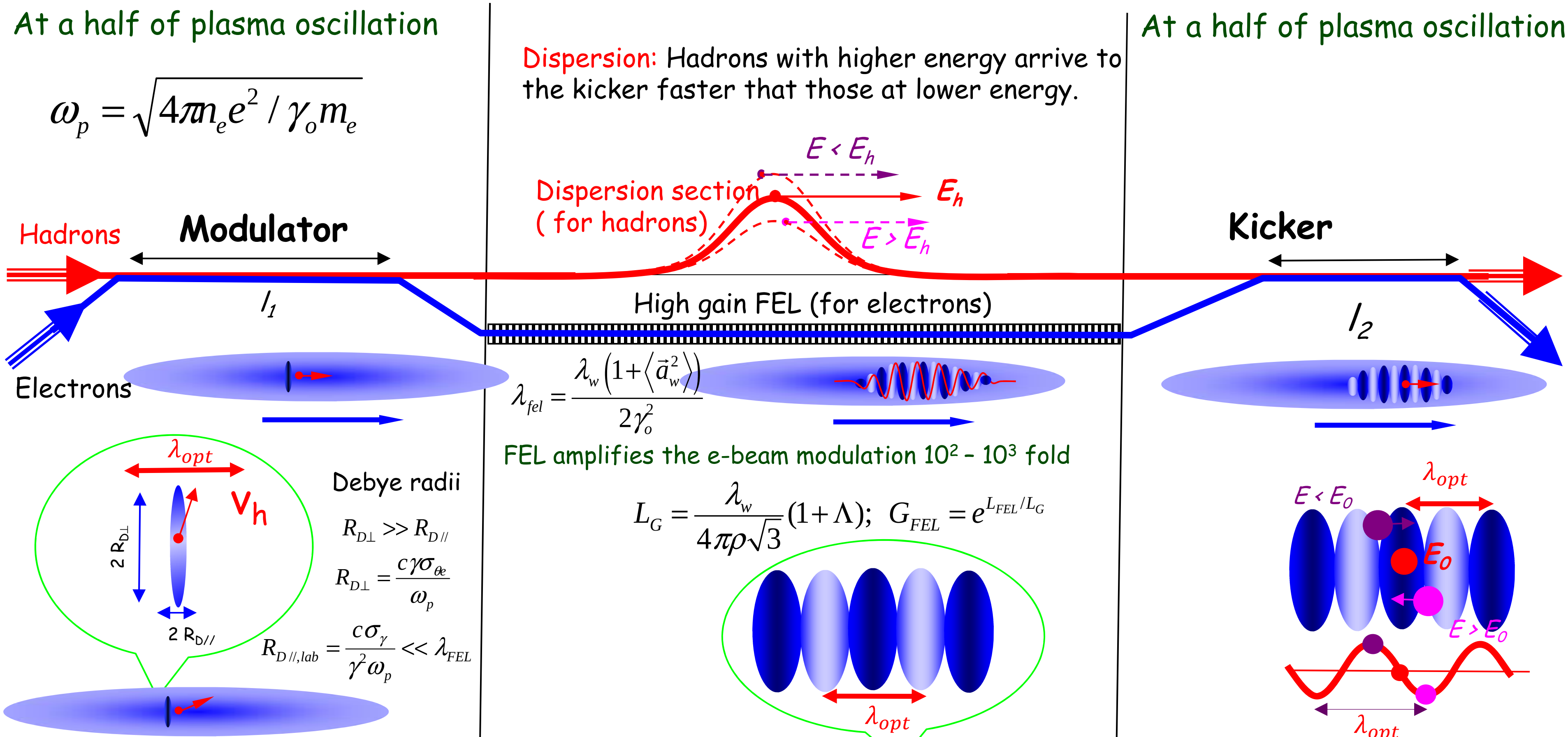
# Outline

- Introduction
  - CeC principle
  - Prototype CeC system in BNL for proof of principle experiment
- Influence of e bunch density and energy profile
  - Model description
  - Results: influences of density and energy profile
  - Beam conditioning: optimizing bunch length and charge
  - Genesis simulation
- Summary

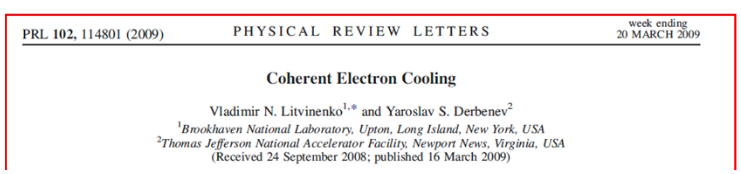
# Coherent Electron Cooling (CeC)




Modulator: Each hadron attracts a cloud of electrons creating a pancake-like modulation of the electron beam density

Free Electron Laser (FEL): The FEL amplifies the electron beam density modulation producing a series of pancakes spaced by the FEL wavelength

Kicker: Phase of the density modulation and corresponding longitudinal electric field in e-beam is adjusted in such way that it decelerate hadrons with higher energies and accelerate hadrons with lower energies. The resulting energy spread is reduced and the hadron beam is cooled.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Coherent Electron Cooling

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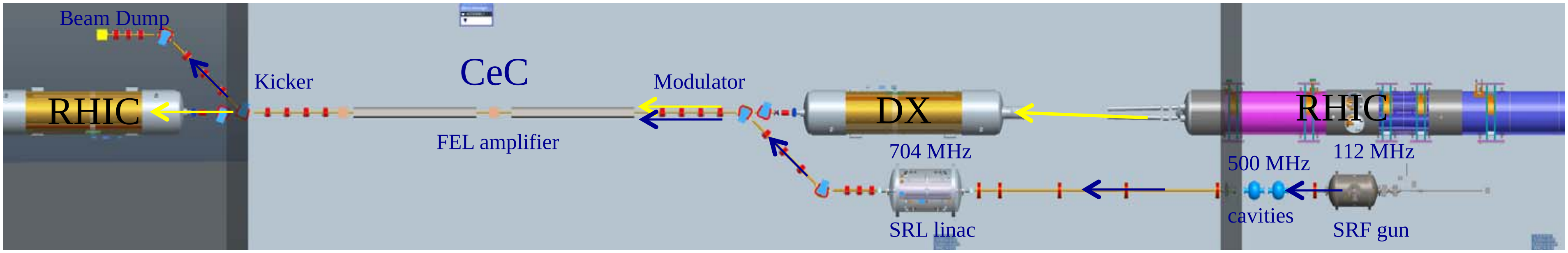
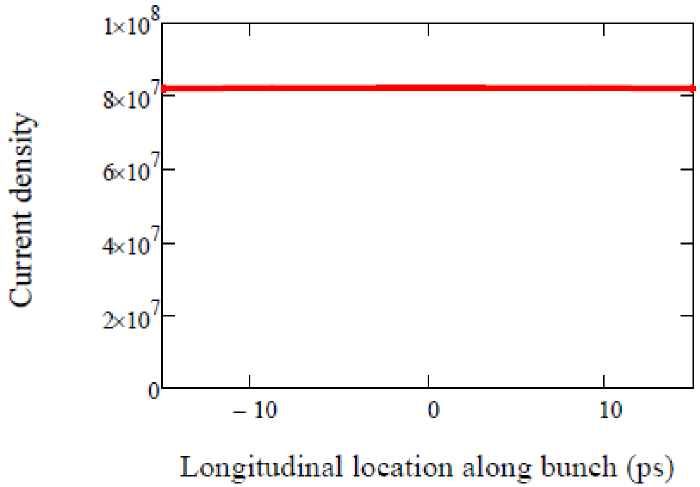
# Coherent Electron Cooling
# Proof-of-Principle experiment is under construction at RHIC's IP2

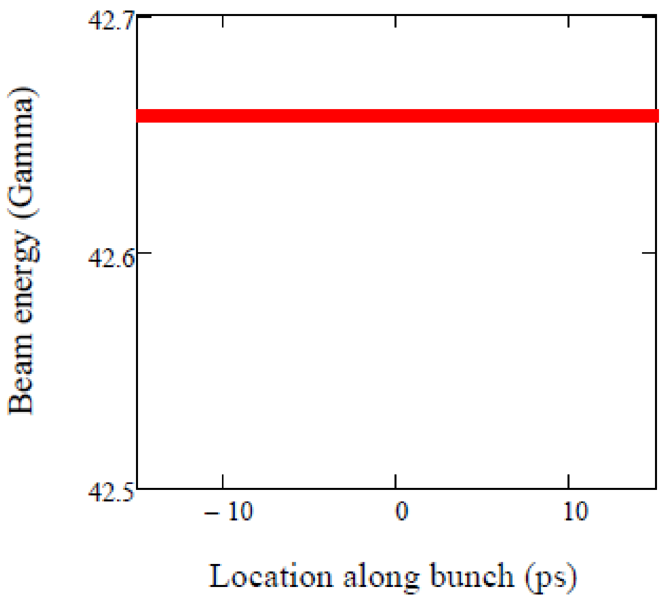



| Parameter | |
|---|---|
| Species in RHIC | Au ions, 40 GeV/u |
| Electron energy | 21.8 MeV |
| Charge per bunch | 1 - 5 nC |
| Rep-rate | 78.2 kHz |
| e-beam current | 0.08 – 0.4 mA |
| e-beam power | 1.7 – 8.5 kW |
| **FEL wavelength** | **13 μm** |

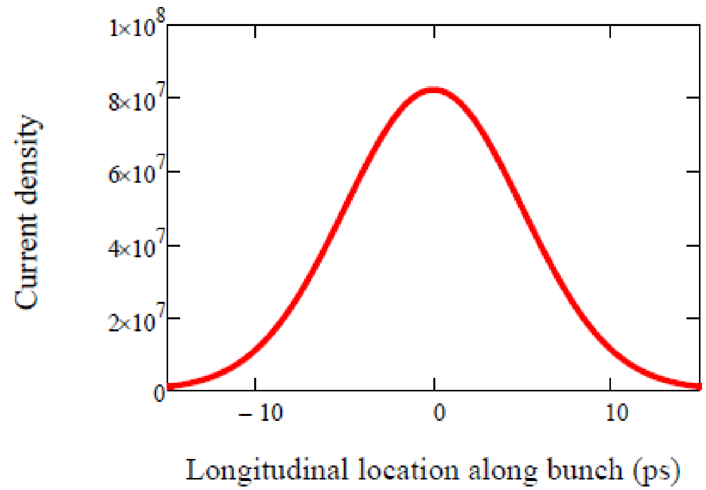
# Ideal v.s. Realistic e Bunch Parameters

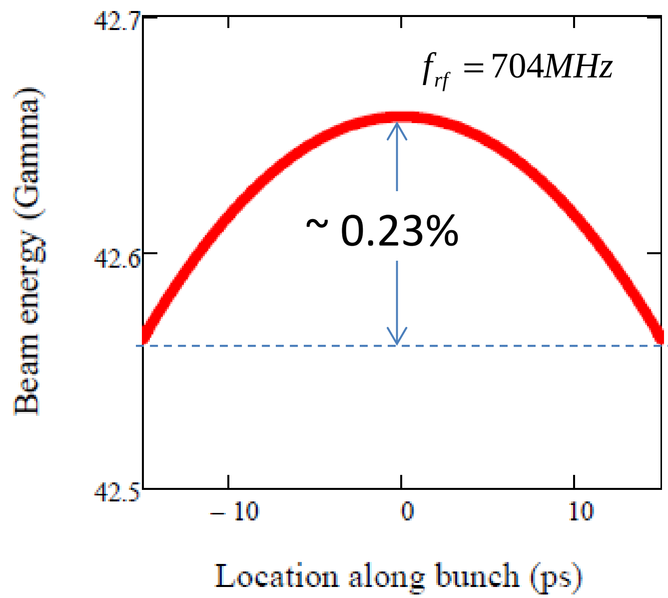
## Ideally:



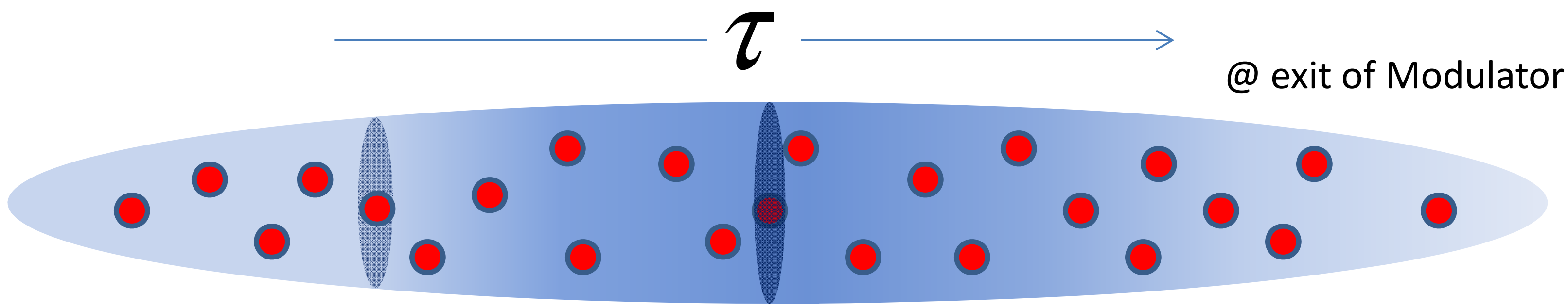




## In reality:



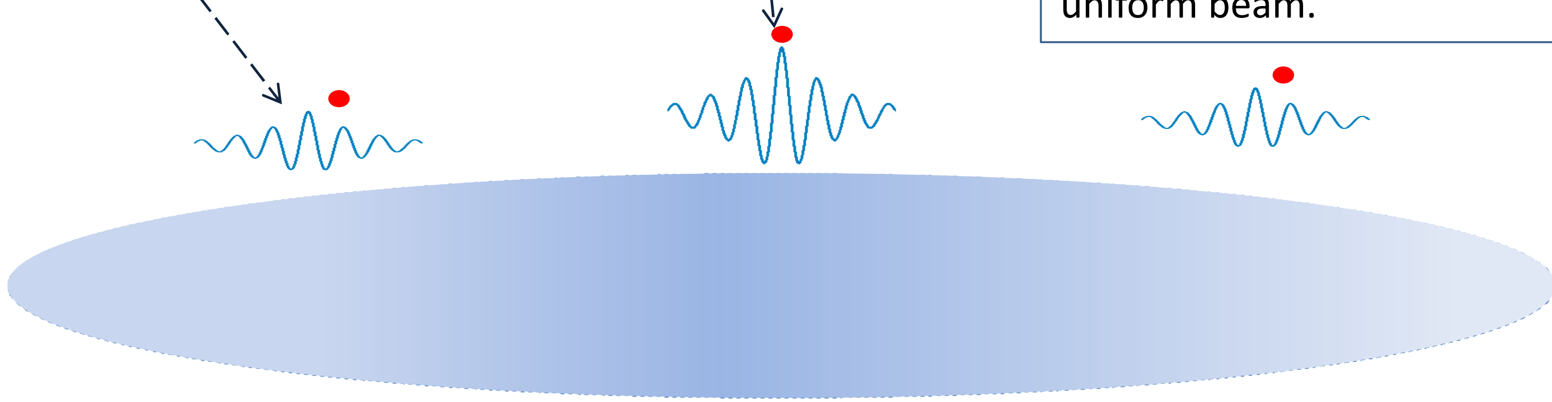
# Illustration of the Influences of Local parameters




Due to charge density and average energy variation along the electron bunch, ions in off-center slices will not be put to the ideal position for cooling.

Ions in the center slice will be put at the right position in the kicker so that they will be cooled most effectively.

**Approach:**
Assuming the local density and energy variation within one coherence length is small, we treat each slice individually using the recently developed FEL theory for uniform beam.



# The Model

We start from the parabolic integro-differential equation derived from 1-D Vlasov equation and 3-D Maxwell equation [1]

$$\left(\nabla_\perp^2 + 2i\frac{\omega}{c}\frac{\partial}{\partial z}\right)\tilde{E}(\vec{r}_\perp, z) = ij_0(\vec{r}_\perp)\int_0^z dz'\left[\frac{e\omega\theta_s^2}{2c^2\varepsilon_0}\tilde{E}(\vec{r}_\perp, z') + \frac{e}{\varepsilon_0\omega}\left(\nabla_\perp^2 + 2i\frac{\omega}{c}\frac{\partial}{\partial z}\right)\tilde{E}(\vec{r}_\perp, z')\right]$$

$$\times\int_{-\infty}^{\infty} e^{i\left(C+\frac{\omega}{c\gamma_z^2 \mathrm{E}_0}P\right)(z'-z)}\frac{\partial}{\partial P}F(P)dP + \frac{i\theta_s\omega}{c\varepsilon_0}e\int_{-\infty}^{\infty}\tilde{f}_1(\vec{r}_\perp, P, 0)e^{-i\left(C+\frac{\omega}{c\gamma_z^2 \mathrm{E}_0}P\right)z}dP$$

For a specific initial perturbation: $\tilde{f}_1(z,t,x,y,P) = \frac{ec}{2\pi\sigma_\perp^2\sqrt{2\pi}\sigma_t}e^{-\frac{r^2}{2\sigma_\perp^2}}e^{-\frac{(z-\beta ct)^2}{2\sigma_z^2}}\delta(P)$

the wave-packet of the electrons can be calculated from [2]

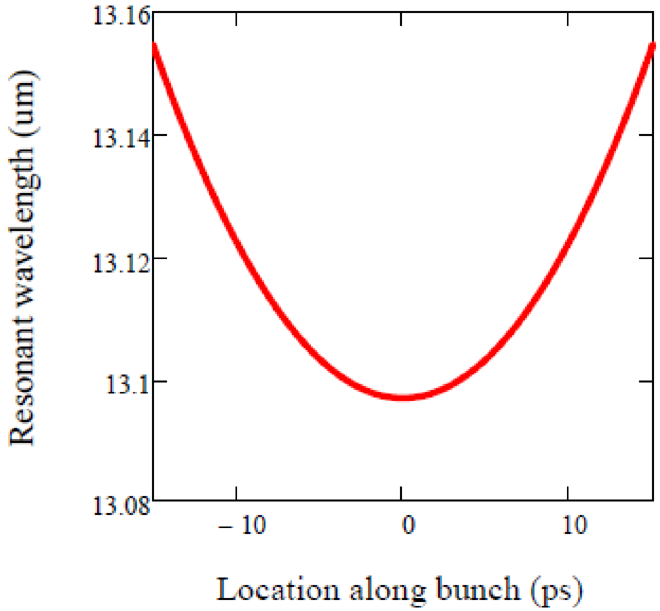
$$\tilde{j}_1(\vec{x},t) = -\frac{ec\Gamma^3\gamma_z^4}{2\pi^2\rho}e^{-i\hat{k}_w\xi}\sum_{(i,j,k,l)}\int_{-\infty}^{\infty}d\hat{C}_{3d}e^{-i2\gamma_z^2(\hat{z}-c\hat{t})\hat{C}_{3d}}I_x\left(\hat{r},\hat{C}_{3d}\right)\frac{\lambda_i\left(B_{jkl}+\frac{\hat{q}^3}{\lambda_i+i\hat{C}_{3d}}\right)e^{\lambda_i\hat{z}} + \frac{i\hat{q}^3\hat{C}_{3d}}{\lambda_i+i\hat{C}_{3d}}e^{-i\hat{C}_{3d}\hat{z}}}{(\lambda_i-\lambda_j)(\lambda_i-\lambda_k)(\lambda_i-\lambda_l)}$$

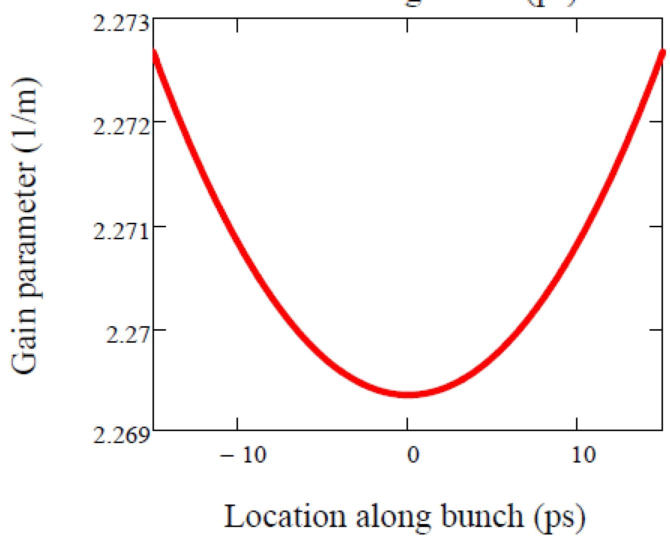

[1] E. Saldin et. al. 'The physics of Free Electron Lasers', (Springer, New York, 1999)

[2] G. Wang et. al. in PAC'11 proceedings, P. 2399

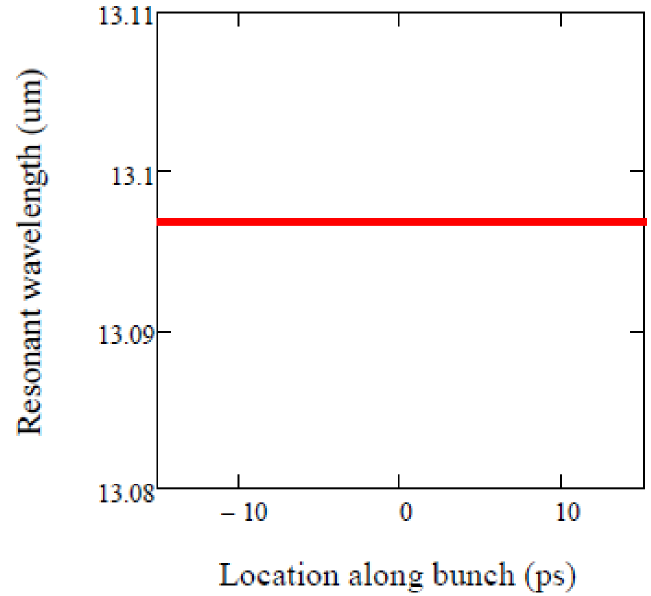
# Influences on Gain parameter and resonant frequency

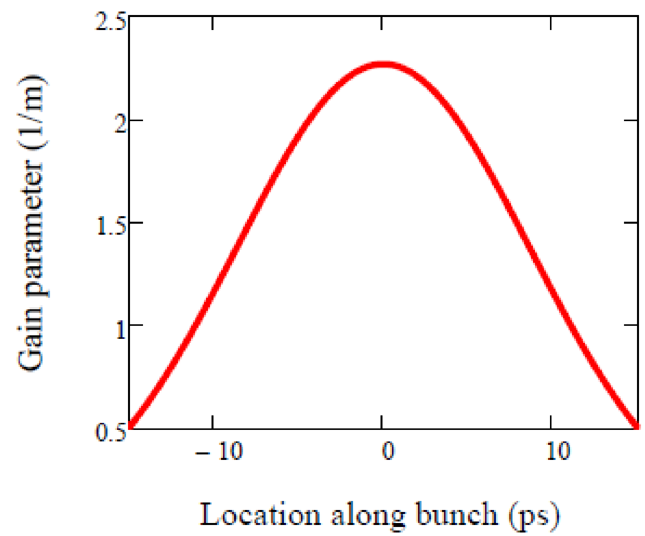
## Influence of Cosine-type energy variation



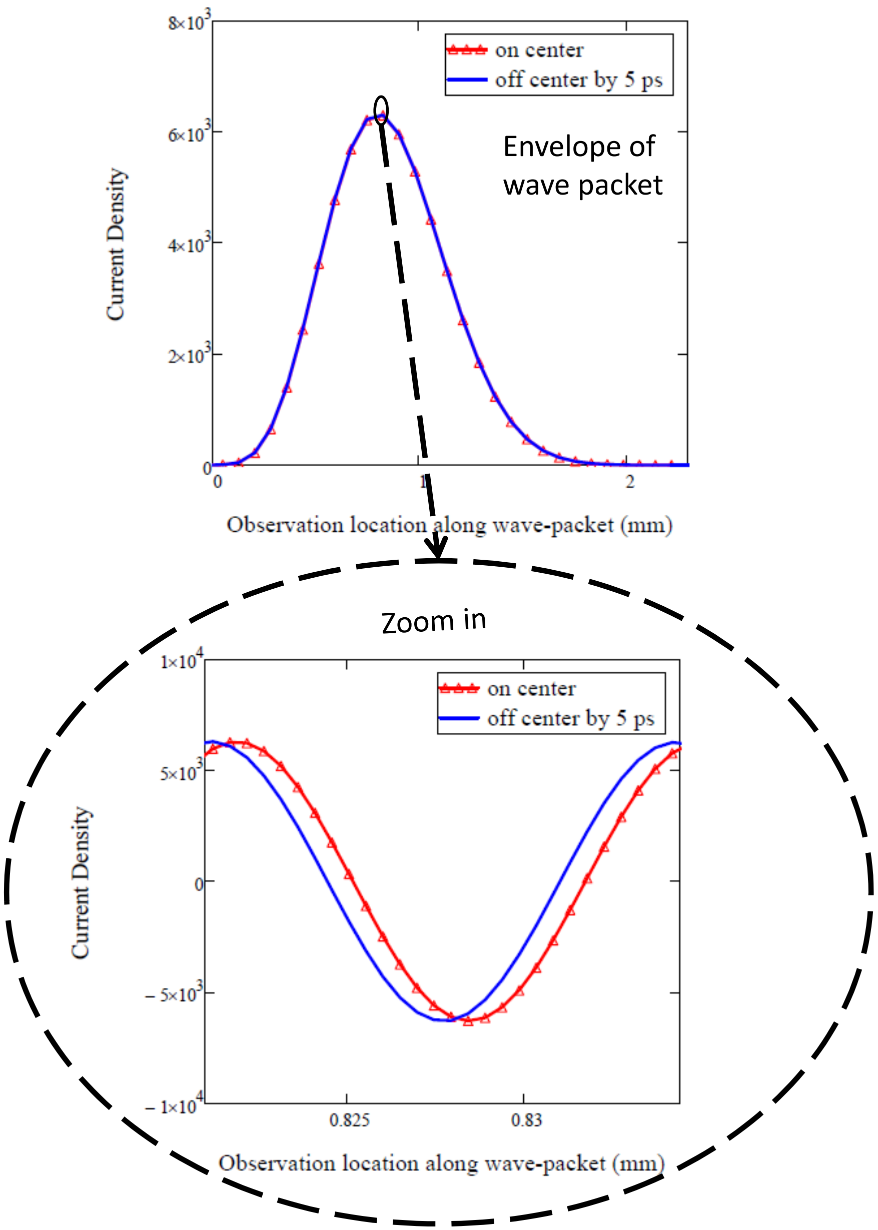

## Influence of Gaussian density variation



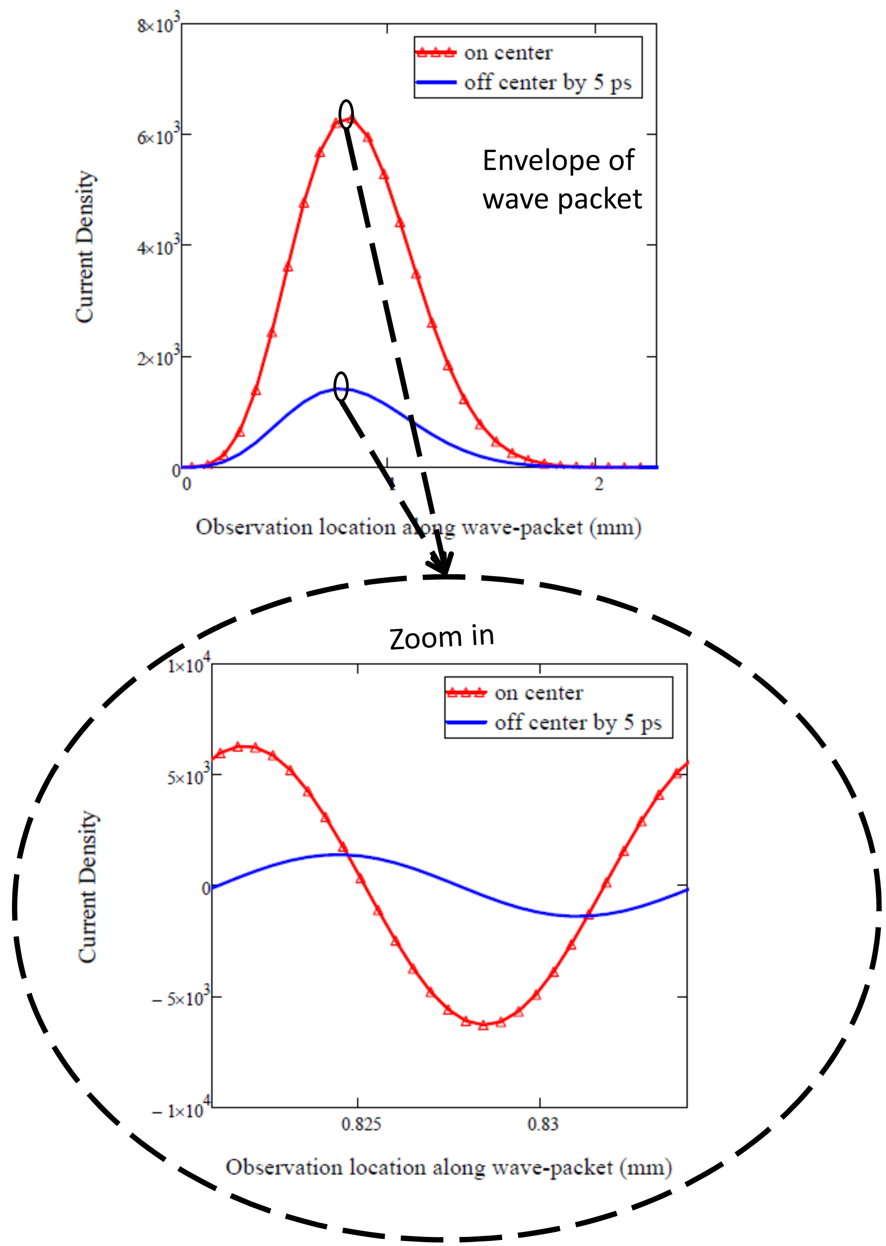



# Comparison of Wave Packets Generated in e Bunch Center and Tail

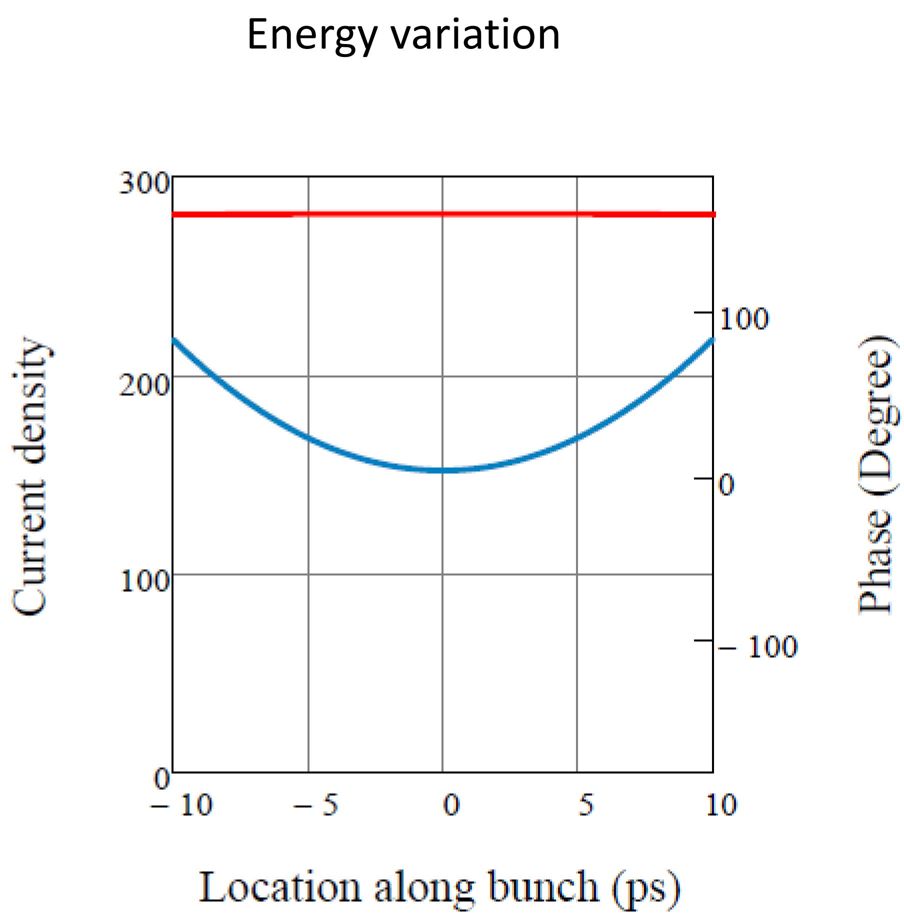
## Energy variation (constant density)

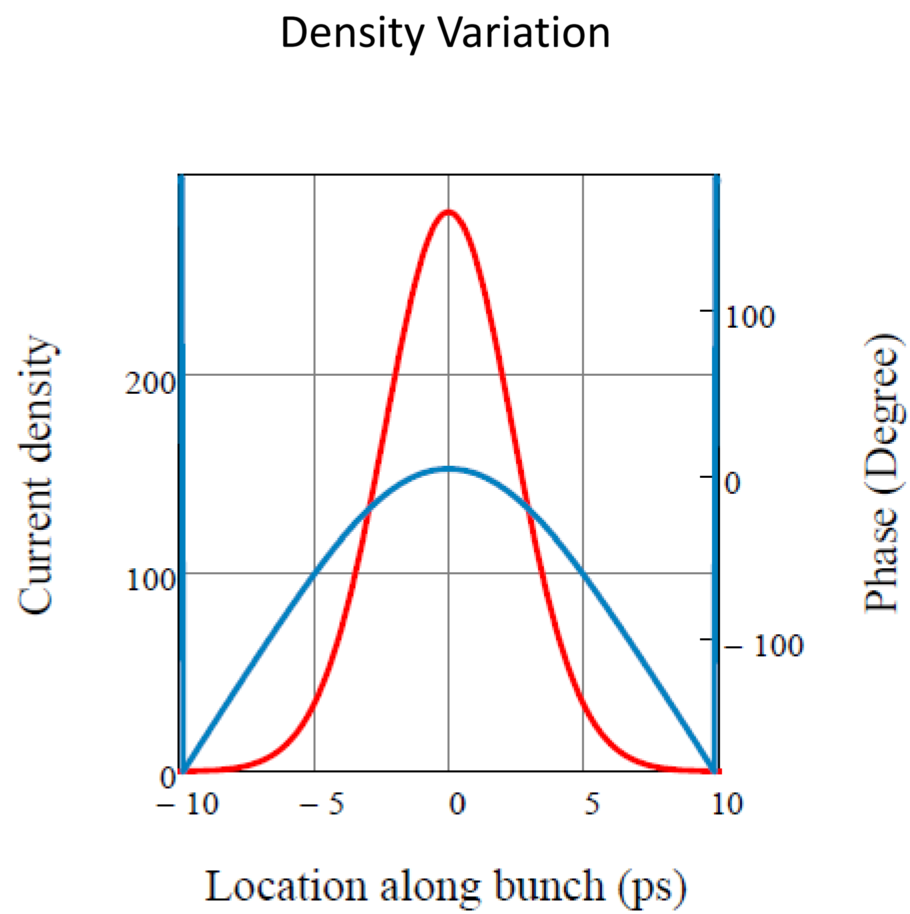



## Density Variation (constant energy)



# Influences on Amplitude and Phase Along e Bunch

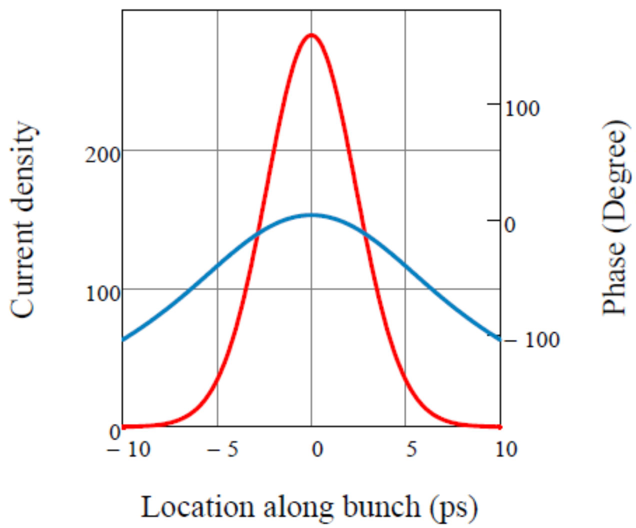


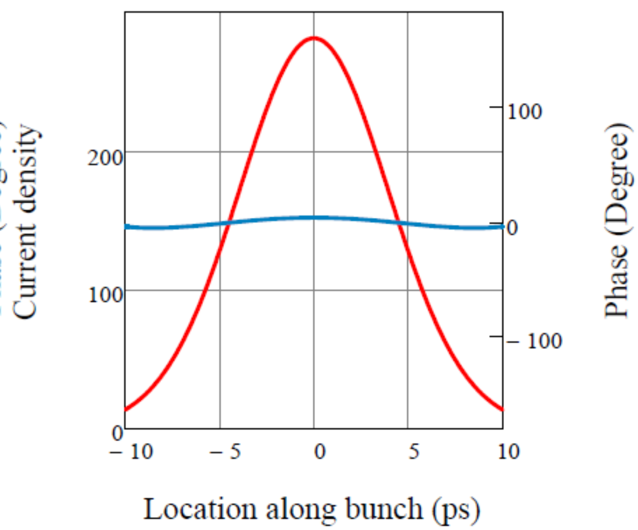

$$\frac{d\psi(\gamma(\tau), j_0(\tau))}{d\tau} = \frac{\partial\gamma}{\partial\tau}\cdot\frac{\partial\psi(\gamma(\tau), j_0(\tau))}{\partial\gamma} + \frac{\partial j_0}{\partial\tau}\cdot\frac{\partial\psi(\gamma(\tau), j_0(\tau))}{\partial j_0} = 0$$

# Beam Conditioning

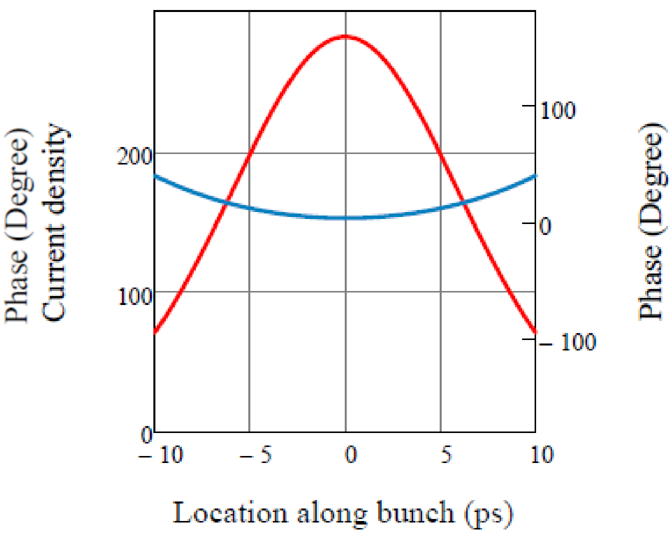
5 ps rms bunch length




8 ps rms bunch length



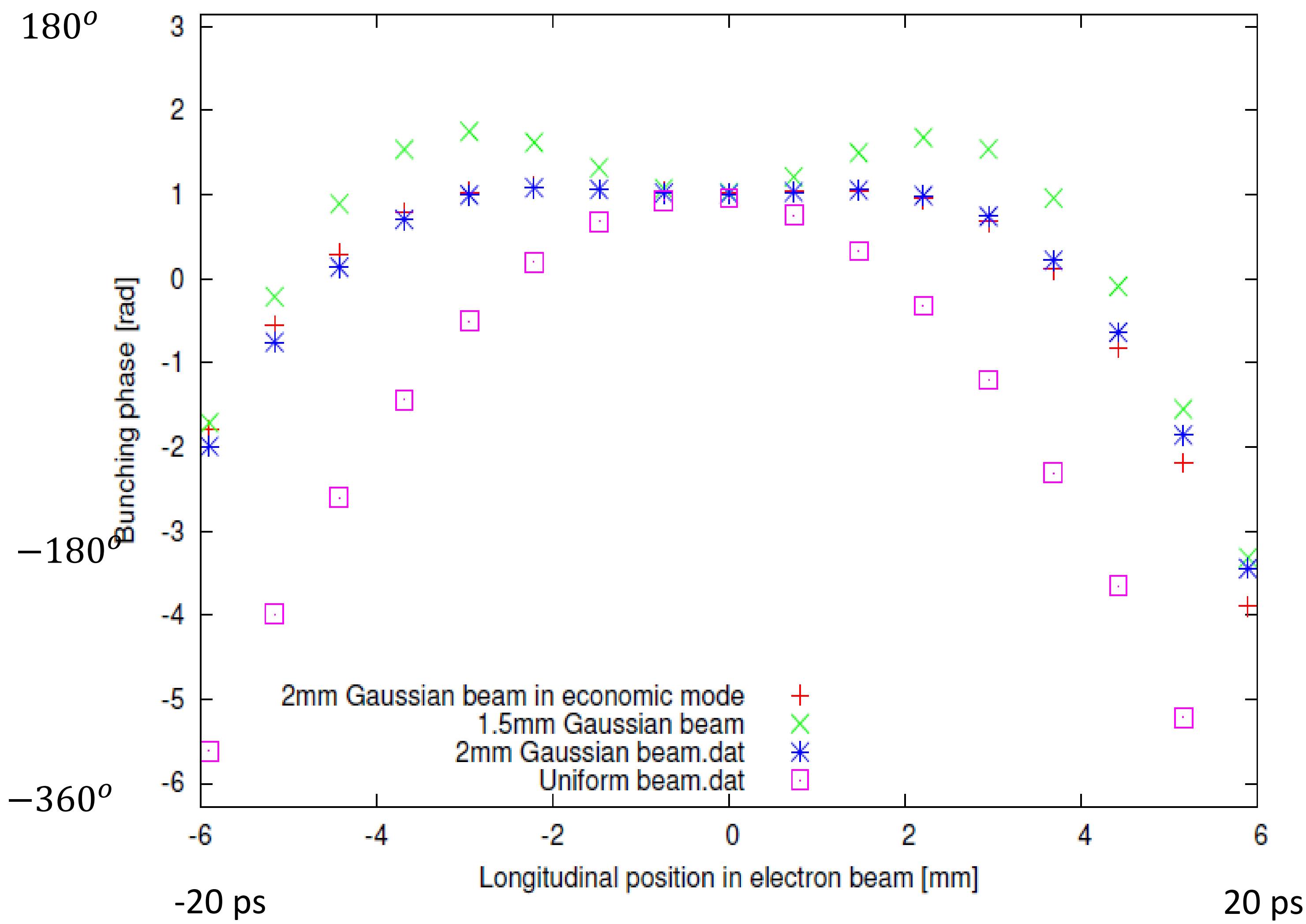

12 ps rms bunch length




- With the same peak current density, there is an optimum bunch length to minimize phase variation of the wave packet with respect to ions located at different portion of the electron bunch.
- For the specific parameters that we consider, the optimum bunch length is around 8 ps, which reduce the phase variation to the level of 5 degree .

# Genesis Simulation

Details are in the paper by Y. Hao, MOPPD038, this conference




The appearance of opposite direction of phase variation is due to definition of bunching factor.

# Summary

- The influences of local electron beam parameters on coherent electron cooling are studied by locally applying an analytical FEL model for uniform electron beam. The approach is valid when the relative variation of electron beam parameters over a FEL coherence length is much smaller than one.
- According to the analysis, phase slippage of the wave packet due to cosine-type energy variation has an opposite sign with respect to that caused by density variation. This property has been used for beam conditioning, i.e. minimizing the phase variation of the wave packets along the electron bunch to achieve optimal cooling.
- In this analysis, we approximate the initial modulation as a 3-D Gaussian pulse. Further improvement involves developing a formalism using an exact solution for Debye screening.
- Genesis simulations are in good agreement with our analytical results

# Backup Slides

# Phase of bunching factor

- Assuming spatial density distribution is

$$\rho(z) = \rho_1 \cos\left(\frac{2\pi z}{\lambda_0} + \varphi\right)$$

the bunching factor is calculated by

$$b = \frac{\rho_1}{\rho_0 \lambda_0} \int_0^{\lambda_0} \cos\left(\frac{2\pi z}{\lambda_0} + \varphi\right) e^{i\frac{2\pi z}{\lambda_0}} dz$$

$$= \frac{\rho_1}{2\rho_0 \lambda_0} \int_0^{\lambda_0} \left[ e^{i\left(\frac{2\pi z}{\lambda_0} + \varphi\right)} + e^{-i\left(\frac{2\pi z}{\lambda_0} + \varphi\right)} \right] e^{i\frac{2\pi z}{\lambda_0}} dz$$

$$= \frac{\rho_1}{2\rho_0} \int_0^{\lambda_0} \left[ e^{i\left(\frac{4\pi z}{\lambda_0} + \varphi\right)} + e^{-i\varphi} \right] dz$$

$$= \frac{\rho_1}{2\rho_0} e^{-i\varphi}$$

# Initial conditions for CeC

In FEL $\hat{z} > 0$ $$\frac{\partial}{\partial z}\tilde{f}_1(P,z) + i\left(C + \frac{\omega}{c\gamma_z^2\varepsilon_0}P\right)\tilde{f}_1(P,z) + \left(iU(z) - e\tilde{E}_z\right)\frac{\partial}{\partial P}f_0 = 0$$

In field free section $\hat{z} < 0$ $$\frac{\partial}{\partial \hat{z}}\tilde{f}_1 + i\left(\hat{C} + \hat{P}\right)\tilde{f}_1 = 0$$

$$\left.\frac{\partial^{(n)}}{\partial \hat{z}^{(n)}}\tilde{j}\left(\hat{z},\hat{P}\right)\right|_{\hat{z}=0} = \int_{-\infty}^{\infty}(-i)^n\left(\hat{C}+\hat{P}\right)^n\tilde{f}_1\left(0,\hat{C},\hat{P}\right)d\hat{P}$$

Phase space density from modulator (beam frame)

$$\tilde{f}_1\left(\vec{k},\vec{v},t\right) = -iZ_i\frac{\omega_p^2}{k^2}\frac{\lambda^2}{\lambda^2+\omega_p^2}\vec{k}\cdot\frac{\partial}{\partial\vec{v}}f_0(\vec{v})\left\{\frac{1-e^{-i\vec{k}\cdot\vec{v}t}}{i\vec{k}\cdot\vec{v}} + \frac{\omega_p}{\lambda\left(\left(\lambda+i\vec{k}\cdot\vec{v}\right)^2+\omega_p^2\right)}\right.$$
$$\left.\times\left[2\omega_p\left(1+\frac{i\vec{k}\cdot\vec{v}}{2\lambda}\right)\left(e^{\lambda t}\cos\left(\omega_p t\right) - e^{-i\vec{k}\cdot\vec{v}t}\right) - e^{\lambda t}\left(\lambda + i\vec{k}\cdot\vec{v} - \frac{\omega_p^2}{\lambda}\right)\sin\left(\omega_p t\right)\right]\right\}$$

# Equation of Motion

1D Vlasov equation for electrons + 3D Maxwell equation for radiation generates

$$\left(\nabla_\perp^2 + 2i\frac{\omega}{c}\frac{\partial}{\partial z}\right)\tilde{E}(\vec{r}_\perp, z) = ij_0(\vec{r}_\perp)\int_0^z dz'\left[\frac{e\omega\theta_s^2}{2c^2\varepsilon_0}\tilde{E}(\vec{r}_\perp, z') + \frac{e}{\varepsilon_0\omega}\left(\nabla_\perp^2 + 2i\frac{\omega}{c}\frac{\partial}{\partial z}\right)\tilde{E}(\vec{r}_\perp, z')\right]$$

$$\times\int_{-\infty}^{\infty} e^{i\left(C+\frac{\omega}{c\gamma_z^2 \mathrm{E}_0}P\right)(z'-z)}\frac{\partial}{\partial P}F(P)dP + \frac{i\theta_s\omega}{c\varepsilon_0}e\int_{-\infty}^{\infty}\tilde{f}_1(\vec{r}_\perp, P, 0)e^{-i\left(C+\frac{\omega}{c\gamma_z^2 \mathrm{E}_0}P\right)z}dP$$

After Fourier transformation to $\vec{r}_\perp$ and carry out $\frac{\partial^3}{\partial\hat{z}^3}$, the paraxial field equation reduces to

$$\tilde{R}^{(4)} + 3\left(i\hat{C}_{3d} + \hat{q}\right)\tilde{R}^{(3)} + \left[\hat{\Lambda}_p^2 + 3\left(i\hat{C}_{3d} + \hat{q}\right)^2\right]\tilde{R}^{(2)} + \left[\left(i\hat{C}_{3d} + 3\hat{q}\right)\hat{\Lambda}_p^2 + \left(i\hat{C}_{3d} + \hat{q}\right)^3 - i\right]\tilde{R}^{(1)} - i\left(i\hat{C}_{3d} + 3\hat{q}\right)\tilde{R}$$

$$= \frac{1}{n_0}\int_{-\infty}^{\infty}\left(\hat{q} - i\hat{P}\right)^3\tilde{f}_1\left(\hat{k}_\perp \hat{P}, 0\right)e^{-i\left(\hat{P}+\hat{C}_{3d}\right)\hat{z}}d\hat{P}$$

$$\tilde{R}(z, k_\perp, C) \equiv e^{i\frac{k_\perp^2 c}{2\omega}z}\tilde{E}(z, k_\perp, C)$$

# Solutions for arbitrary initial condition

Electron current density is related to radiation field by

$$\tilde{j}_1\left(\hat{z},\hat{C},\hat{k}_\perp\right) = -\frac{c\Gamma}{2\pi\theta_s}\left[i\hat{k}_\perp^2 + \frac{\partial}{\partial\hat{z}}\right]\tilde{E}\left(\hat{z},\hat{C},\hat{k}_\perp\right)$$

$$\tilde{j}_1\left(\hat{z},\hat{C},\hat{k}_\perp\right) = -e^{-i\hat{k}_\perp^2\hat{z}}\left\{\frac{2c\varepsilon_0\Gamma}{\theta_s}\sum_{i=1}^{4}\lambda_i A_i e^{\lambda_i\hat{z}} + \sum_{(i,j,k,l)}\frac{ec\int_{-\infty}^{\infty}\frac{\left(\hat{q}-i\hat{P}\right)^3\cdot\left[\lambda_i e^{\lambda_i\hat{z}} + i\left(\hat{P}+\hat{C}_{3d}\right)e^{-i\left(\hat{P}+\hat{C}_{3d}\right)\hat{z}}\right]\tilde{f}_1\left(\hat{k}_\perp,\hat{P},0\right)}{\lambda_i + i\left(\hat{P}+\hat{C}_{3d}\right)}d\hat{P}}{\left(\lambda_i-\lambda_j\right)\left(\lambda_i-\lambda_k\right)\left(\lambda_i-\lambda_l\right)}\right\}$$

$$\left(\lambda + i\hat{C}_{3d} + \hat{q}\right)\left\{\lambda^3 + 2\left(i\hat{C}_{3d} + \hat{q}\right)\lambda^2 + \left[\hat{\Lambda}_p^2 + \left(i\hat{C}_{3d} + \hat{q}\right)^2\right]\lambda - i\right\} + 2\hat{q}\left[\hat{\Lambda}_p^2\lambda - i\right] = 0$$

$$\begin{pmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{pmatrix} = -\frac{\theta_s}{2\varepsilon_0 c\Gamma}\begin{pmatrix} 1 & 1 & 1 & 1 \\ \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 \\ \lambda_1\left(\lambda_1 - i\hat{k}_\perp^2\right) & \lambda_2\left(\lambda_2 - i\hat{k}_\perp^2\right) & \lambda_3\left(\lambda_3 - i\hat{k}_\perp^2\right) & \lambda_4\left(\lambda_4 - i\hat{k}_\perp^2\right) \\ \lambda_1\left(\lambda_1 - i\hat{k}_\perp^2\right)^2 & \lambda_2\left(\lambda_2 - i\hat{k}_\perp^2\right)^2 & \lambda_3\left(\lambda_3 - i\hat{k}_\perp^2\right)^2 & \lambda_4\left(\lambda_4 - i\hat{k}_\perp^2\right)^2 \end{pmatrix}^{-1}\begin{pmatrix} -\frac{2\varepsilon_0 c\Gamma}{\theta_s}\tilde{E} \\ \tilde{j}_1 \\ \tilde{j}_1^{(1)} \\ \tilde{j}_1^{(2)} \end{pmatrix}_{\hat{z}=0}$$

# An example to check validity continue

$\hat{z} < 0$

$$\tilde{f}_1(z,t,x,y,P) = \frac{ec}{2\pi\sigma_\perp^2\sqrt{2\pi}\sigma_t} e^{-\frac{r^2}{2\sigma_\perp^2}} e^{-\frac{(z-\beta ct)^2}{2\sigma_z^2}} \delta(P)$$

$$\tilde{j}_1(\vec{x},t) = -\frac{ec\Gamma^3\gamma_z^4}{2\pi^2\rho} e^{-i\hat{k}_w\xi} \sum_{(i,j,k,l)} \int_{-\infty}^{\infty} d\hat{C}_{3d} e^{-i2\gamma_z^2(\hat{z}-c\hat{t})\hat{C}_{3d}} I_x(\hat{r},\hat{C}_{3d}) \frac{\lambda_i\left(B_{jkl} + \frac{\hat{q}^3}{\lambda_i + i\hat{C}_{3d}}\right)e^{\lambda_i\hat{z}} + \frac{i\hat{q}^3\hat{C}_{3d}}{\lambda_i + i\hat{C}_{3d}} e^{-i\hat{C}_{3d}\hat{z}}}{(\lambda_i-\lambda_j)(\lambda_i-\lambda_k)(\lambda_i-\lambda_l)}$$

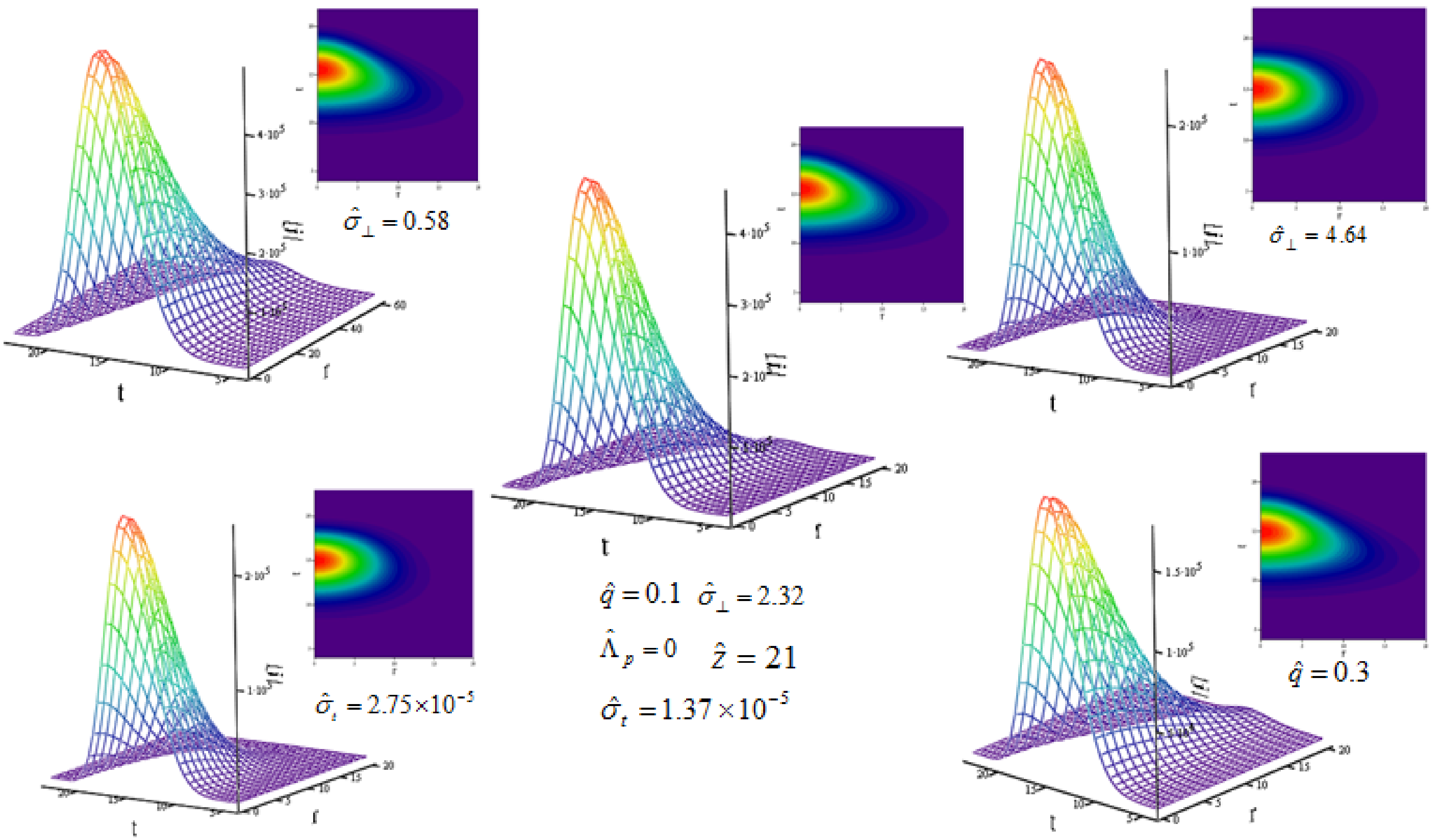
$$B_{jkl} \equiv \lambda_j\lambda_k + \lambda_j\lambda_l + \lambda_k\lambda_l + i\hat{C}_{3d}(\lambda_j + \lambda_k + \lambda_l) - \hat{C}_{3d}^2$$

$$I_x(\hat{r},\hat{C}_{3d}) \equiv \int_0^\infty e^{-\left(\frac{\hat{\sigma}_\perp^2}{2} - i\xi\right)x} e^{\frac{-\hat{\sigma}_z^2}{2}(x+\hat{C}_{3d}-\hat{k}-\hat{k}_w)^2} J_0(\hat{r}_\perp\sqrt{x})dx \qquad \xi = -\left[\hat{z} + 2\gamma_z^2(\hat{z} - c\hat{t})\right]$$

For infinitely short perturbation, i.e. $\hat{\sigma}_z = 0$

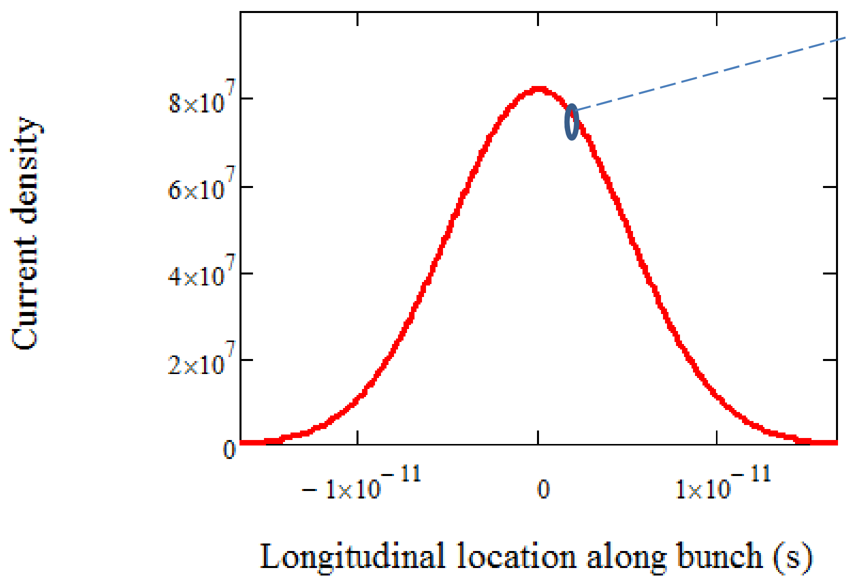
$$\tilde{j}_1(\vec{x},t) = \frac{-ec\Gamma^3\gamma_z^4 e^{-i\hat{k}_w\xi}}{2\pi^2\rho} \frac{e^{-\frac{1}{4}\frac{\hat{r}^2}{\hat{\sigma}_\perp^2/2-i\xi}}}{\hat{\sigma}_\perp^2/2 - i\xi} \sum_{(i,j,k,l)} \int_{-\infty}^{\infty} e^{-i2\gamma_z^2(\hat{z}-c\hat{t})\hat{C}_{3d}} \frac{\lambda_i \cdot \left(B_{jkl} + \frac{\hat{q}^3}{\lambda_i + i\hat{C}_{3d}}\right)e^{\lambda_i\hat{z}} + \frac{i\hat{q}^3\hat{C}_{3d}}{\lambda_i + i\hat{C}_{3d}} e^{-i\hat{C}_{3d}\hat{z}}}{(\lambda_i-\lambda_j)(\lambda_i-\lambda_k)(\lambda_i-\lambda_l)} d\hat{C}_{3d}$$

# An example to check validity (continue)

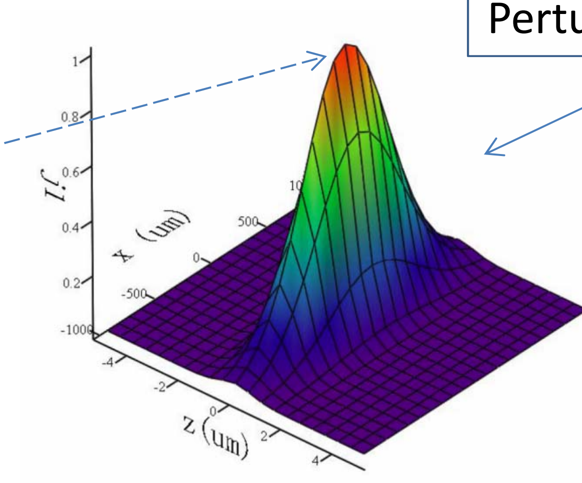


# Electron Density Profile and Initial Perturbation

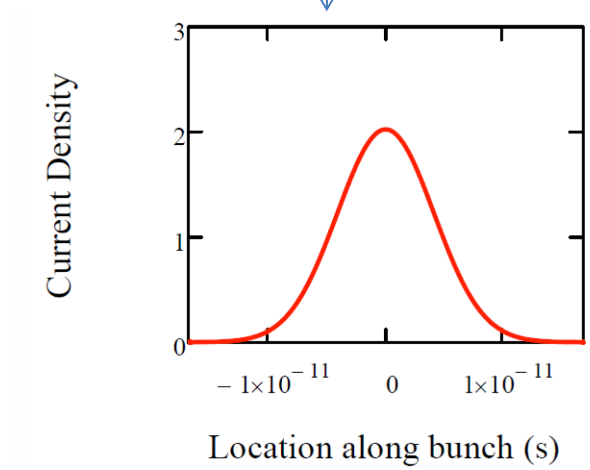
Background electron distribution




Perturbation profile



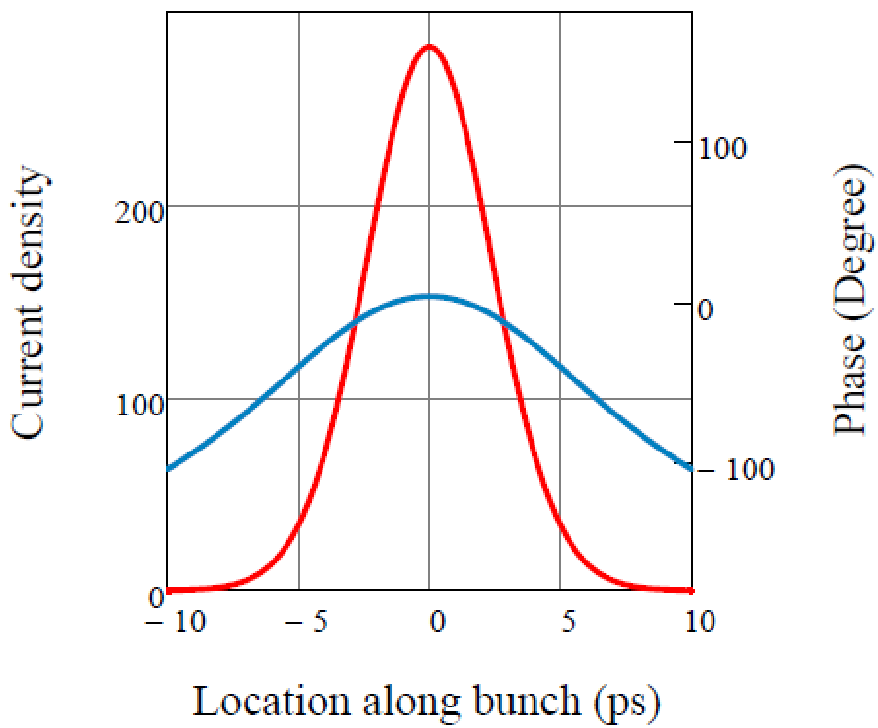

Peak amplitude of initial modulation along bunch
(variation is due to local Debye lengthes)



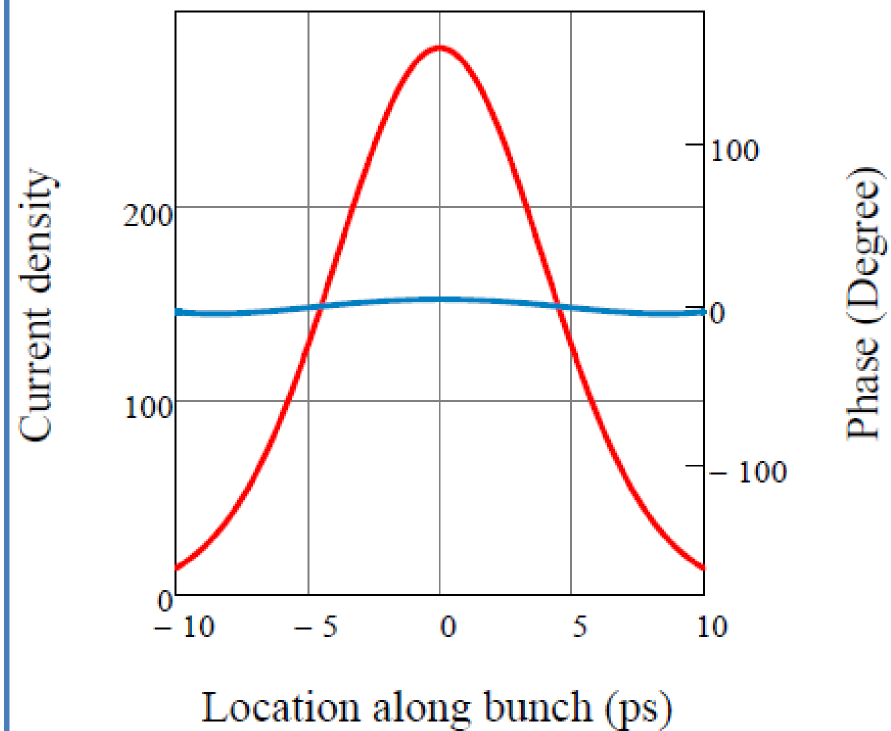

For simplicity, we approximate the initial modulation with a 3-D Gaussian pulse. Both transversal and longitudinal width of the pulse are taken as the local Debye lengths.

# Beam Conditioning

Without beam conditioning




With beam conditioning




Increasing bunch length from 5 ps to 8.3 ps, and increasing bunch charge from 1nC to 1.67 nC reduces the phase shift across the bunch to less than 4 degrees with slightly reduced gain at the center slice.