THYB01

Beam-beam Limit in Hadron Colliders

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Thanks to Y. Alexahin, W. Fischer, W. Herr, V. Lebedev, T. Sen, R. Tomas, A. Valishev, D. Zhou, F. Zimmermann

- Beam-beam tune shift (tune shift due to beam-beam interaction)

$$\xi = \frac{N_p r_p \beta^*}{4\pi\gamma\sigma_r^2}$$

N_p:proton bunch population
β^*: beta function at IP
σ_r: beam size
f_{rep}: collision repetition

- Luminosity

$$L = \frac{N_{p_1} N_{p_2}}{4\pi\sigma_r^2} f_{rep} = \frac{N_p \gamma f_{rep}}{r_p \beta^*} \xi_{IP}$$

$$\xi_{IP} = \frac{r_p \beta^* L}{\gamma_p N_{\bar{p}} f_{col}}$$

Beam-beam limit

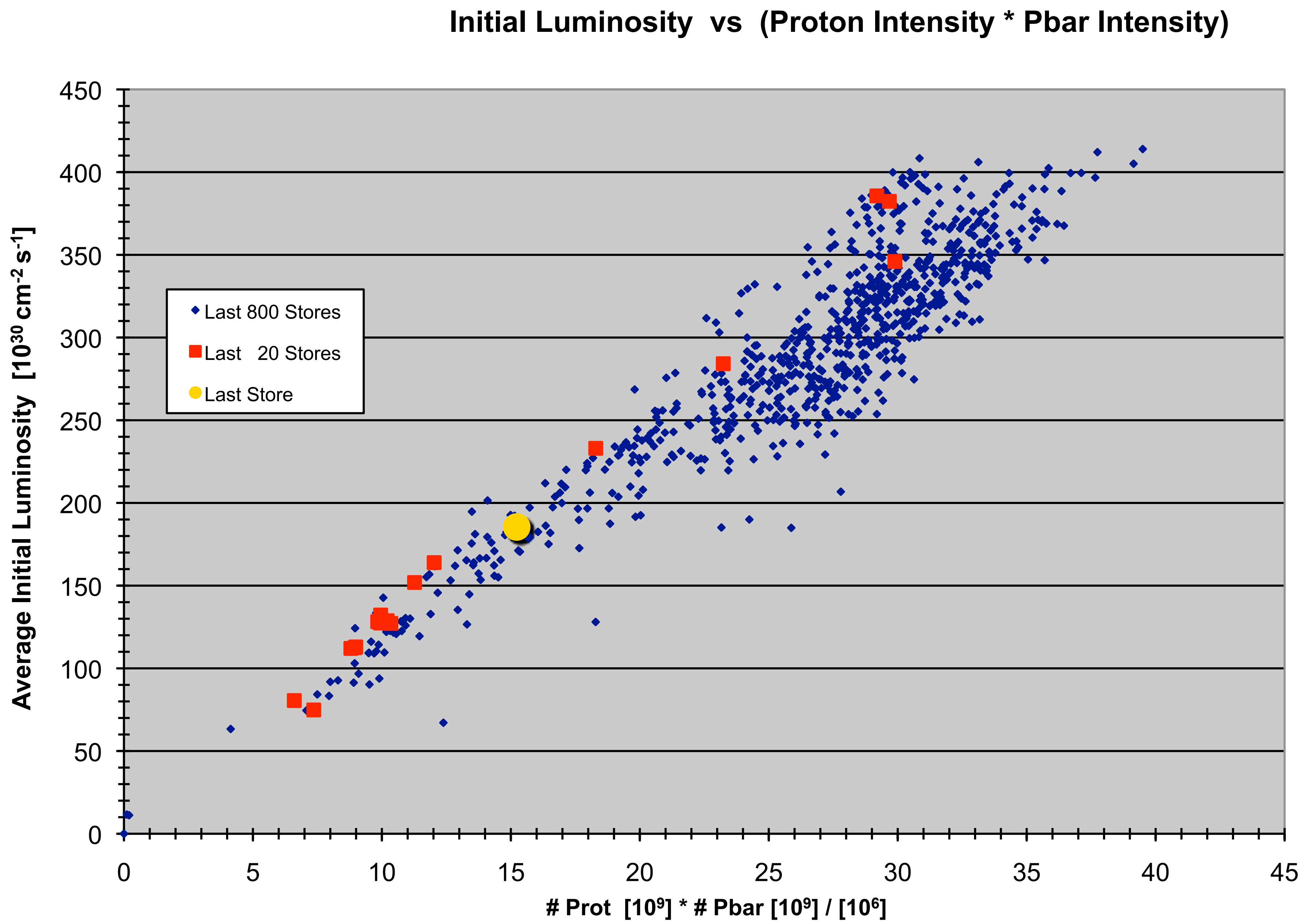
- Beam-beam tune shift is saturated for increasing the bunch population: i.e. emittance growth arises. Luminosity increases proportional to N_p not N_p^2.
- Emittance growth, which results luminosity degradation, arise (even keeping current).
- Short beam life time at collision. We do not discuss here.

Table 1: Summary of Proton colliders.

	Tevatron	**RHIC**	**LHC**
Circumf. (m)	6,283	3,834	26,658
Energy (GeV)	980	250	3,500
Emit. (μm)	20(p)/4(pbar)	20	2
beta*	0.28	0.6	1.0
Bunch length (m)	0.48	0.6	0.38
Tune (x/y/z)	20.577/20.570 /0.0007	28.67/29.68 /0.00036	64.31/59.32 /0.0019
Bunch population	$2.9\mathrm{x}10^{11}$(p) $1.1\mathrm{x}10^{11}$(pbar)	$1.65\ \mathrm{x}10^{11}$	$1.9\mathrm{x}10^{11}$
Number of bunches	36	107	1380
Beam-beam parameter	0.03/2IP	0.005/IP	0.034*/2IP
Lumi. ($\mathrm{cm}^{-2}\mathrm{s}^{-1}$)	$4.1\mathrm{x}10^{32}$	$1.45\mathrm{x}10^{32}$	$3.6\mathrm{x}10^{33}$

* Beam-beam parameter for LHC is obtained in a dedicated experiment [2].

Tevatron



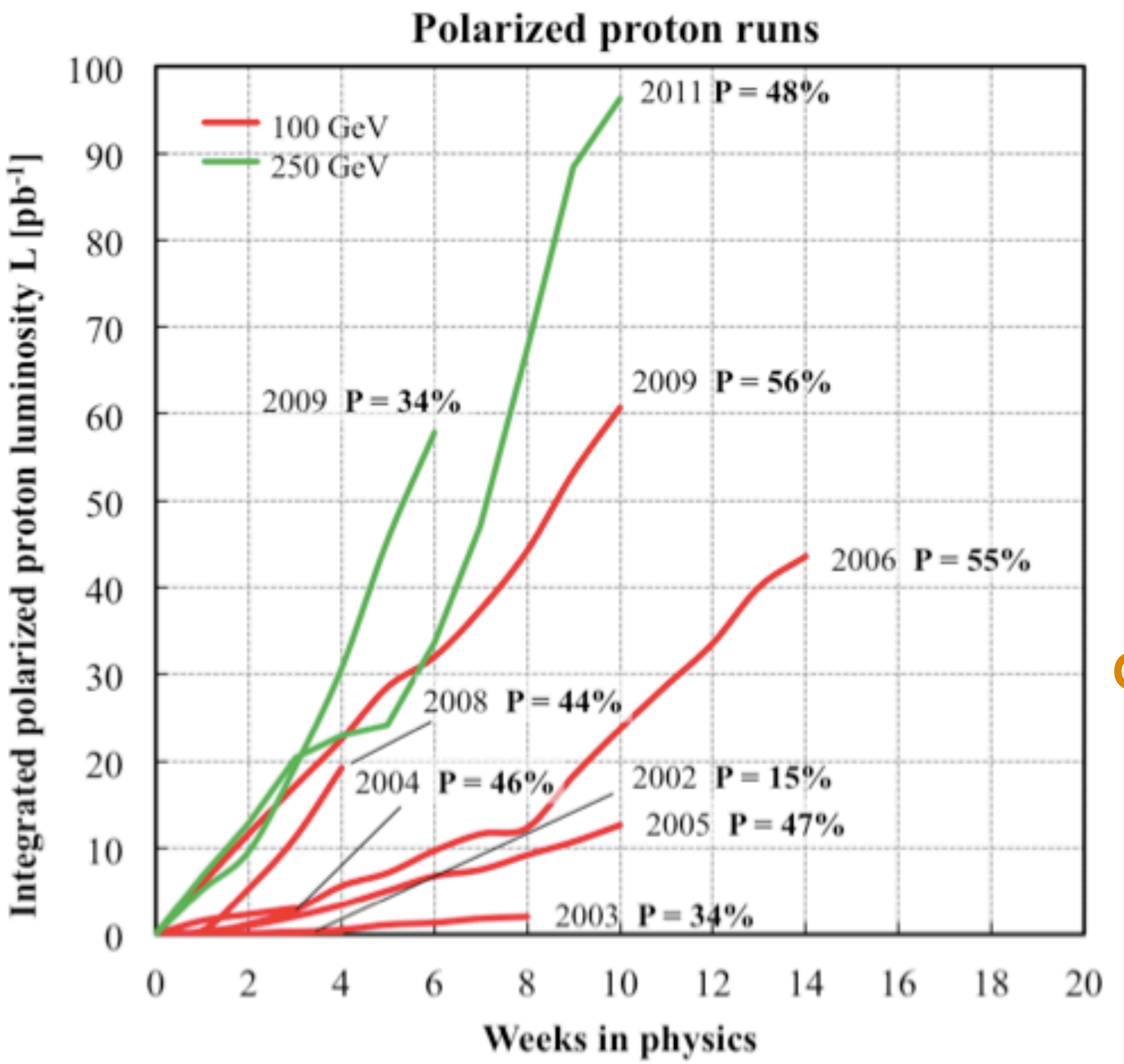
Collision between 36x36 bunches.

$$\xi_{IP} = \frac{r_p \beta^* L}{\gamma_p N_{\bar{p}} f_{col}} \frac{1}{F_{HG}}$$

$\beta^*=0.28$m, $\sigma z=0.48$m, $F_{HG}<1$

RHIC   Two ring

RHIC polarized protons – luminosity and polarization



$L_{peak}=145\mathrm{x}10^{30}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$

At 250 GeV in 2011
L_{avg} = $90\mathrm{x}10^{30}\mathrm{cm}^{-2}\mathrm{s}^{-1}$
P_{avg} = 48%

L_{avg} +60% rel. to 2009
P_{avg} +40% rel. to 2011

Tune operating point is constrained between 2/3-7/10 to maintain the proton polarization.

Courtesy W. Fischer

$FOM = LP^4$
(longitudinally polarized beams)

$$\xi_{IP} = \frac{r_p \beta^* L}{\gamma_p N_{\bar{p}} f_{col}} \frac{1}{F_{HG}} = 0.005 \quad F_{HG} = 0.7$$

LHC 3.5GeV

Observations: head-on beam-beam effects I

- First dedicated experiment with few bunches
- Test maximum beam-beam parameter (at injection energy) - head-on only
  - Intensity $1.9 \cdot 10^{11}$ p/bunch
  - Emittances 1.1 - 1.2 μm
  - Achieved:
    $\xi = 0.017$ for single collision ($\approx$ 5 times nominal !)
    $\xi = 0.034$ for two collision points (IP1 and IP5)
  - No obvious emittance increase or lifetime problems during collisions (maximum ξ not yet found)
- ⚠ No long range encounters present !

The unattainable, here becomes action

- Courtesy W. Herr, presentation at Chamonix 2012.

Study how high beam-beam parameter can be achieved

Possible mechanism for the luminosity degradation

- Crossing angle and offset
- IR coupling, dispersion,chromaticity
- IR nonlinearity
- Long range beam-beam and lifetime
- Coherent instability
- External noise
- Physical aperture, squeezing beta...

Parameters
Target high luminosity LHC

- Focus single bunch beam-beam limit
- E=7 TeV, $\varepsilon=2.7\times10^{-10}$ m ($\gamma\varepsilon=2\mu$m).
- $\beta^*=0.55$ m, $\sigma_z=0.0755$ m.
- $(\nu_x, \nu_y, \nu_z)\sim(64.31, 59.32, 0.0019)$
- $N_p=1.68\times10^{11}$ (x2, x2.5, x3...)

$\xi_{tot}=0.02, 0.04, 0.05, \ldots$

$$\xi = \frac{N_p r_p \beta^*}{4\pi\gamma\sigma_r^2}$$

- 2 IP, Super-periodicity

Beam-beam limit in a simple toy model

- Round beam+linear arc represented by 6x6 matrix

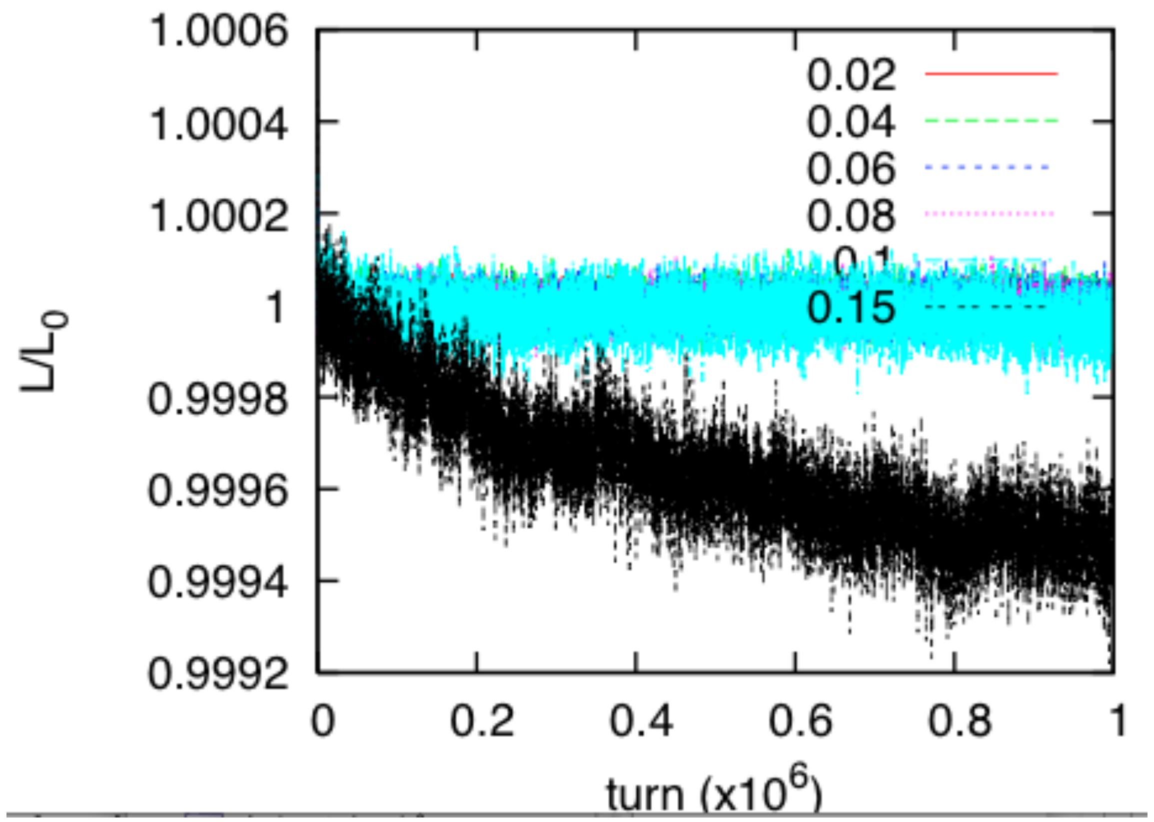
$$
\begin{aligned}
\Delta p_r &= \frac{2N_p r_p}{\gamma}\frac{1}{r}\left[1-\exp\left(-\frac{r^2}{2\sigma_r^2}\right)\right] \\
\Delta p_z &= \frac{N_p r_p}{\gamma}\frac{1}{\sigma_r^2}\exp\left(-\frac{r^2}{2\sigma_r^2}\right)\frac{d\sigma_r^2}{dz}.
\end{aligned}
$$

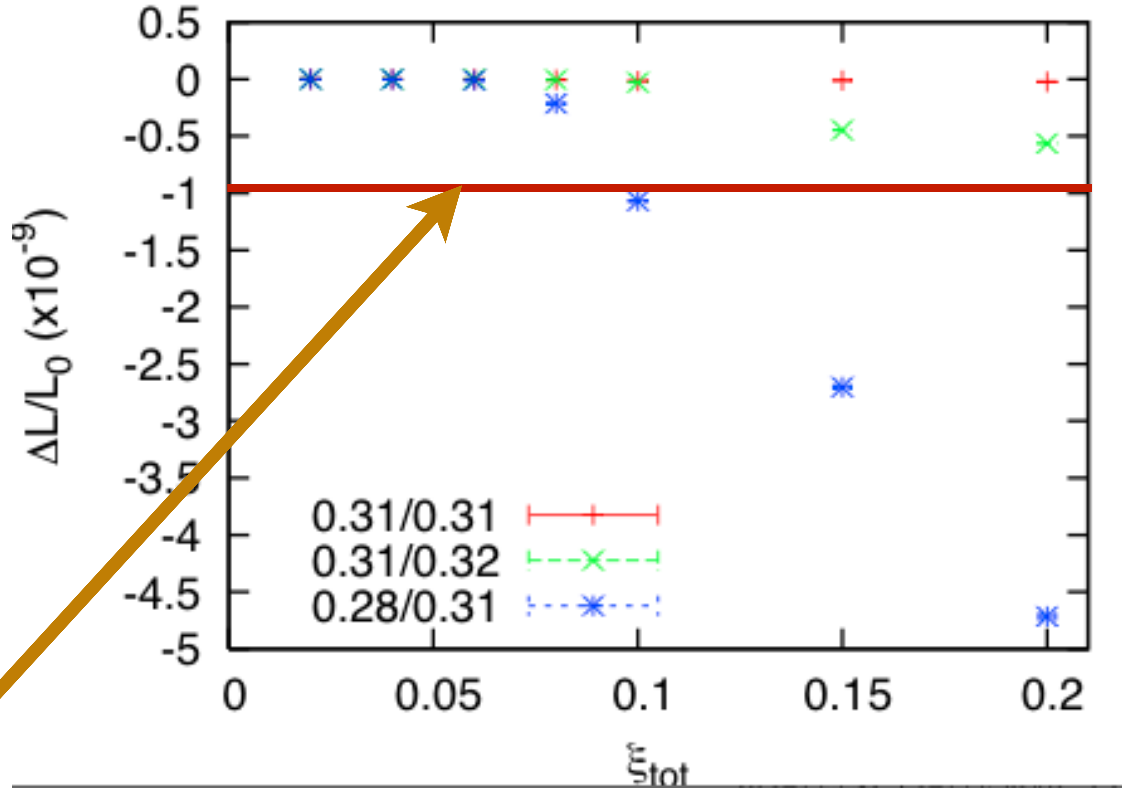
$$
M = \begin{pmatrix} M_x & 0 & 0 \\ 0 & M_y & 0 \\ 0 & 0 & M_z \end{pmatrix} \qquad M_i = \begin{pmatrix} \cos\mu_i & \beta_i \sin\mu_i \\ -\sin\mu_i/\beta_i & \cos\mu_i \end{pmatrix}
$$

- weak-strong model
- Tracking 10^6 turns
- 2 IP, Super-periodicity 2:

Breaking superperiodicity degrades the performance in the most case.

Luminosity decrement for the toy model





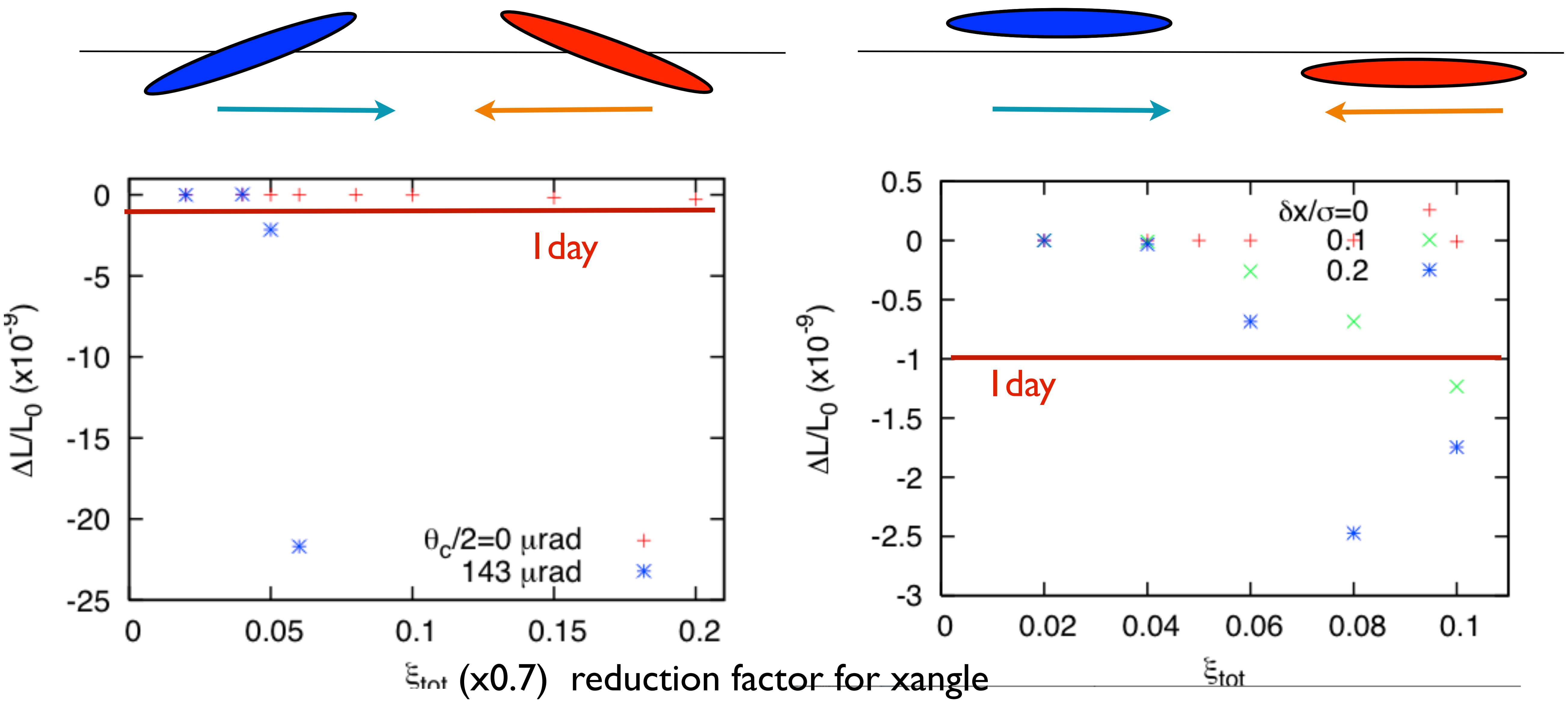
- $\Delta L/L_0=-1\times10^{-9}$: 1day luminosity lifetime
- The beam-beam limit is 0.2!? High integrability, an equal tune in a round beam or $\boxed{\nu_x=0.5+\alpha \text{ in a flat beam.}}$

K. Ohmi et al., EPAC06, MOPLS032

- This is not surprising, because the system is approximately two degree of freedom (r-s) or (y-s).
- Similar result is obtained in e+e- colliders for $\nu_x=0.5+\alpha$ with crossing angle zero.

Crossing angle and static offset

- Breaking symmetry, appear odd order terms in beam-beam potential, then degrade the luminosity.



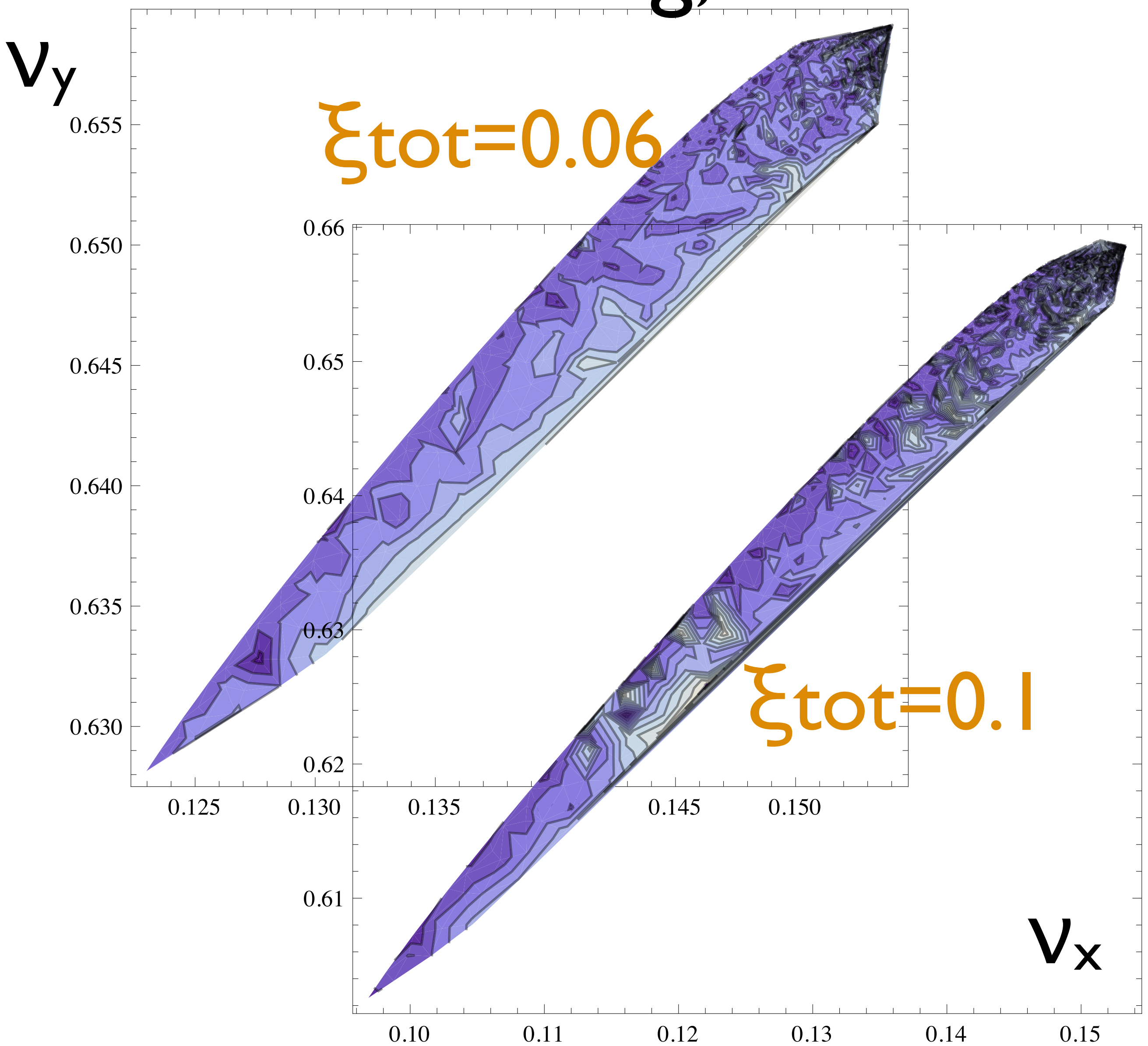
The beam-beam limit is 0.035-0.05.

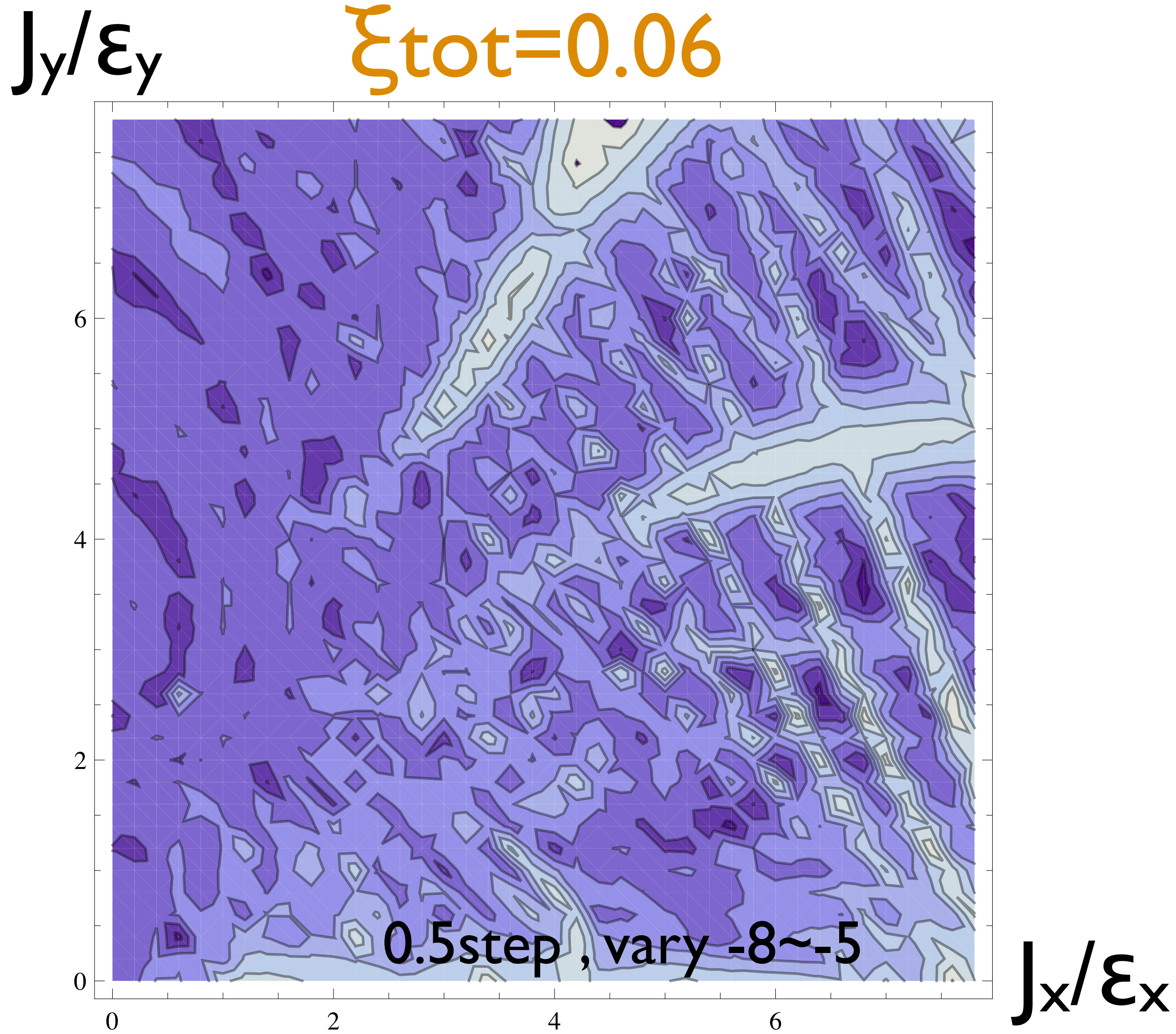
Why crossing angle and offset degrade the luminosity

- Frequency map analysis, plot diffusion index

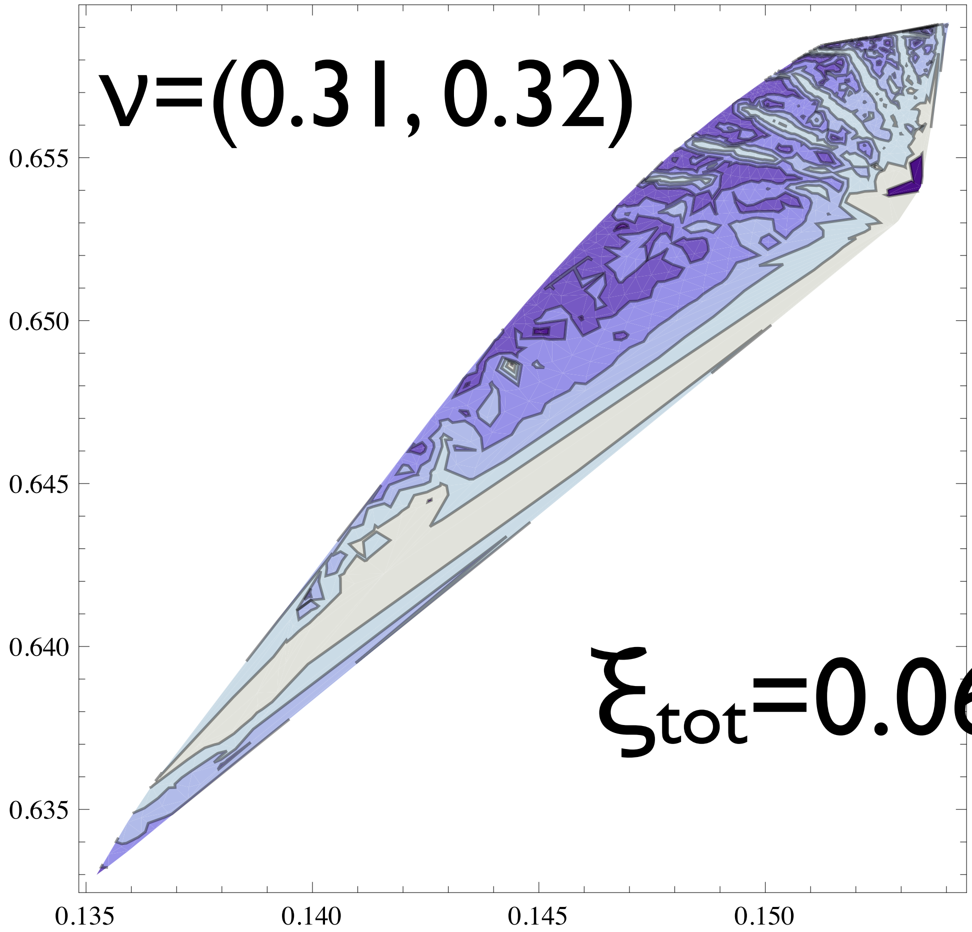
$$D = \frac{1}{2}\log_{10}(\langle\delta\nu_x\rangle^2 + \langle\delta\nu_y\rangle^2)$$

- No crossing, no offset

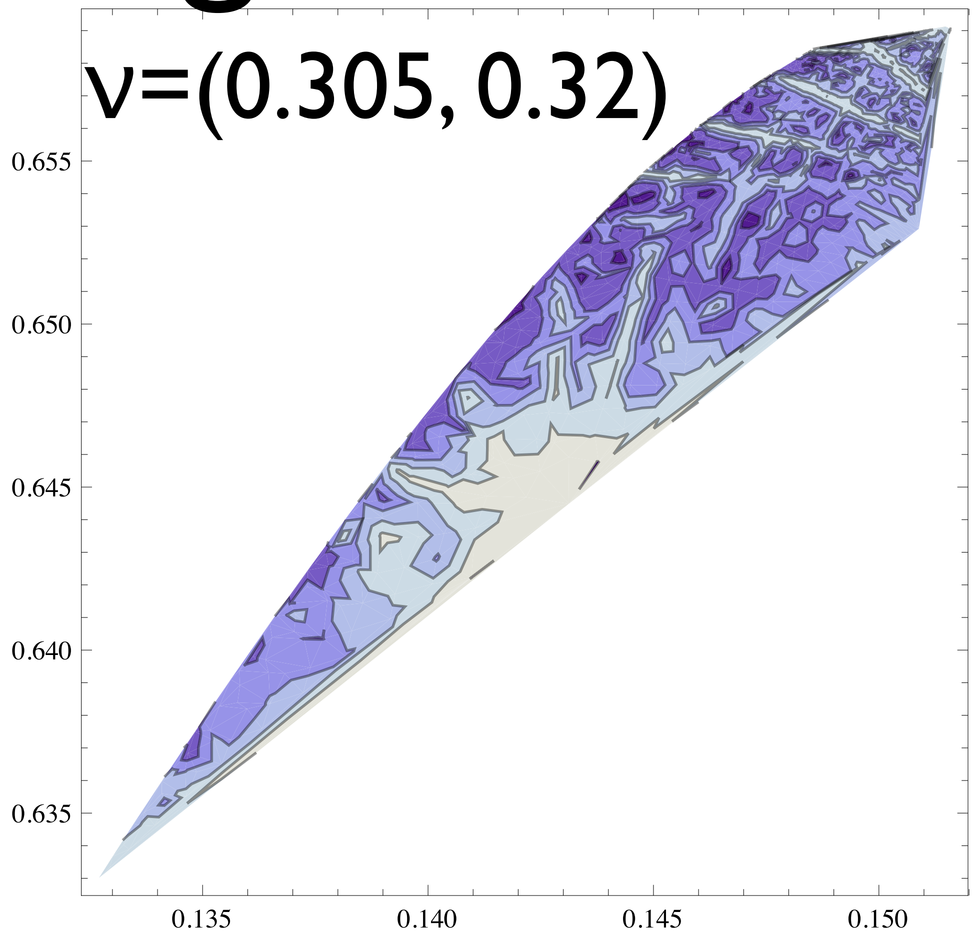


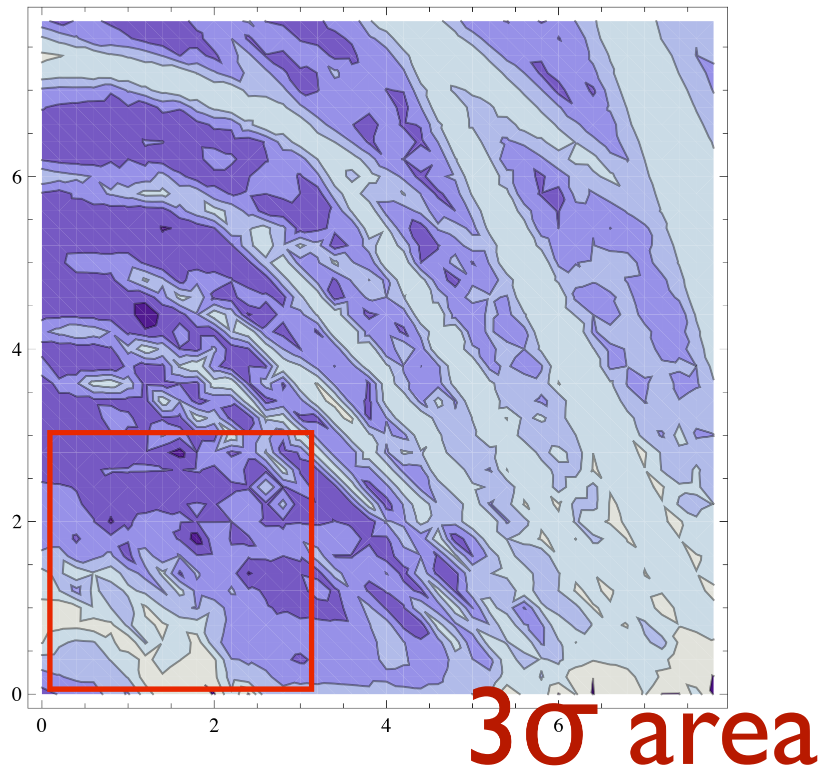


FMA for crossing collision



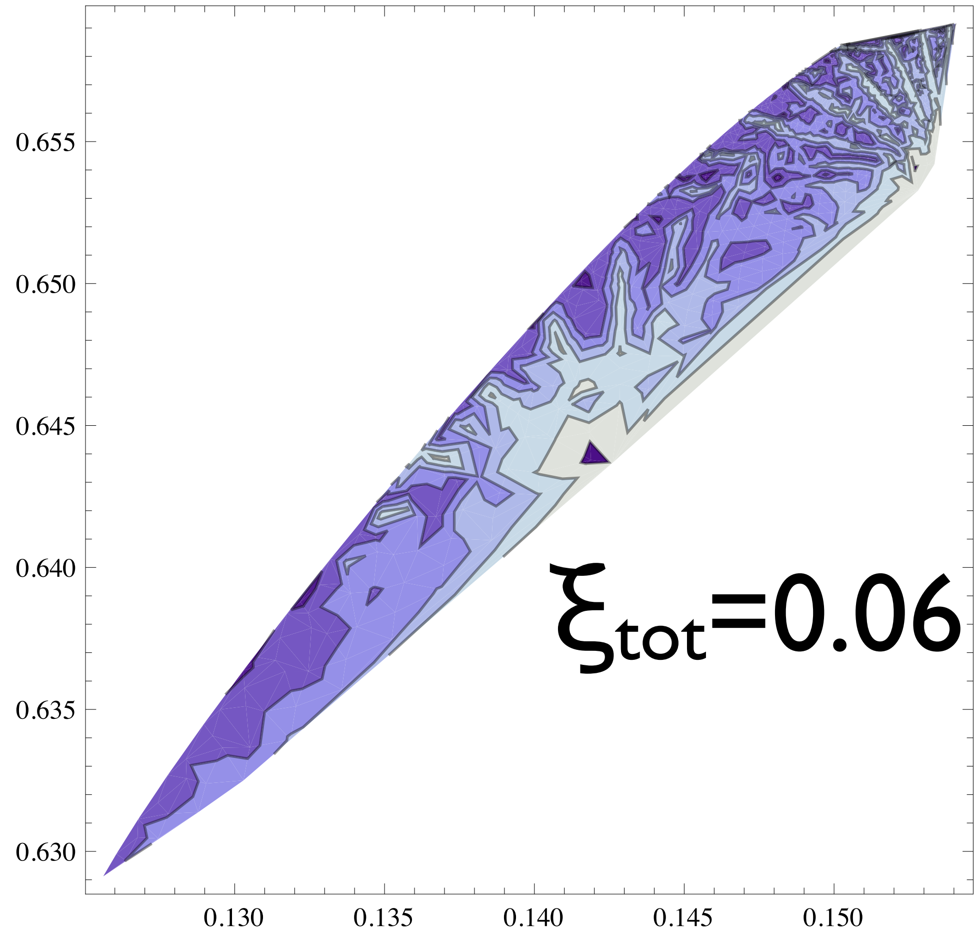
ξ_{tot}=0.06x0.7

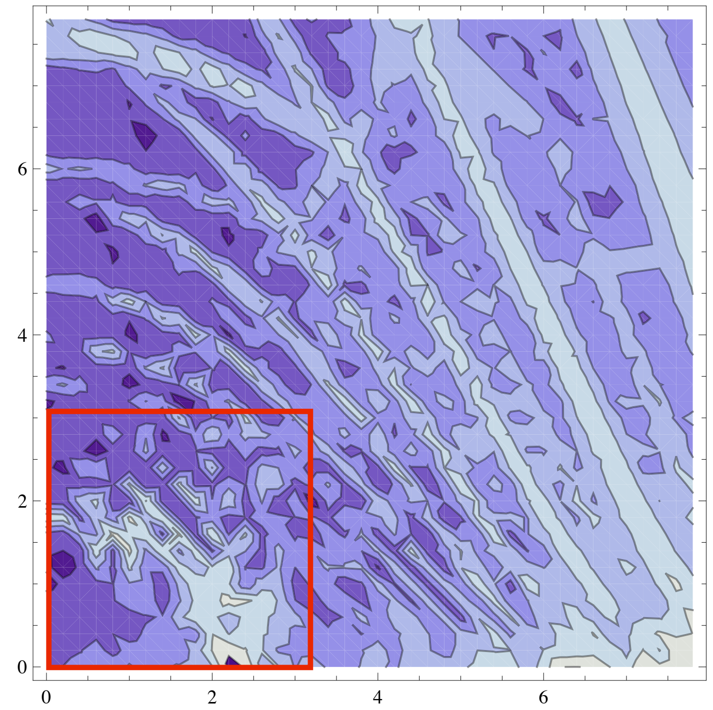




- Linear coupling resonance is seen but no effect on the luminosity
- Diffusive zone appears near ~2σ.
- Luminosity degradation is caused by 7-th order resonances.

FMA for offset collision



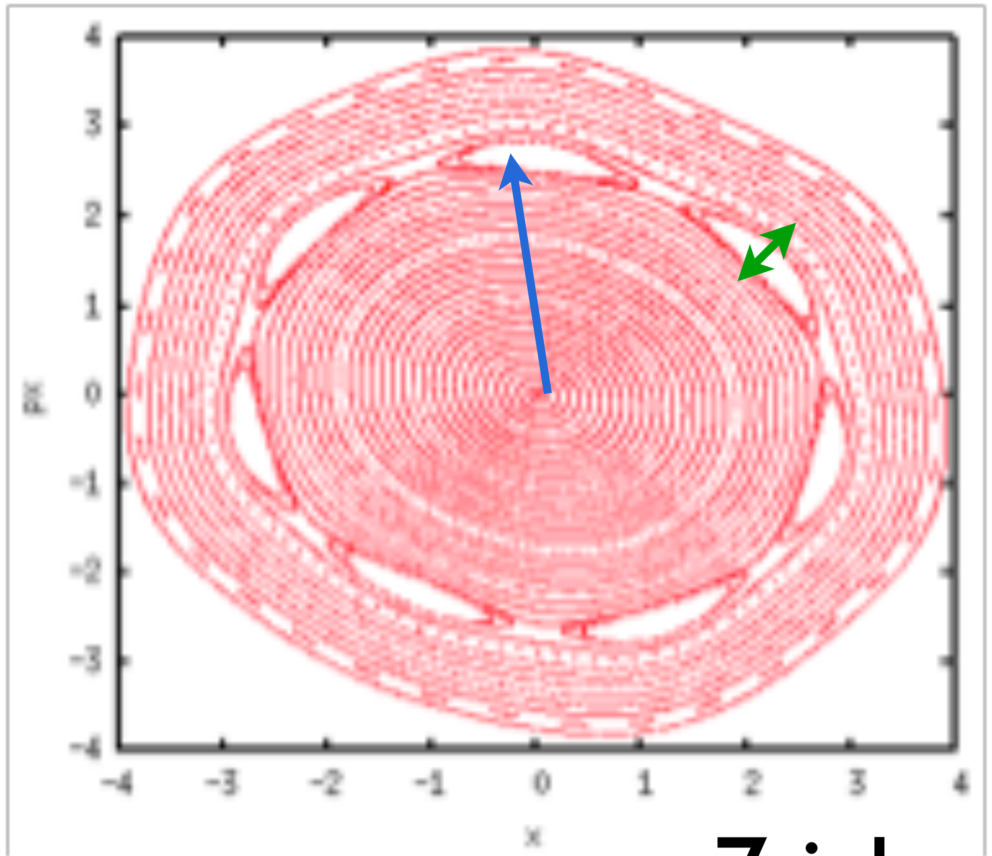


3σ area

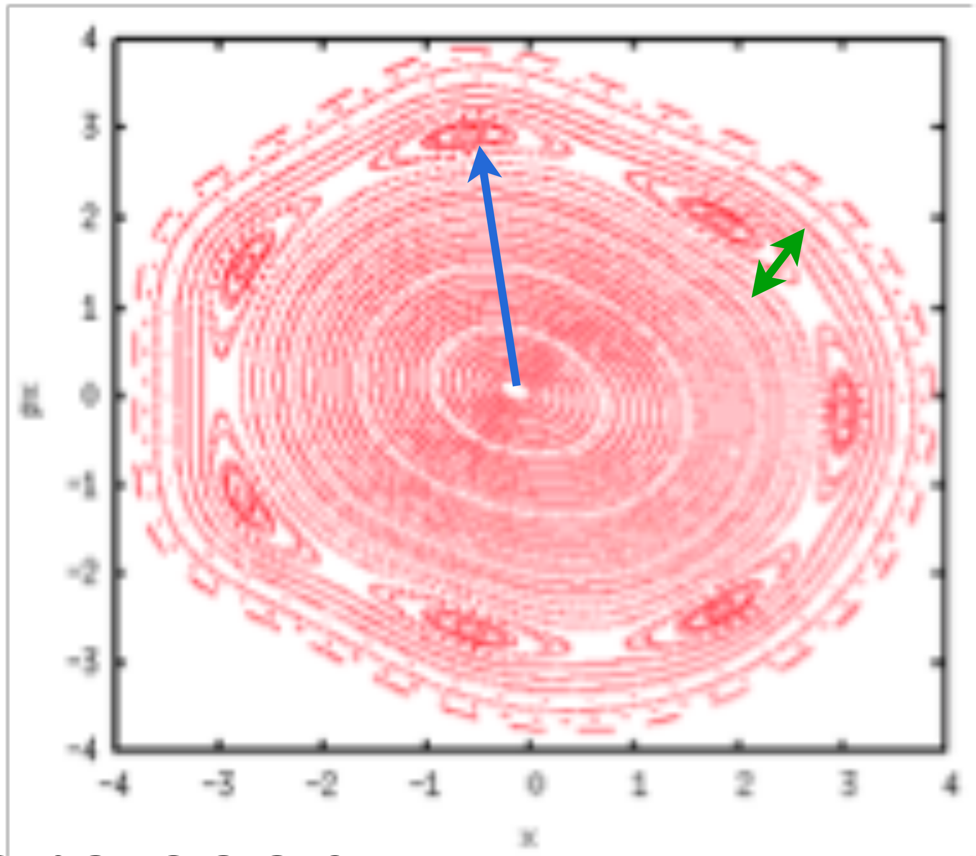
- Similar as crossing collision.
- 7-th order resonances appear.
- Diffusive zone correspond to 7-th order resonances appears.
- Symmetry breaking for x (offset) or p_x (crossing).

Phase space plot

crossing



offset



7 islands are seen.

Both are similar structure,

resonance radius and width.

Taylar map analysis

- Taylar map is obtained for the simple beam-beam model.
- The map is factorized by $M\exp(-:H:)$
- Fourier expansion for the betatron phase.

$$H = H_{00}(J_x, J_y) + \sum_{m_x, m_y} G_{m_x, m_y}(J_x, J_y)\exp(m_x\phi_x + m_y\phi_y)$$

$$\begin{aligned}
H_{00}(J_x, J_y) = {} & 2.01375\times10^{42}J_x^6 + 1.21356\times10^{43}J_x^5J_y - 4.82225\times10^{33}J_x^5 \\
& +2.79527\times10^{43}J_x^4J_y^2 - 2.2716\times10^{34}J_x^4J_y + 1.14155\times10^{25}J_x^4 \\
& +1.29819\times10^{44}J_x^3J_y^3 - 7.27118\times10^{34}J_x^3J_y^2 + 4.63551\times10^{25}J_x^3J_y \\
& -2.70542\times10^{16}J_x^3 + 1.66781\times10^{44}J_x^2J_y^4 - 1.48894\times10^{35}J_x^2J_y^3 \\
& +1.18738\times10^{26}J_x^2J_y^2 - 8.66053\times10^{16}J_x^2J_y + 6.10249\times10^{7}J_x^2 + 1.79409\times10^{44}J_xJ_y^5 \\
& -1.71884\times10^{35}J_xJ_y^4 + 1.68734\times10^{26}J_xJ_y^3 - 1.57801\times10^{17}J_xJ_y^2 + 1.36494\times10^{8}J_xJ_y \\
& +1.07216\times10^{44}J_y^6 - 1.2051\times10^{35}J_y^5 + 1.3982\times10^{26}J_y^4 - 1.59505\times10^{17}J_y^3 \\
& +1.74099\times10^{8}J_y^2
\end{aligned}$$

$$\begin{aligned}
G_{70}(J_x, J_y) = {} & -(1.02037\times10^{36} + 3.66357\times10^{37}i)J_x^{7/2}J_y^2 \\
& -(9.42572\times10^{35} + 2.37993\times10^{37}i)J_x^{9/2}J_y \\
& +(6.12286\times10^{26} + 1.98798\times10^{28}i)J_x^{7/2}J_y \\
& +(-3.22114\times10^{35} - 5.54337\times10^{36}i)J_x^{11/2} \\
& +(3.8924\times10^{26} + 1.0181\times10^{28}i)J_x^{9/2} \\
& -(2.88738\times10^{17} + 1.01787\times10^{19}i)J_x^{7/2}
\end{aligned}$$

Resonance line and width

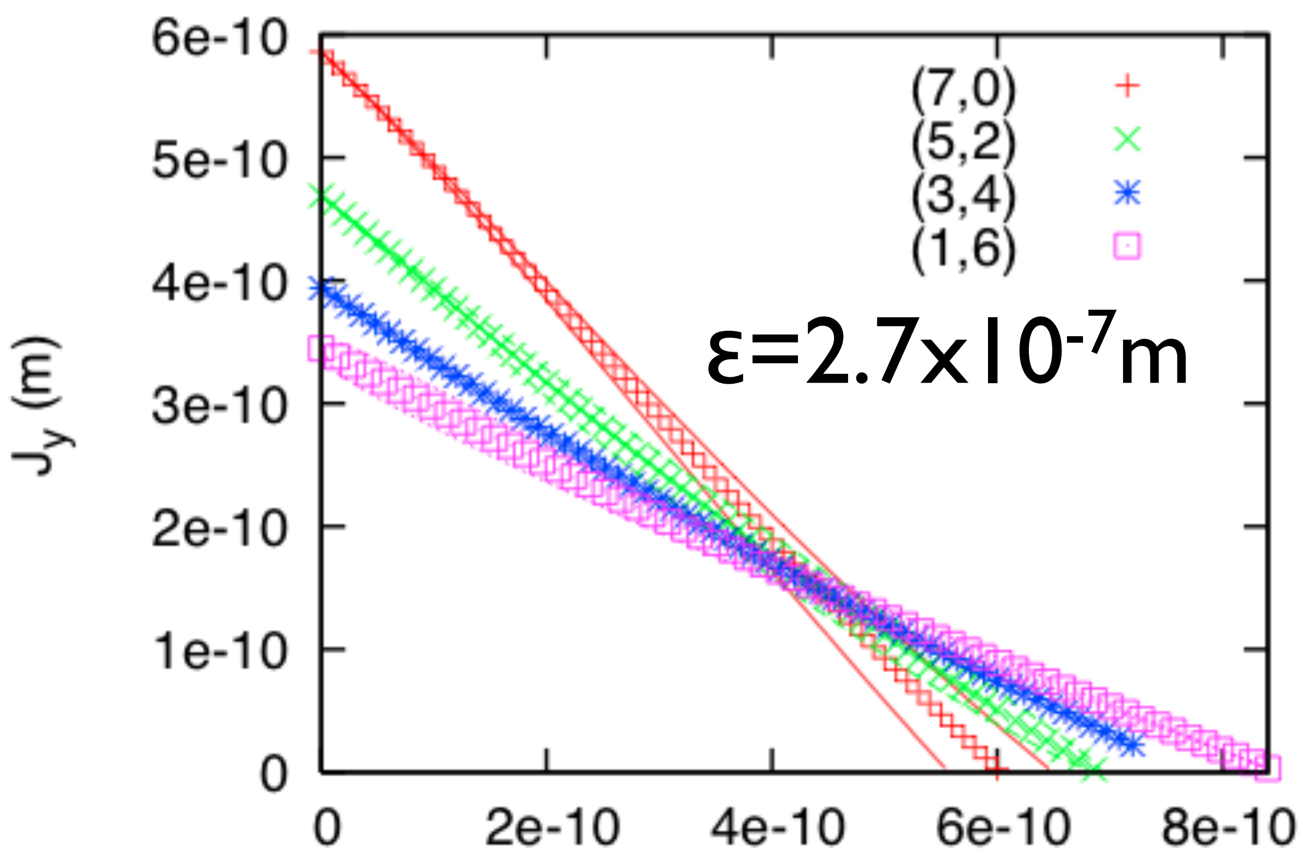
- Resonance line in J space

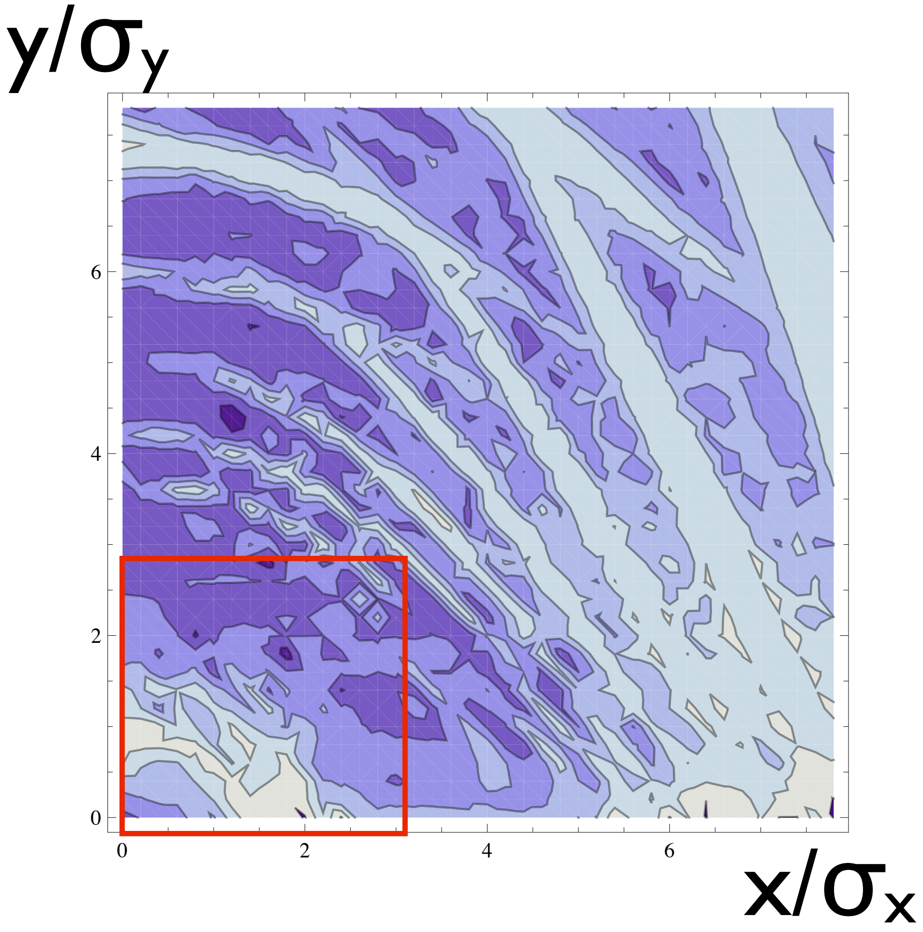
$$m_x\left(\nu_{x0}+\left.\frac{\partial H_{00}}{\partial J_x}\right|_{J=J_R}\right)+m_y\left(\nu_{y0}+\left.\frac{\partial H_{00}}{\partial J_y}\right|_{J=J_R}\right)=n$$

- Resonance width

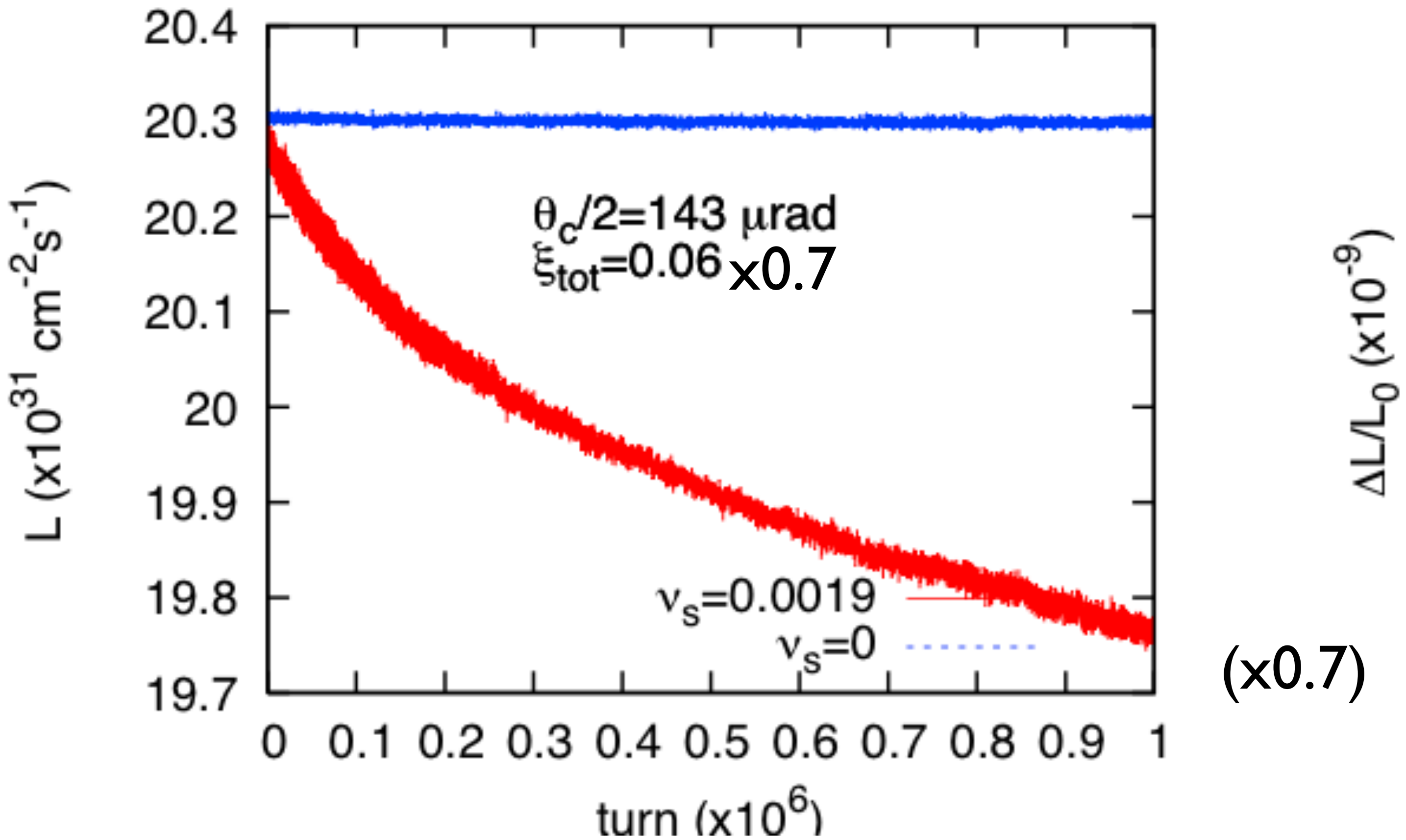
$$\Delta J_x = 2m_x\sqrt{\frac{G_{m_x,m_y}}{\Lambda}} \qquad \Lambda = m_x^2\frac{\partial^2 H_{00}}{\partial J_x^2}+m_x m_y\frac{\partial^2 H_{00}}{\partial J_x \partial J_y}+m_y^2\frac{\partial^2 H_{00}}{\partial J_y^2}$$

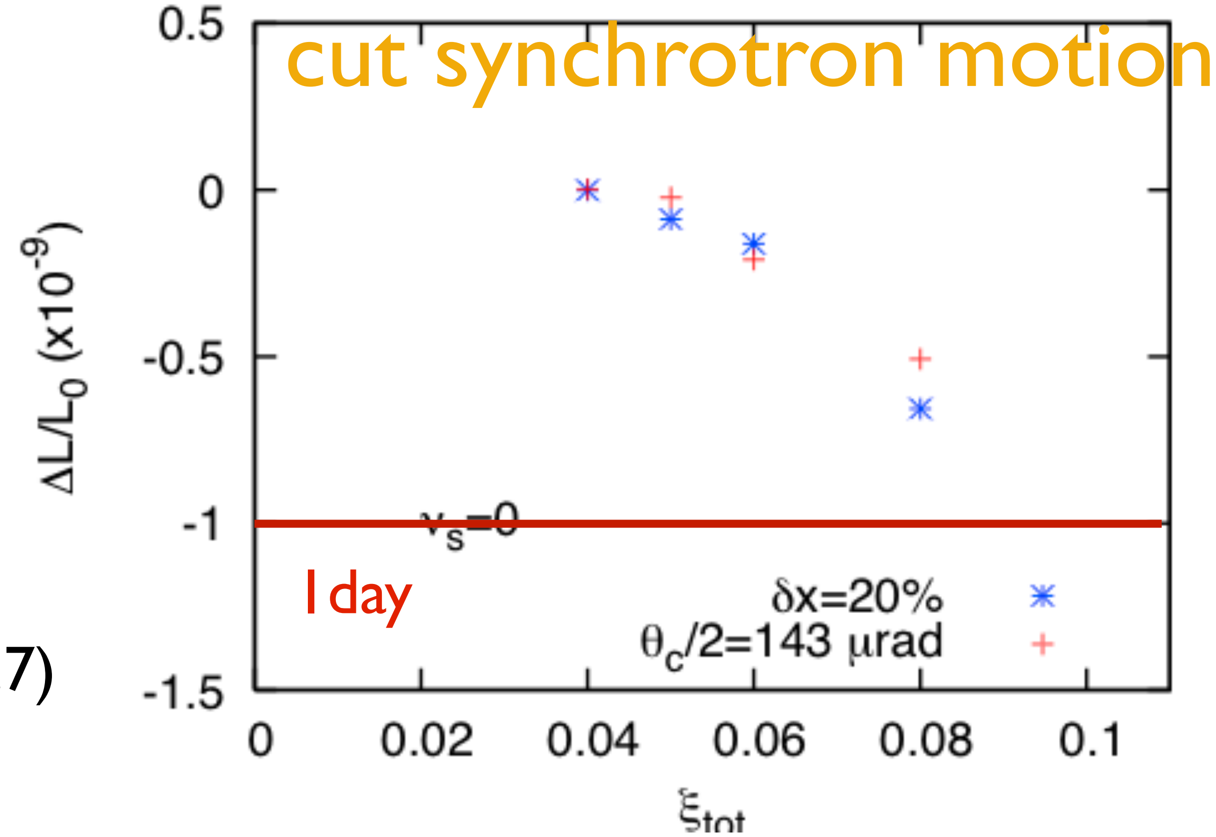
- Similar behavior as FMA



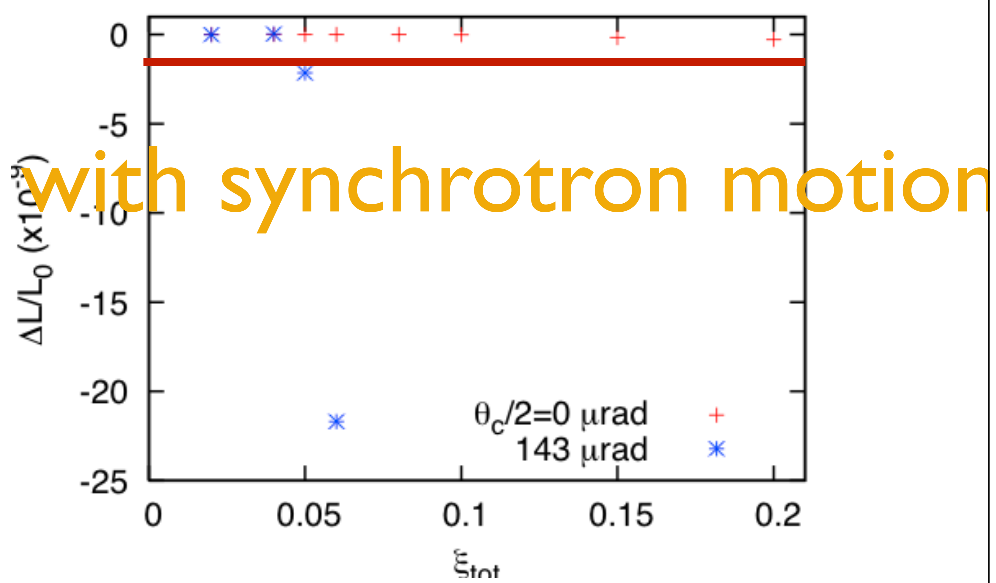


Synchrotron motion

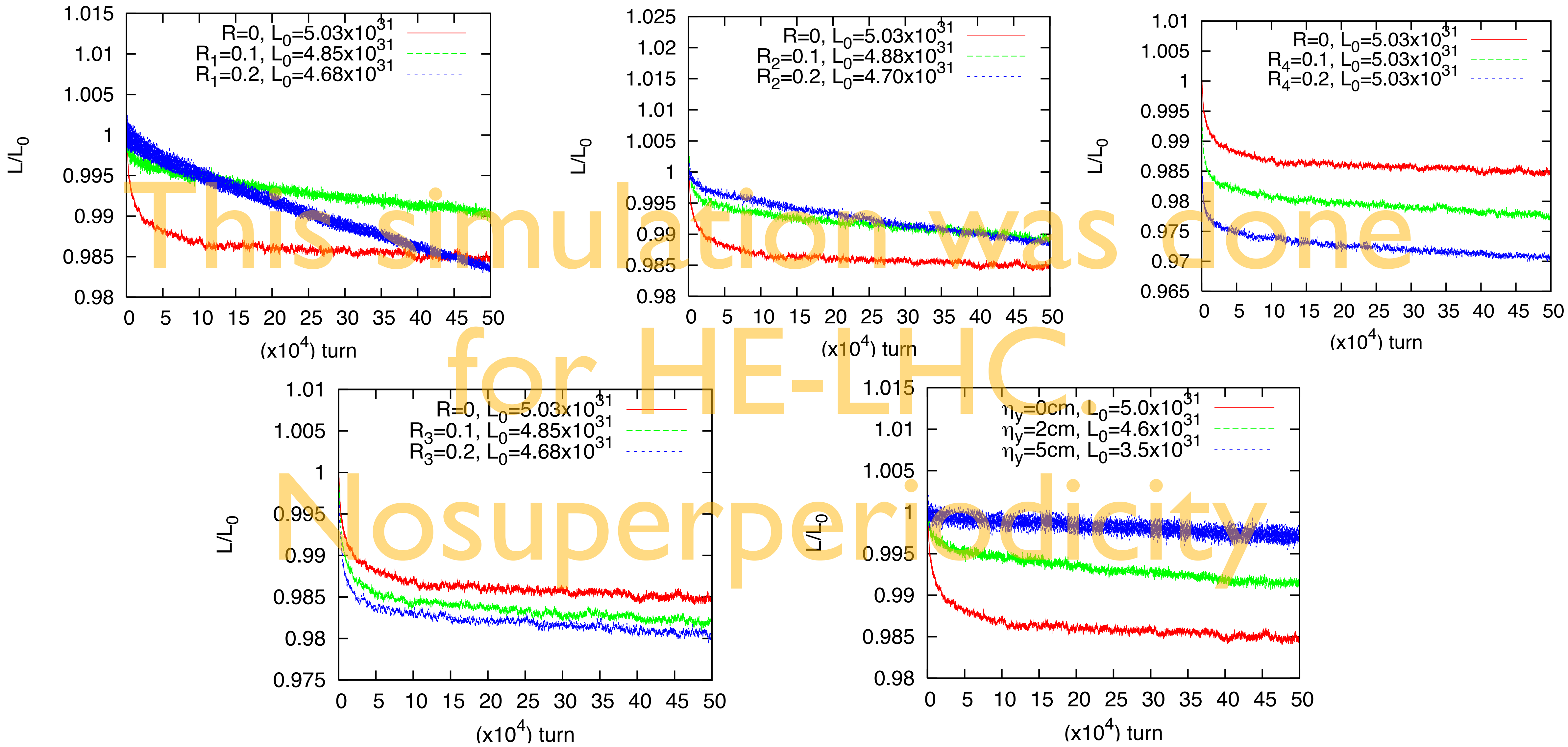




- Synchrotron motion enhances the luminosity degradation. The 7-th order resonances modulated by the synchrotron motion induce stronger emittance growth.



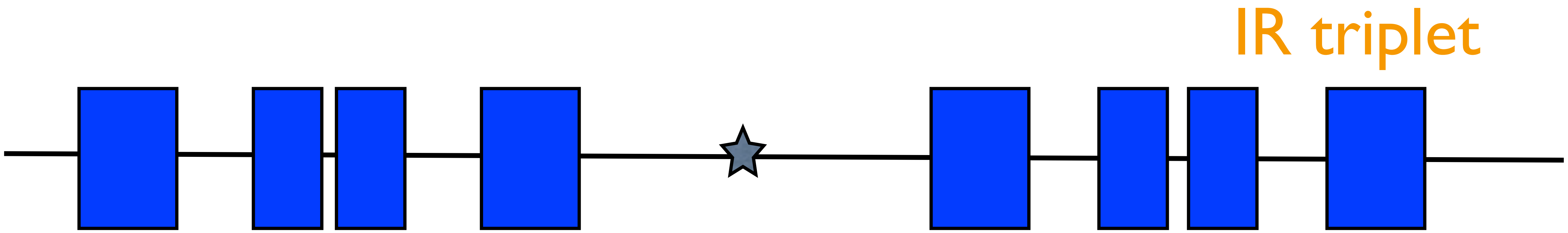
Effect of x-y coupling for incoherent emittance growth



- ξ/IP=(0.017,0.021)
- Clear degradation is seen only for R1,R2=0.2.
- The sensitivity is 10^2-10^3 looser.

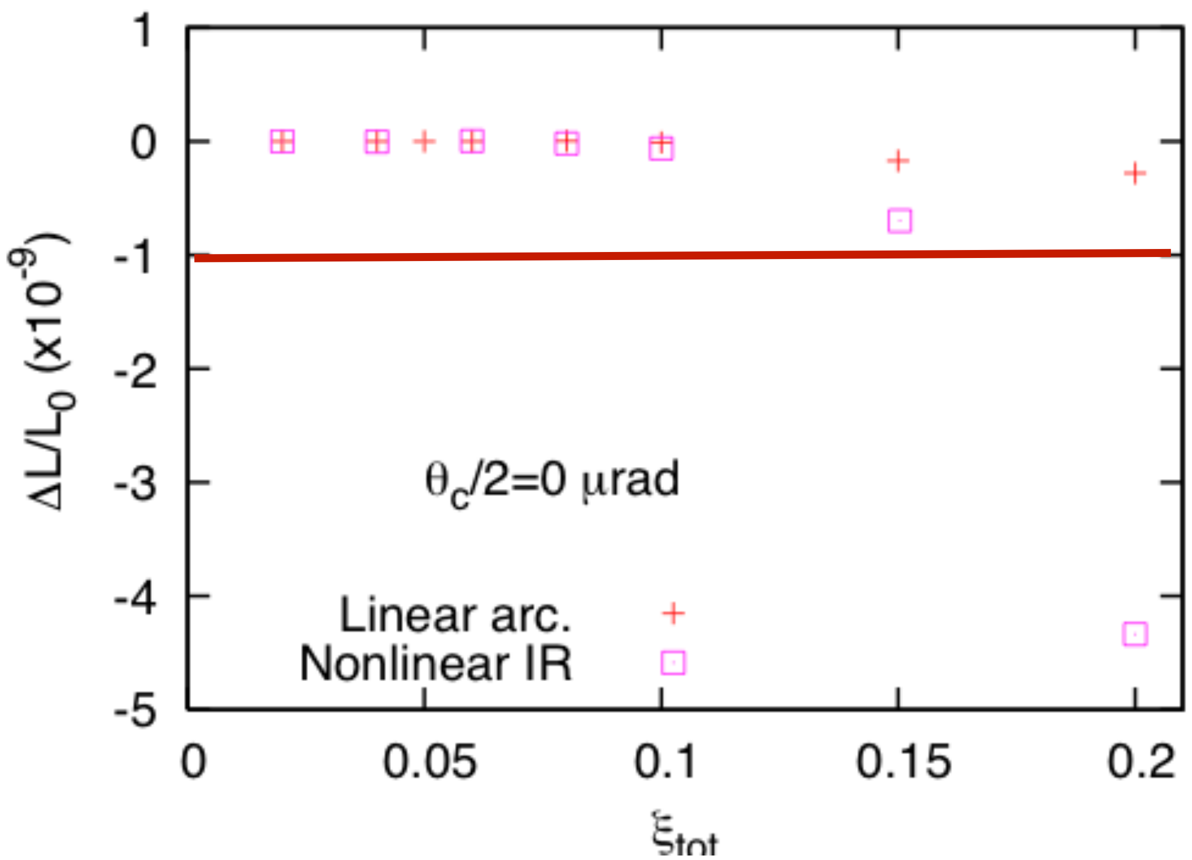
IR Quadrupole nonlinearity

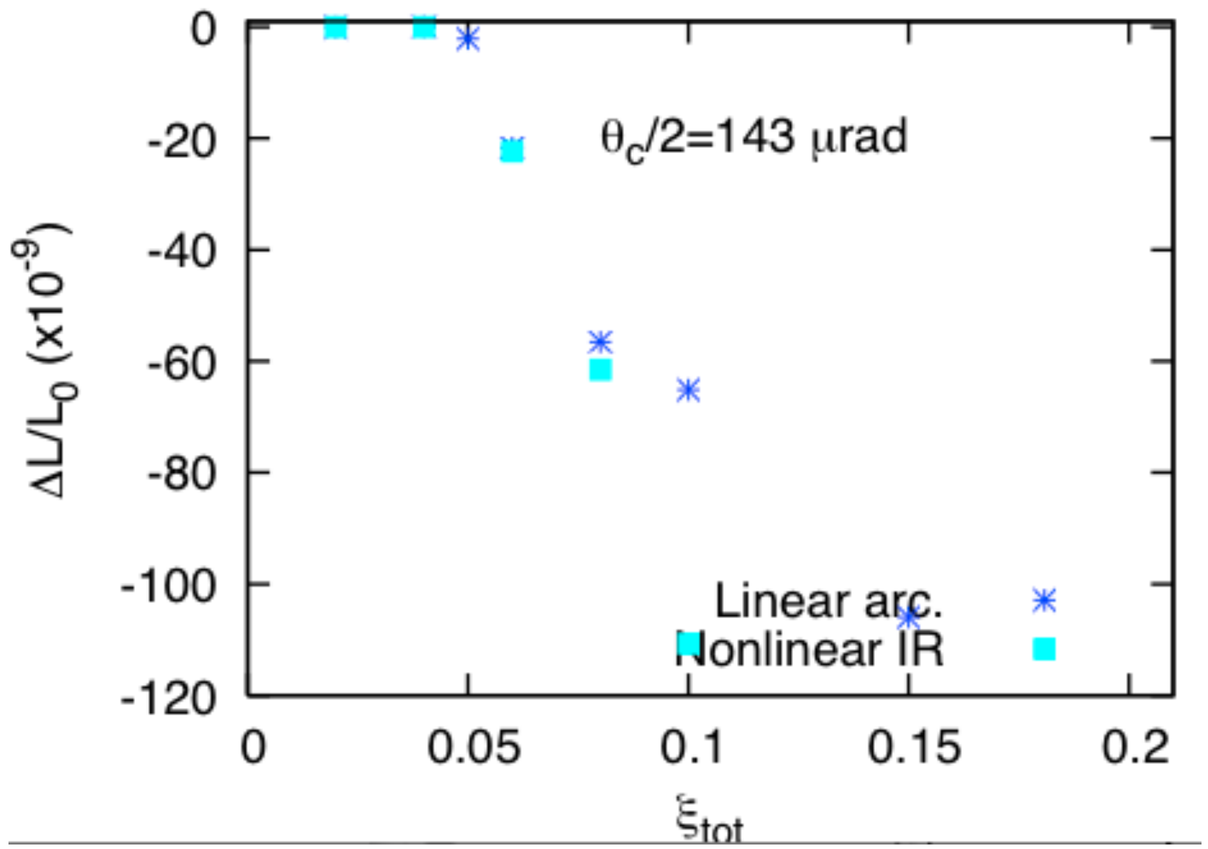
- Triplet of IP1, MQXA.1R(L)1, MQXB.A2R(L)1, MQXB.B2R(L)1, MQXA.3R(L)1
- Multipole components of these magnets are dominant for the limit of the dynamic aperture.
- Kinematic term and fringe field was negligible in LHC.
- Study with a model containing the 8 IR magnets.



Multipole table of Quadrupole is given by R.Tomas.

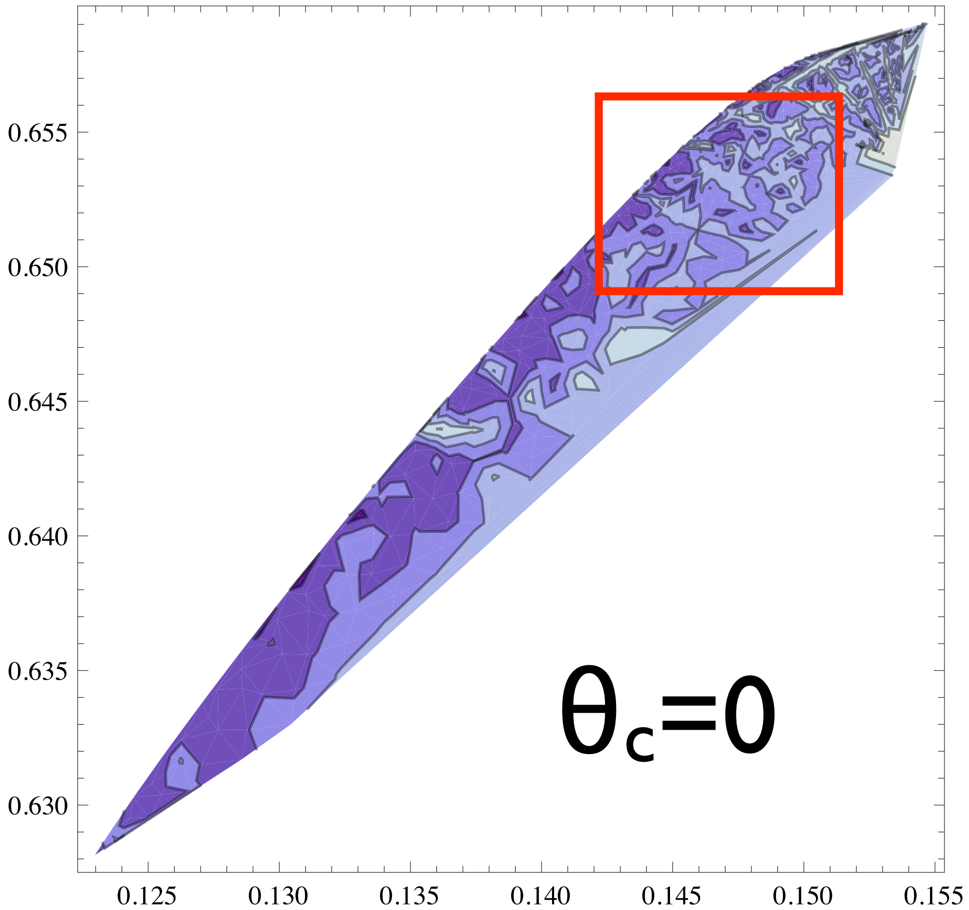
IR nonlinearity

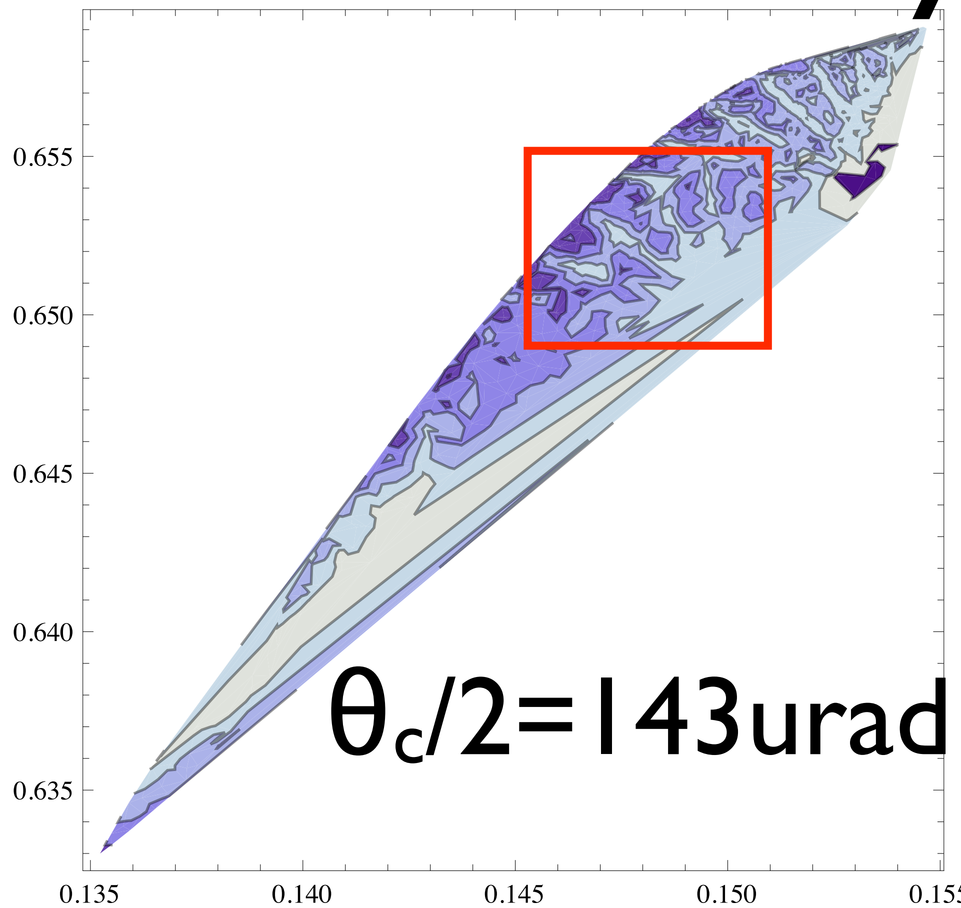




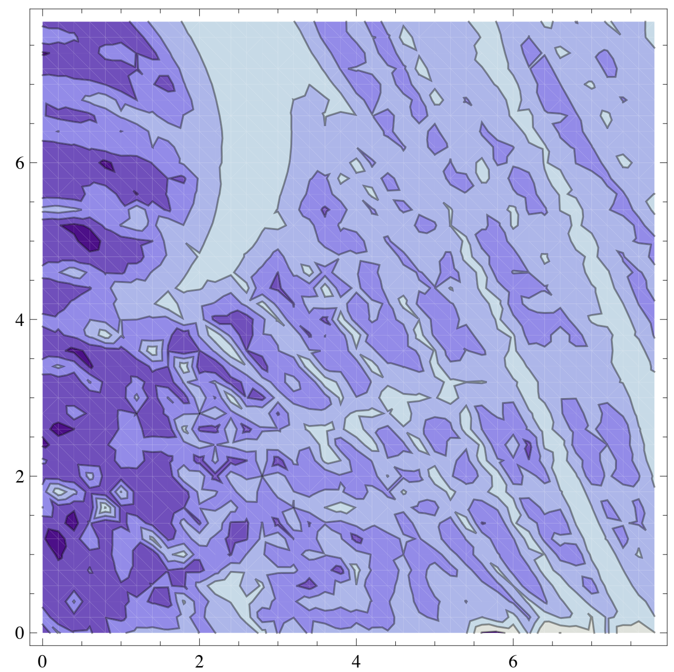
- IR nonlinearity affects the luminosity at large beam-beam parameter, $\xi_{tot}>0.1$.
- The difference is not remarkable $\xi_{tot}<0.1$.

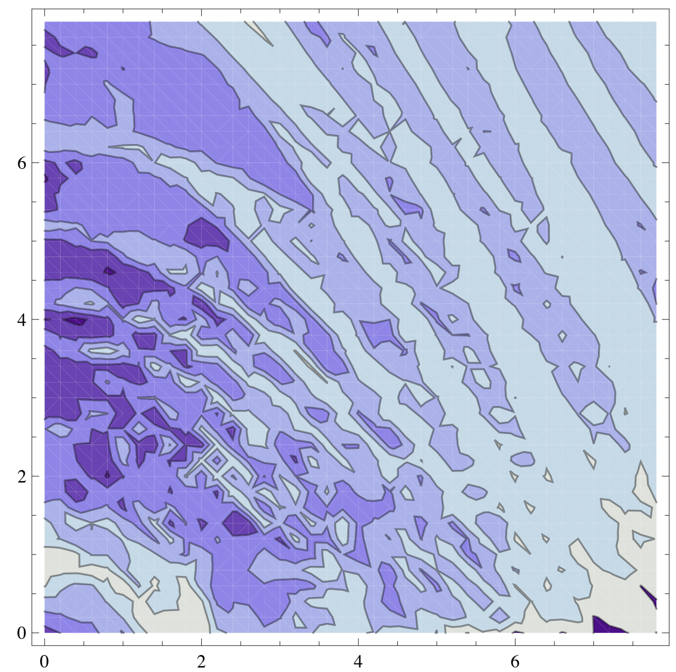
Resonances due to IR nonlinearity





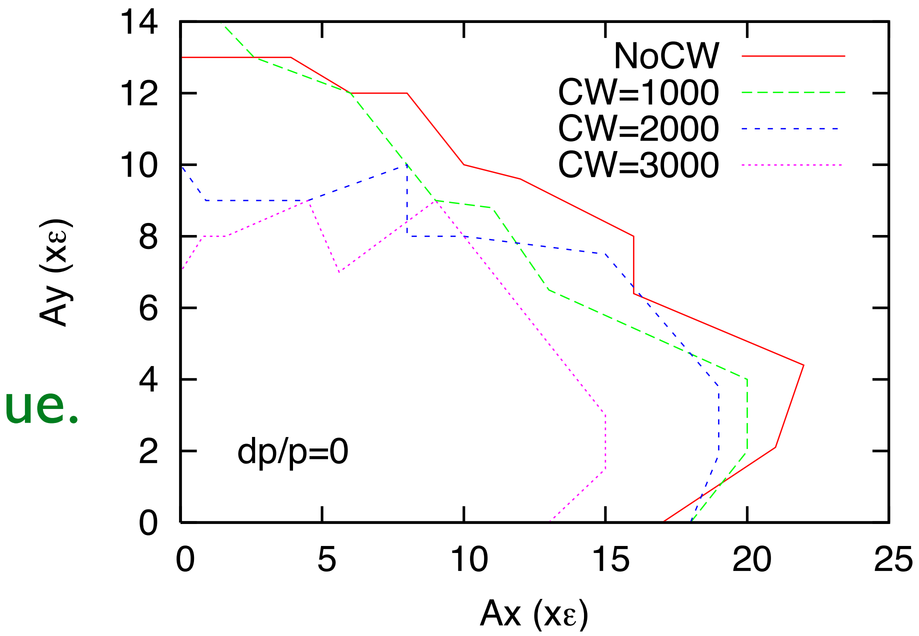
- 10-th order resonances appear due to IR nonlinearity.



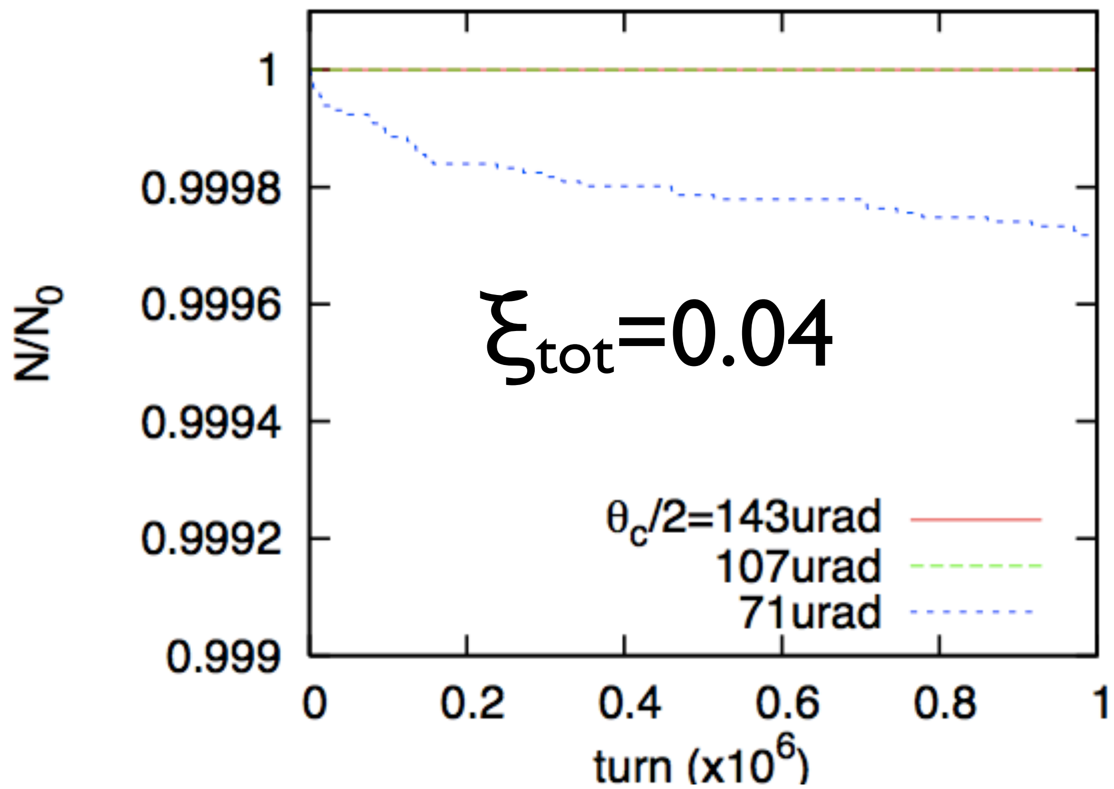


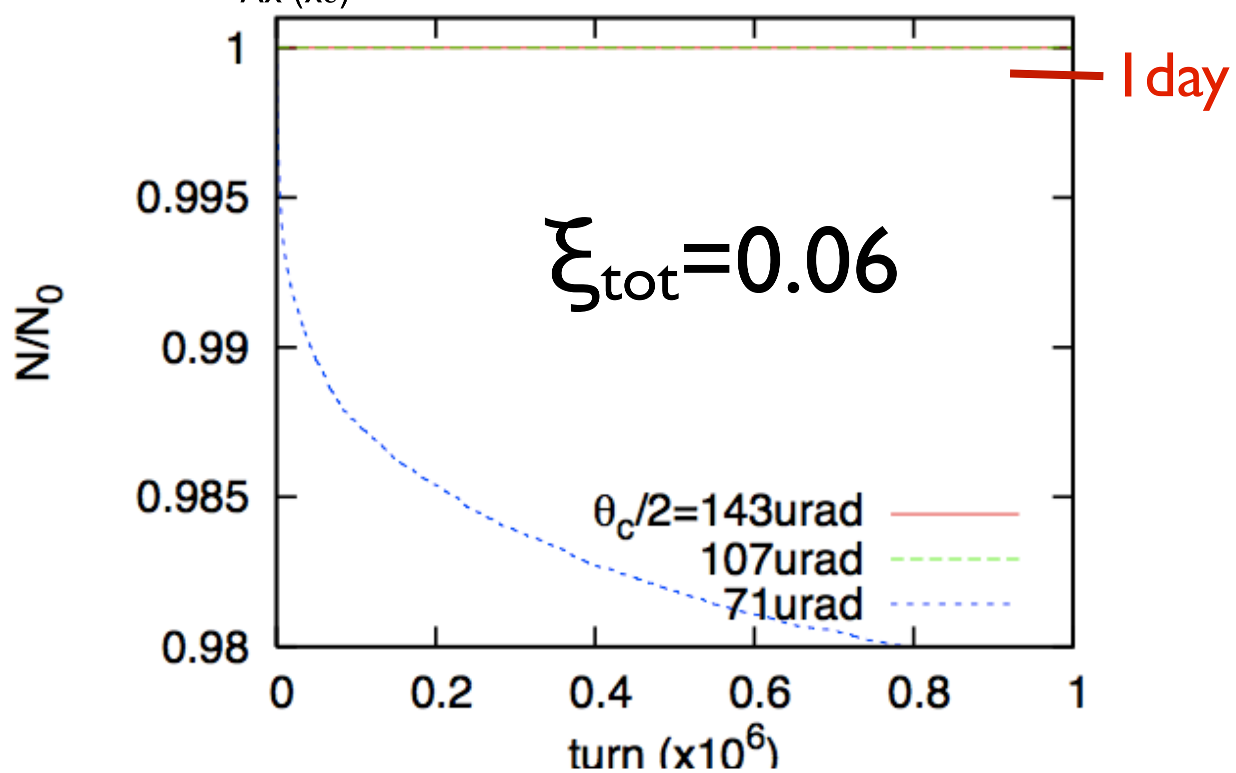
IR nonlinearity lifetime, parasitic interaction

- Finite dynamic aperture due to IR nonlinearity.
- Parasitic interaction

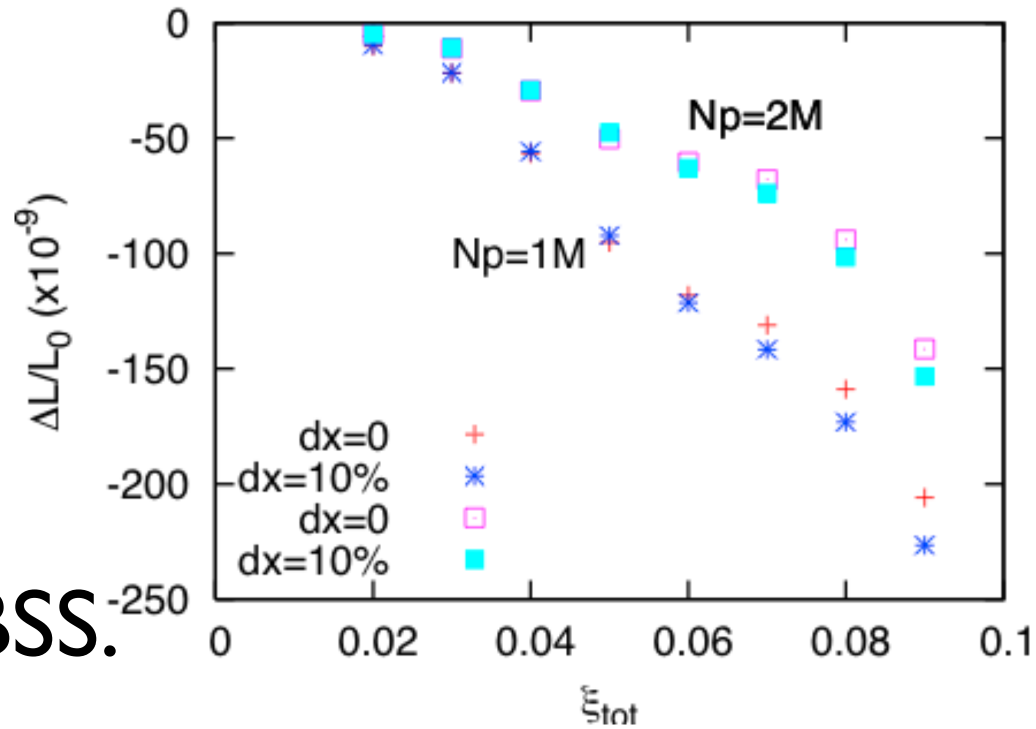


This aperture is wider than the normal value.
The life time may be underestimated.





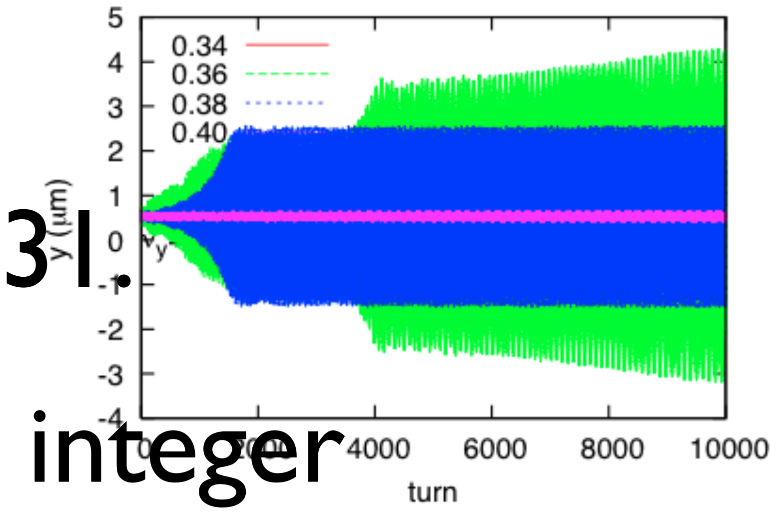
Incoherent emittance growth in strong-strong simulation



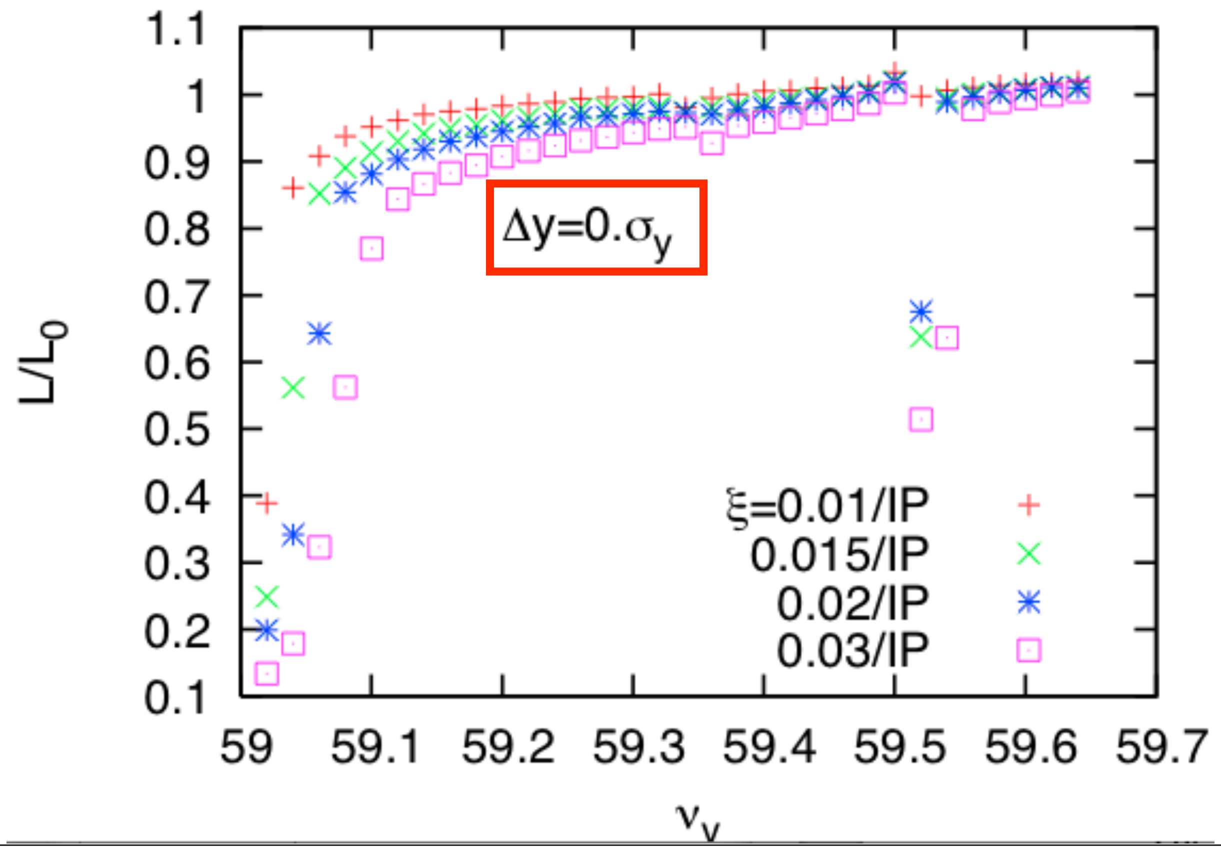
- Strong-strong simulation code, BBSS.
- The luminosity decrement depends on the macro-particle statistics. The statistical offset noise degrade the luminosity artificially.
- The difference due to the static offset is seen.
- Simulation with high statistics is necessary. it is possible but consuming.

Coherent beam-beam instability

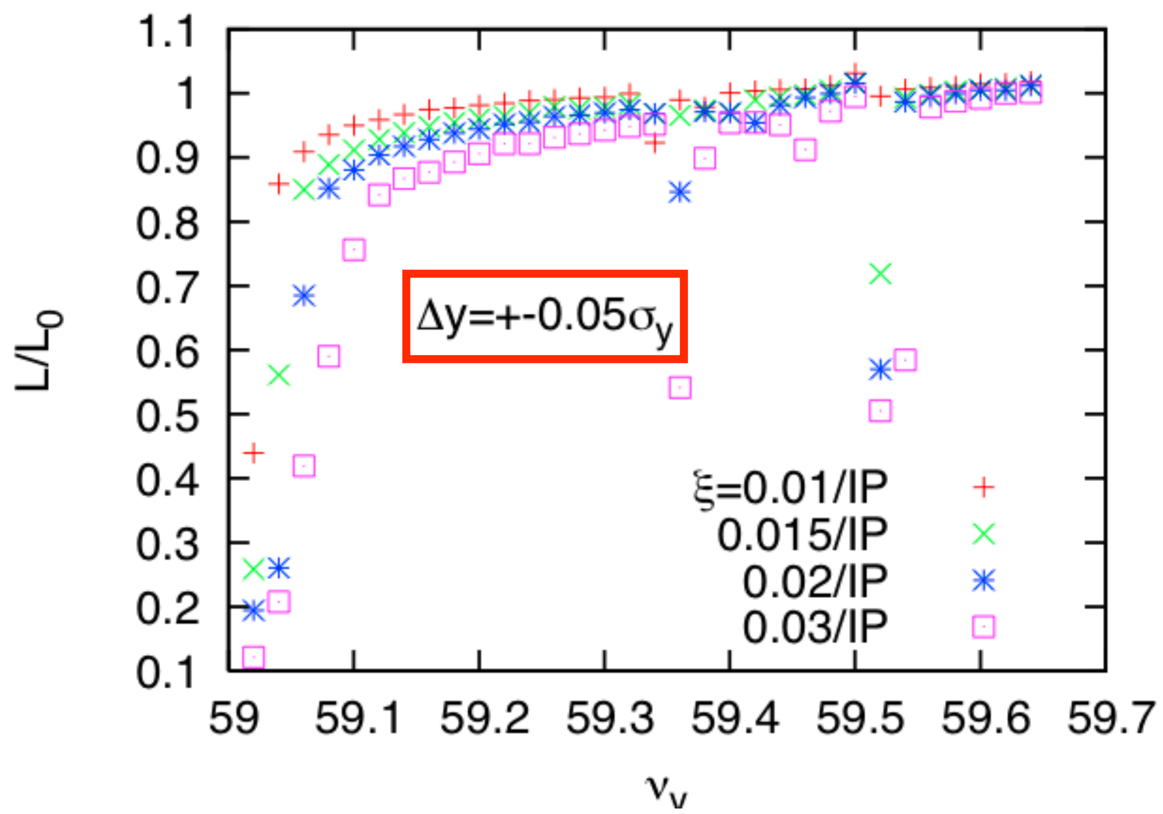
- Strong-strong simulation (BBSS)
- Vertical tune scan with keeping ν_x=0.31.
- Coherent instabilities are seen above integer and half integer, π mode instability.
- Instability is seen ν_y=0.38



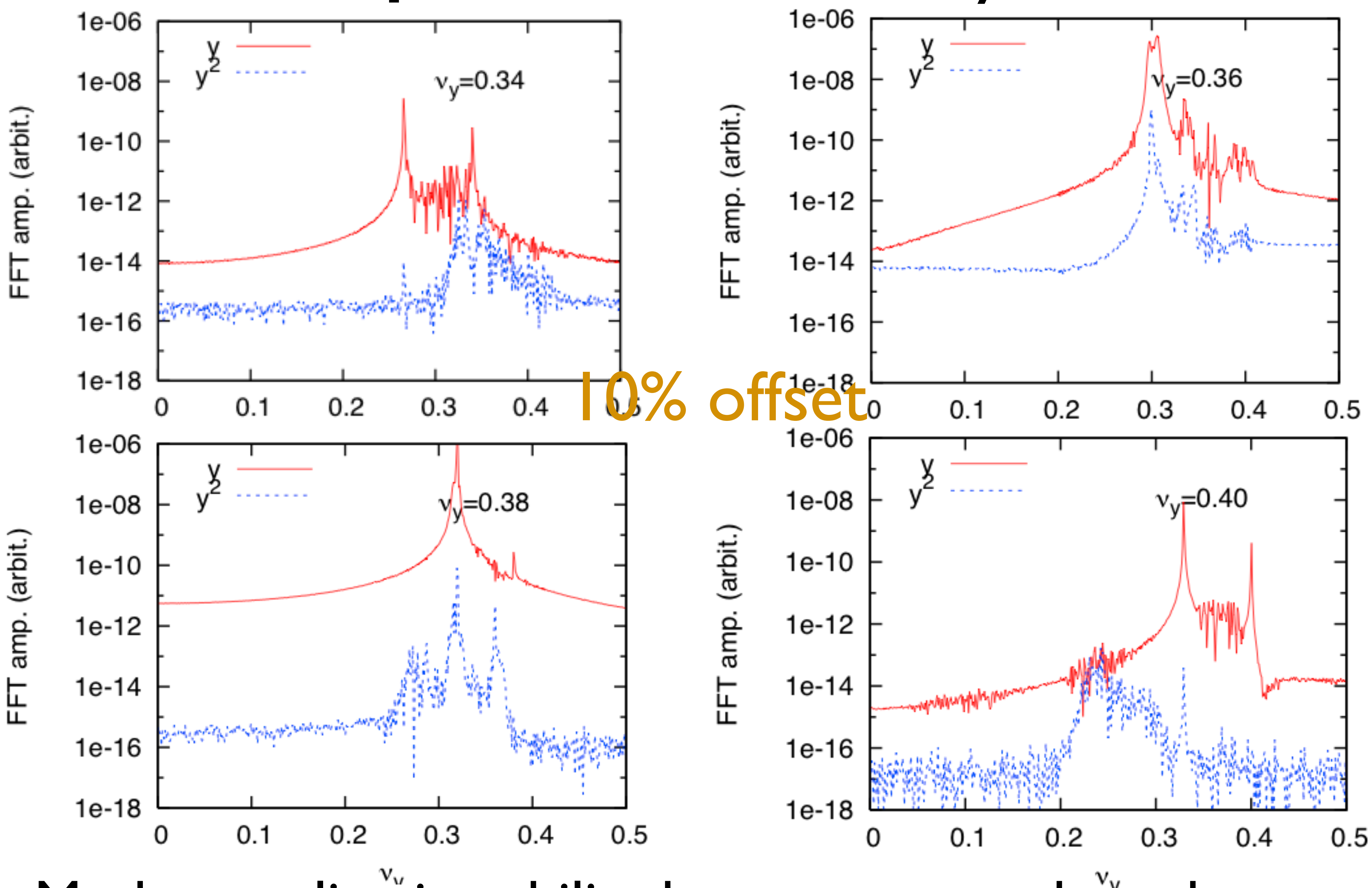
RHIC is operated at ν=2/3+α.



small offset



FFT spectra near ν_y=0.38



- Mode coupling instability between π mode and quadrupole mode.
- The coupling of π and quadrupole modes is induced by a collision offset in a perturbation theory by Y. Alexahin.

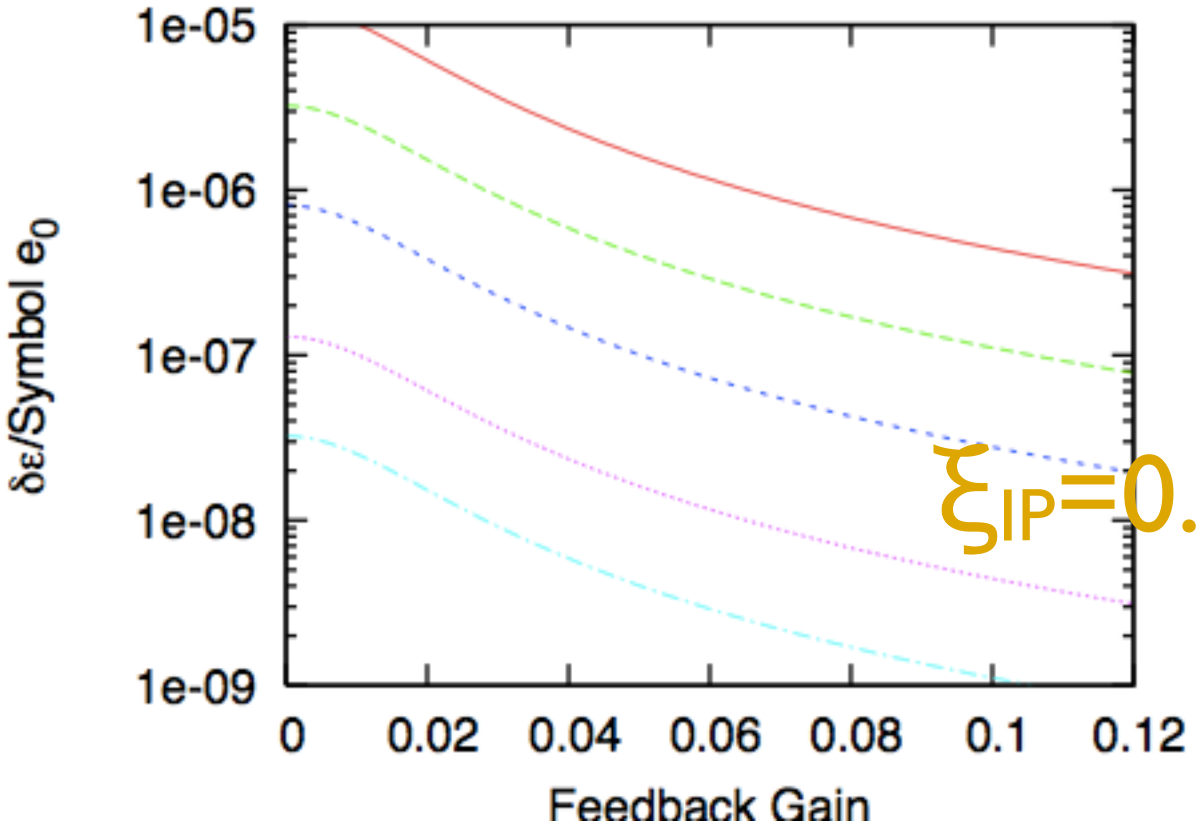
External noise I

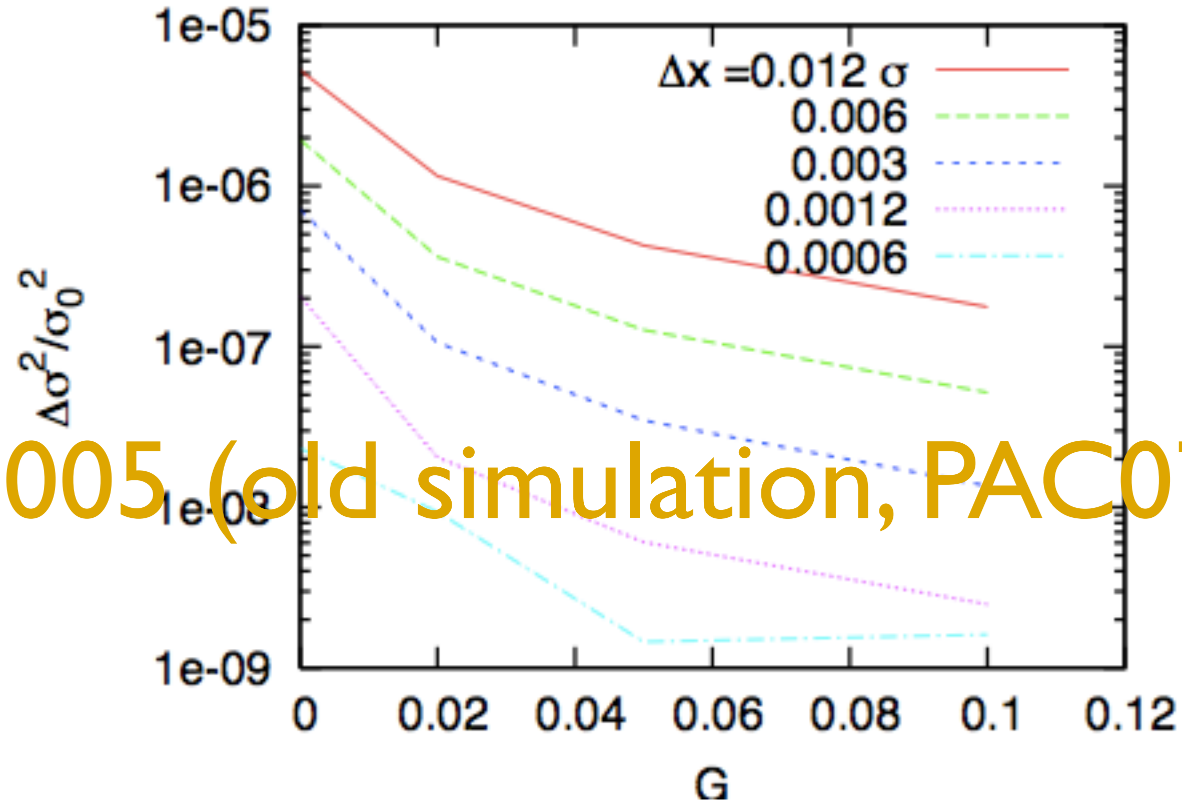
T. Sen, J. Ellison, PRL77, 1051 (1996)
Y. Alexahin, NIMA391, 73 (1996)

- Coherent dipole noise induces π mode oscillation. Smearing the oscillation causes an emittance growth.

$$\frac{\delta\varepsilon}{\varepsilon} \approx \frac{K}{\left(1+\frac{G}{2\pi|\xi|}\right)^2}\frac{\delta x^2}{\sigma_x^2} \qquad K = 0.089$$

by Y. Alexahin





ξ_{IP}=0.005 (old simulation, PAC07)

Analytic

Strong-strong simulation

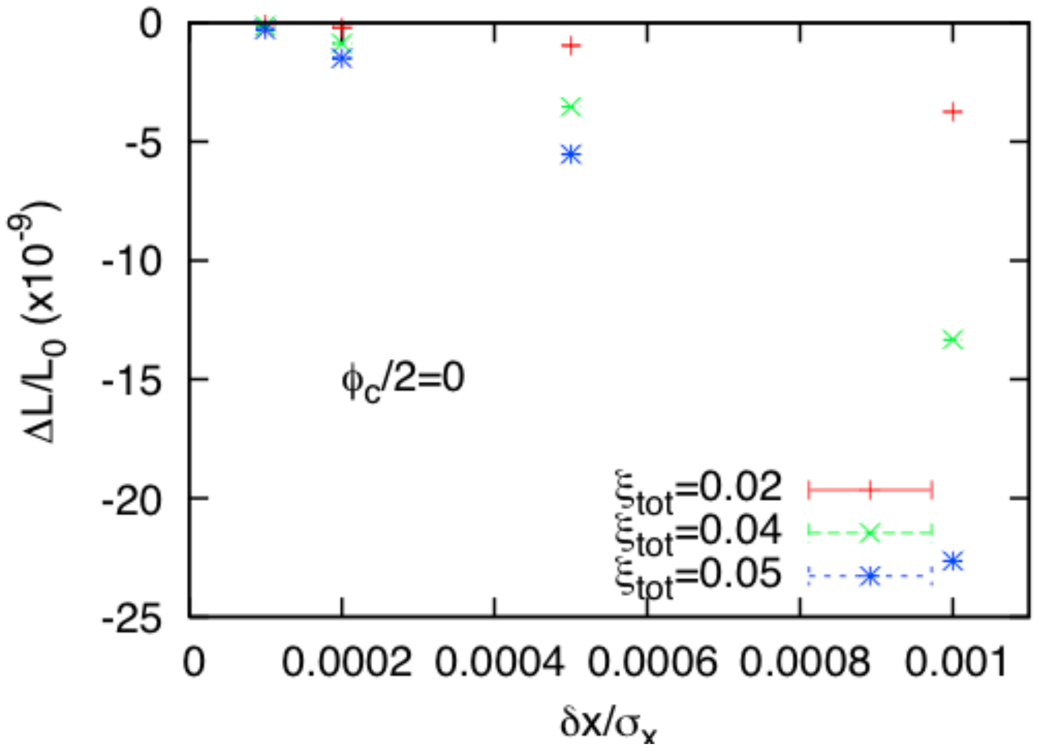
External noise II

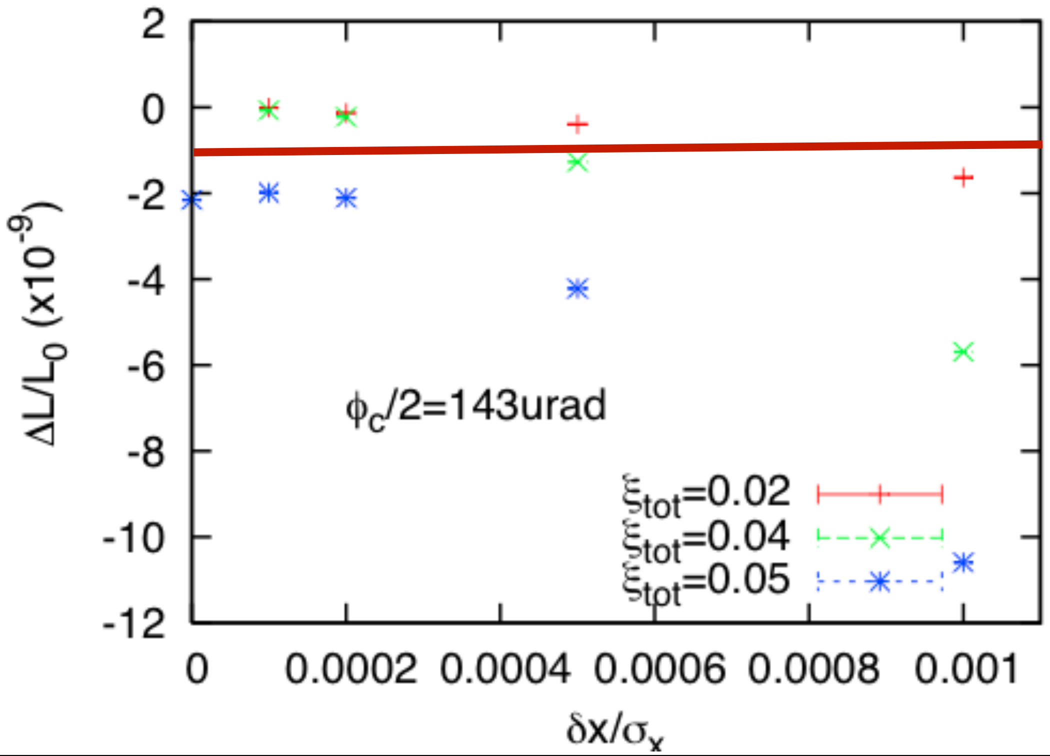
- Weak-strong model, in which strong beam modulates with $\delta x/\sigma_x$ and $\delta y/\sigma_y$, is used.

$$\langle \delta J^2(J) \rangle = \frac{\left(\frac{N_p r_p}{\gamma_p}\delta x\right)^2}{8 - 4/\tau_{cor}} \sum_{k=0}^{\infty} \frac{\sinh\theta (2k+1)^2 G_k^2(J/2\varepsilon)}{\cosh\theta - \cos 2\pi(2k+1)\nu_x}$$

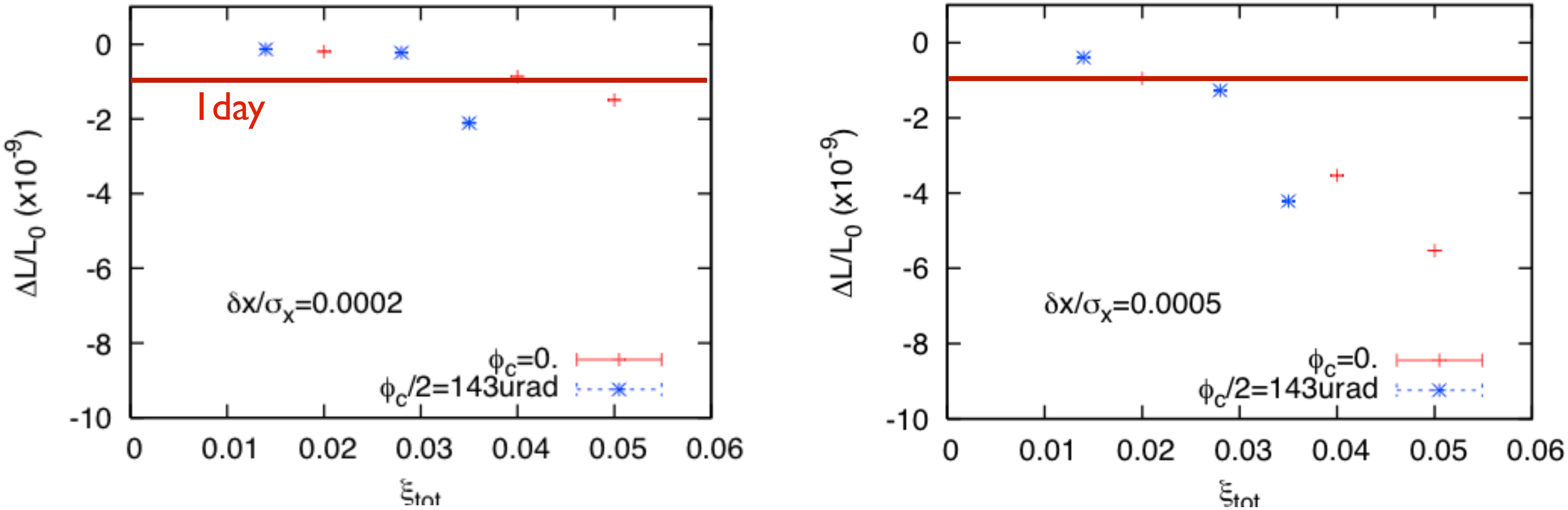
by T. Sen

- Quadratic dependence on $\delta x/\sigma_x$ and ξ.
- Critical noise amplitude, 0.02% for ξ=0.04-0.05.
- Strong beam G=1, weak beam G=0 in the Alexahin's model. K=0.023-0.4 at $\delta x/\sigma_x$ =0.01.





External Noise & crossing angle



- ξ dependence in the critical $\delta x/\sigma_x$=0.02%-0.05%.
- The luminosity decrement for the noise is independent to the crossing angle. The decrement due to the crossing angle is dominant at ξ_{tot}>0.035.

Intrabeam

- Emittance growth time is 105 hours (hor.)for the nominal LHC, $\varepsilon=5\text{x}10^{-10}$ m, and $N_p=1.15\text{x}10^{11}$.
- The growth time is 40 hour for ξtot=0.02 ($\varepsilon=2.7\text{x}10^{-10}$ m, and $N_p=1.63\text{x}10^{11}$). It is 16 hour for ξtot=0.05, when Np/ε x2.5
- Incoherent fluctuation directory causes an emittance growth.

$$\frac{\delta\varepsilon}{\varepsilon} \approx \frac{\delta x^2}{\sigma_x^2}$$

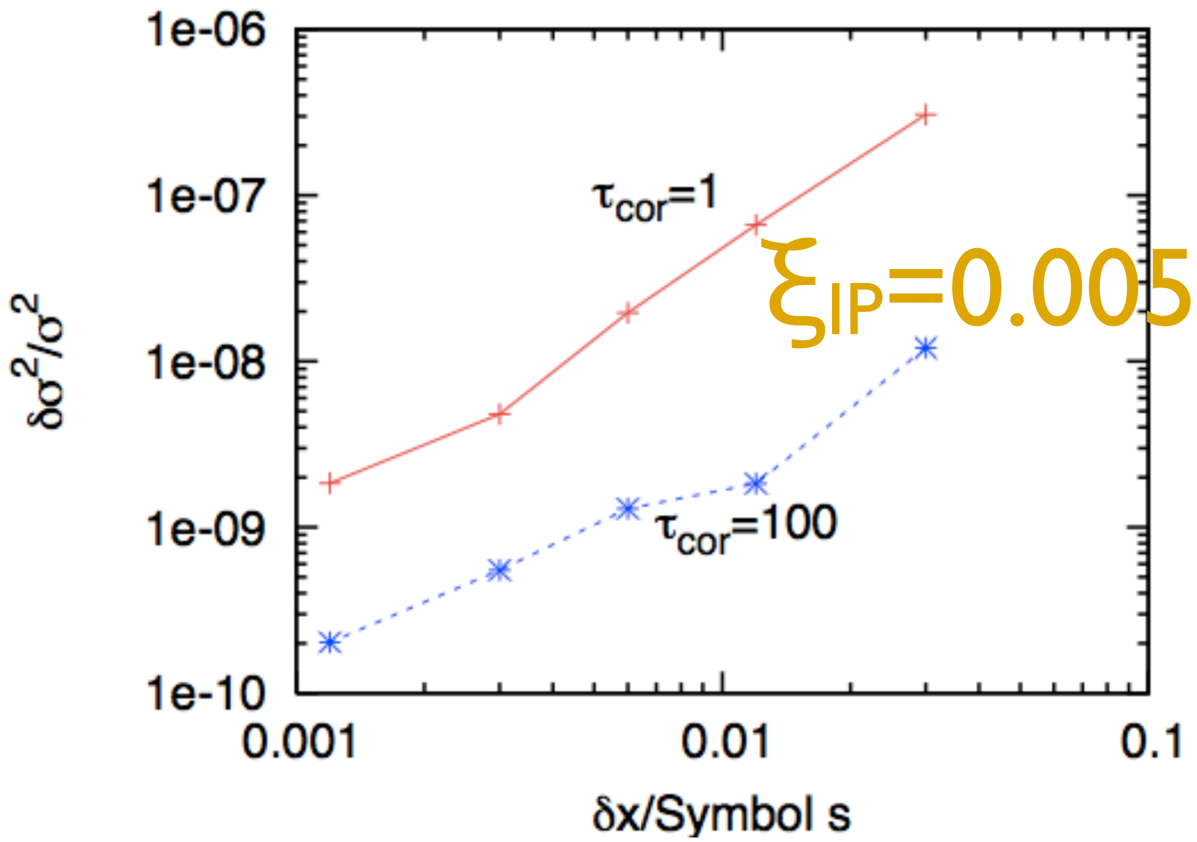
- Fluctuations give unexpected emittance growth in a nonsolvable case as is studied in KEKB (very high ξ>0.1).

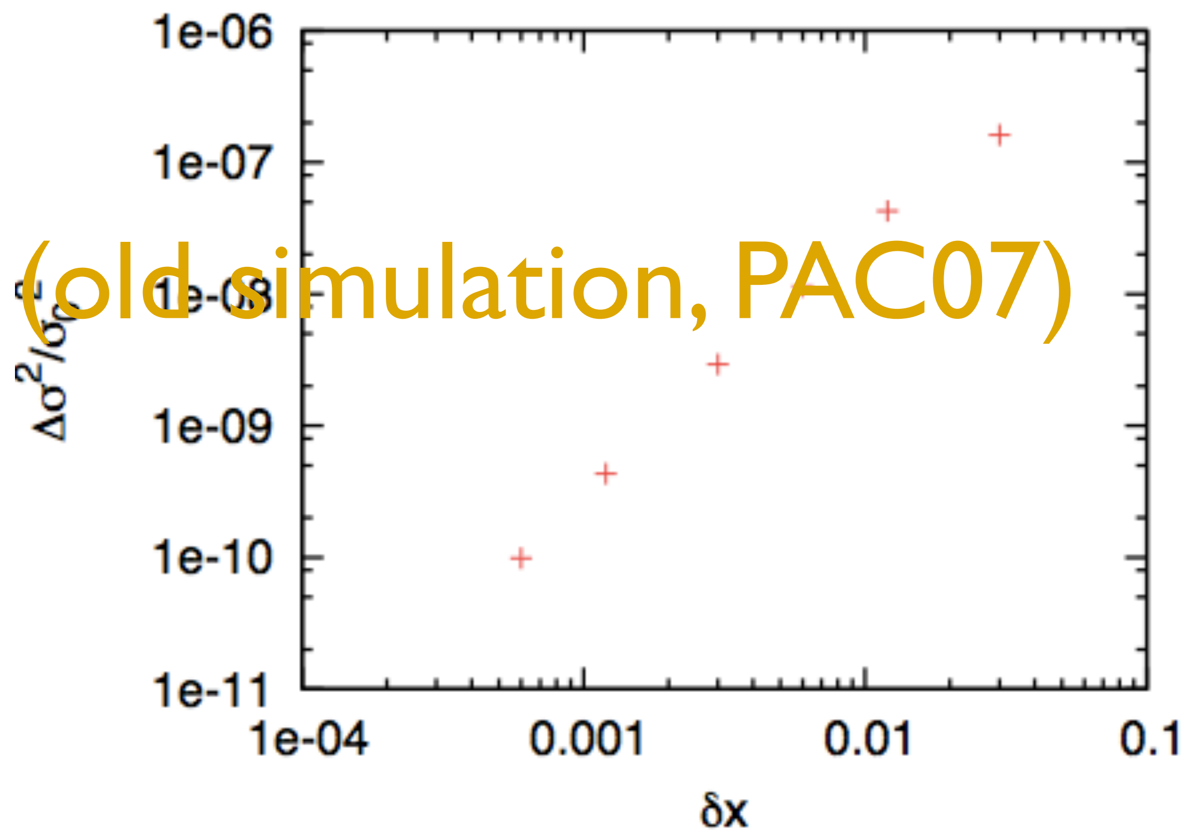
Summary and conclusions

- We discuss the beam-beam limit in LHC with every possible mechanism.
- The results show a hurdle ξ_{tot} ~0.035-0.05 for offset error or crossing angle.
- The superperiodicity, assumed in this presentation, is breaking in LHC. More resonances appear. π separation is worse than Superperiodicity 2 in my experience.
- Is $\xi_{tot}>0.05$ possible? Probably yes, if perfect machine is constructed. Squeeze beta, aperture, superperiodicity, lattice nonlinearity, one more collision point...
- Sensitivity for errors and noises increases $\sim\xi_{tot}^2$. $\delta x/\sigma_x \sim 0.02\%$ is tolerance for fast turn-by-turn noise.

Thank you for your attention

ξ_{IP}=0.005 (old simulation, PAC07)





- G=1/τ_{cor} is

Diffusion in the head-on collision

- Radiation excitation enhances beam enlargement.
- In Gaussian model, enlargement is small.
- Accuracy of PIC is excellent as far as diffusion.