

An Alternative 1D Model for CSR with Chamber Shielding

Demin Zhou

Accelerator Physics Group, Accelerator Laboratory, KEK

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Outline

1. Introduction

2. Numerical calculation of CSR

3. “1D” model for CSR power spectrum

4. 1D model for simulation of CSR effects

5. Summary

1. Introduction

Parabolic equation (PE) in Frenet-Serret coordinate system:

$$\frac{\partial \vec{E}_{\perp}}{\partial s} = \frac{i}{2k} \left[\nabla_{\perp}^2 \vec{E}_{\perp} - \frac{1}{\epsilon_0} \nabla_{\perp} \rho_0 + 2k^2 \left(\frac{x}{R(s)} - \frac{1}{2\gamma^2} \right) \vec{E}_{\perp} \right]$$

T. Agoh, PRST-AB 7, 054403 (2004).

Longitudinal field and impedance:

$$E_s = \frac{i}{k} \left(\nabla_{\perp} \cdot \vec{E}_{\perp} - \mu_0 c J_s \right) \quad Z(k) = -\frac{1}{q} \int_0^{\infty} E_s(x_c, y_c) ds$$

Contributors:

- G. Stupakov and I.A. Kotelnikov (Mode expansion, PRST-AB 2003, 2009)
- T. Agoh and K. Yokoya (Mesh and Eigenfunction expansion, PRST-AB 2004, 2009)
- D.R. Gillingham and T.M. Antonsen (Time-domain PE, PRST-AB 2007)
- K. Oide (Mesh + Eigen solver, PAC 2009)
- D. Zhou (Mesh + Finite difference, JJAP 2012)
- L. Wang (Mesh + Finite element, 2012)

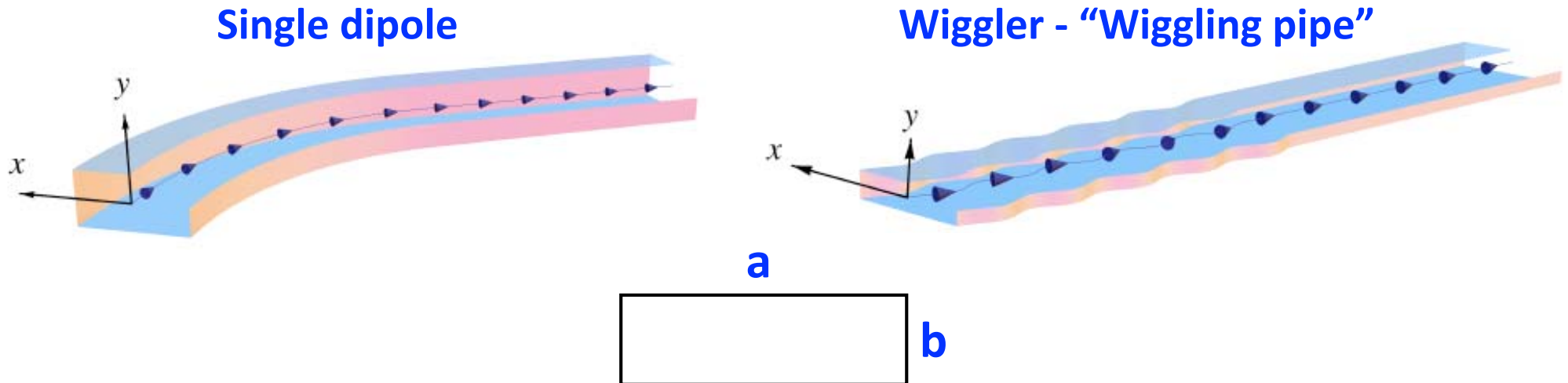
1. Introduction

CSRZ code (D. Zhou): Uniform rectangular cross-section

Field separation:

$$\vec{E}_\perp = \vec{E}_\perp^r + \vec{E}_\perp^b \rightarrow \frac{\partial \vec{E}_\perp^r}{\partial s} = \frac{i}{2k} \left[\nabla_\perp^2 \vec{E}_\perp^r + 2k^2 \left(\frac{x}{R(s)} - \frac{1}{2\gamma^2} \right) (\vec{E}_\perp^r + \vec{E}_\perp^b) \right]$$

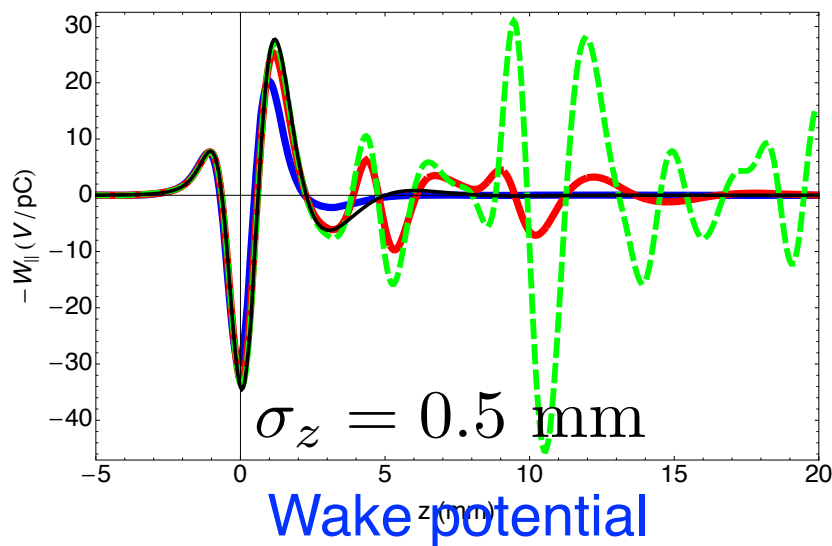
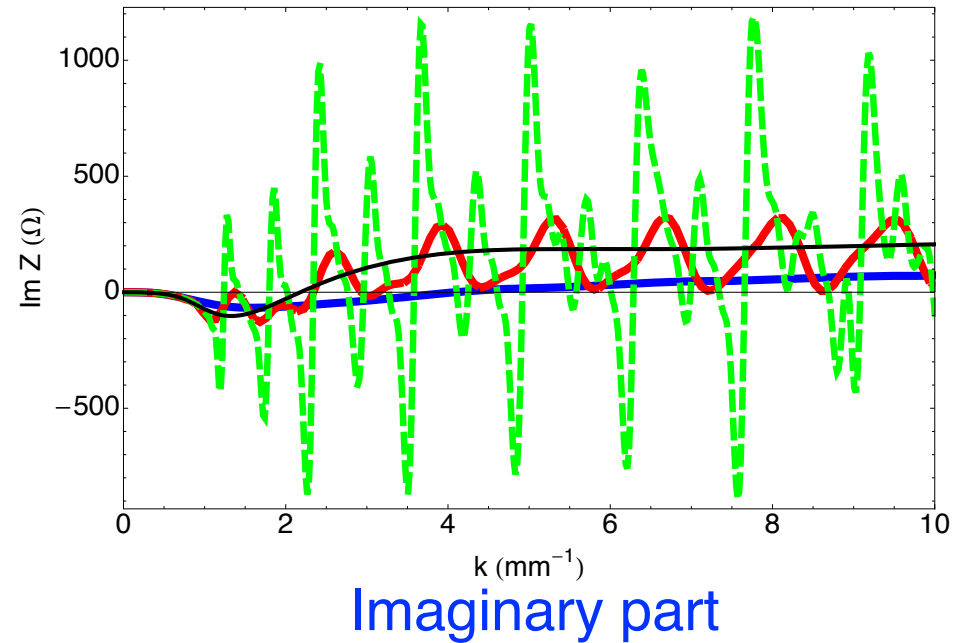
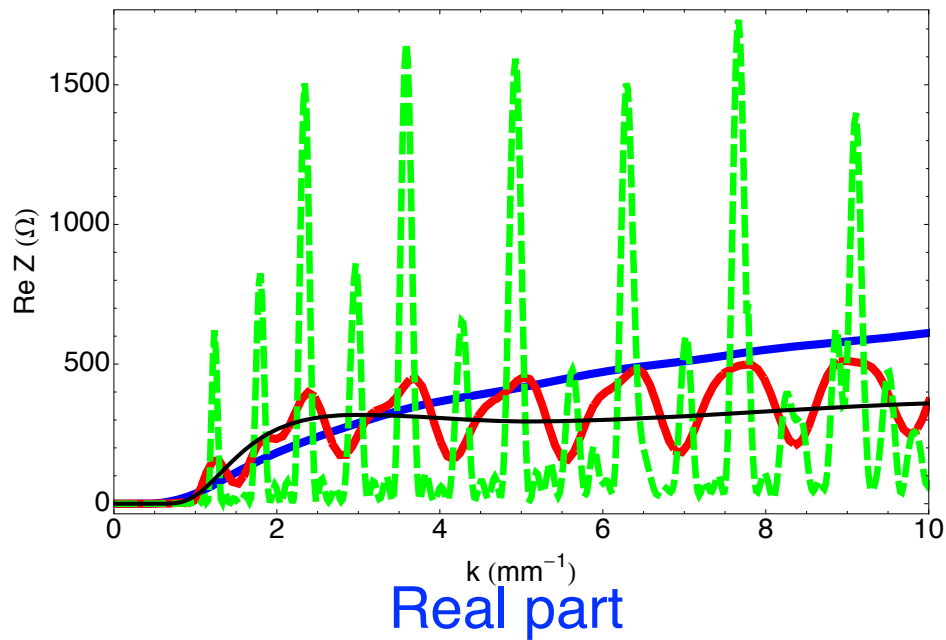
The curvature is variable (a series of dipoles, wiggler, etc.):



2. Numerical calculation of CSR

A single bend: excited modes of toroidal chamber

$a/b=60/30$ mm, $R=5$ m, $L_{\text{bend}}=0.5/2/8$ m



$$L_c \approx 2\sqrt{2Rx_b}$$

Blue solid: $L_{\text{bend}}=0.5$ m

Red dashed: $L_{\text{bend}}=2$ m

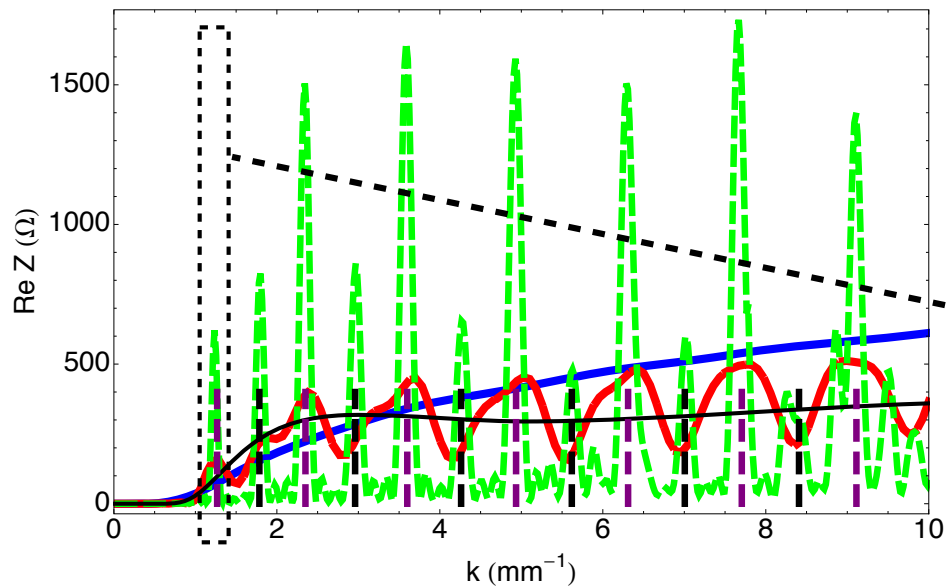
Green dotted: $L_{\text{bend}}=8$ m

Black solid: Parallel-plates model

2. Numerical calculation of CSR

Resonance poles = Eigen modes

$a/b=60/30$ mm, $L_{\text{bend}}=8$ m, $R=5$ m

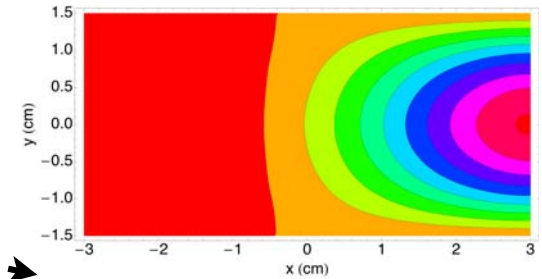


Freq. & index

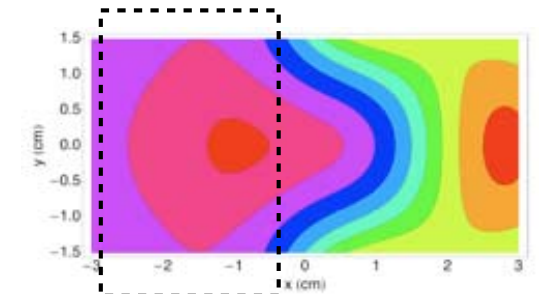
$$k = 1230 \text{ m}^{-1}$$

$$(m, p) = (0, 1)$$

Im. E_x^r :

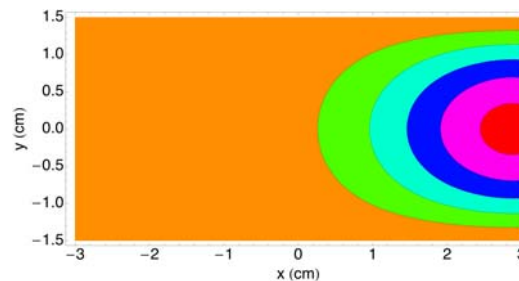


Re. E_x^r :



Overtaking fields

Mode pattern:

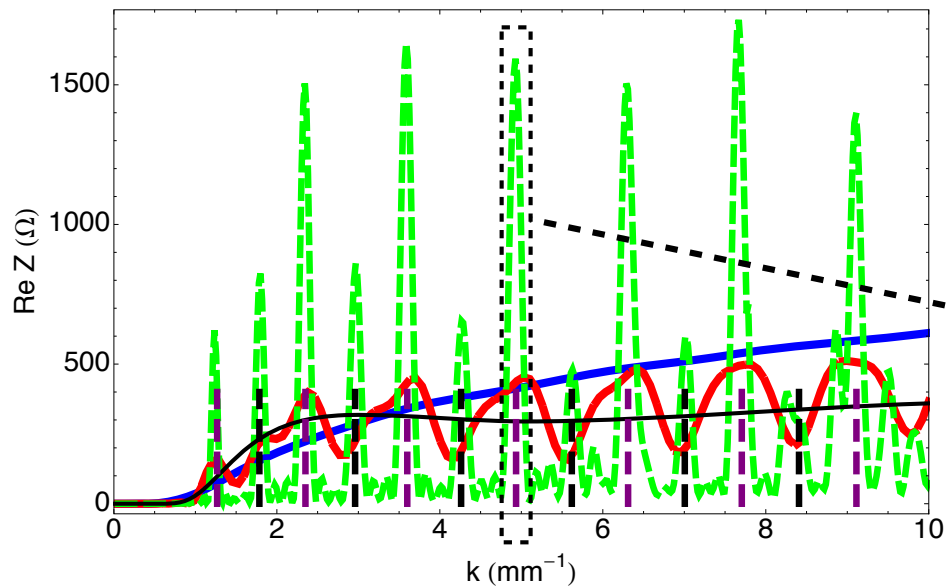


$$\mathbf{E}_x \text{ mode: } E_x(x, y) = E_{x0} \text{Ai} \left(k_y^2 \kappa^2 - x/\kappa \right) \sin \left[k_y \left(y + b/2 \right) \right]$$

2. Numerical calculation of CSR

Resonance poles = Eigen modes

$a/b=60/30$ mm, $L_{\text{bend}}=8$ m, $R=5$ m

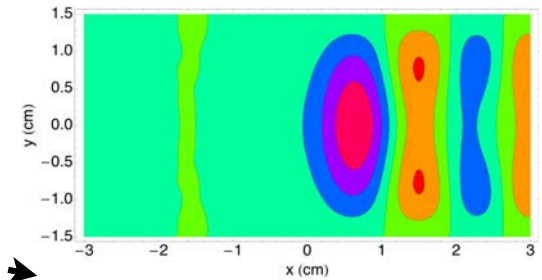


Freq. & index

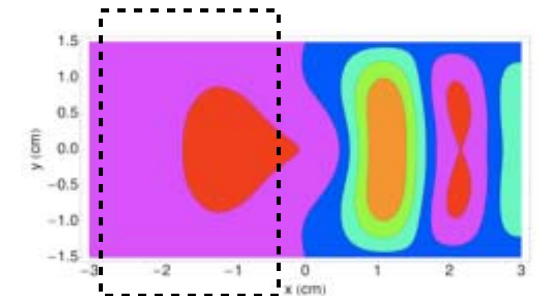
$$k = 4930 \text{ m}^{-1}$$

$$(m, p) = (3, 1)$$

Im. E_x^r :

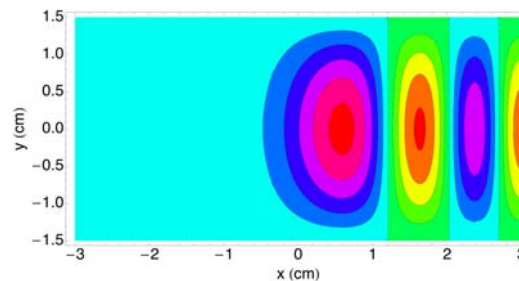


Re. E_x^r :



Overtaking fields

Mode pattern:

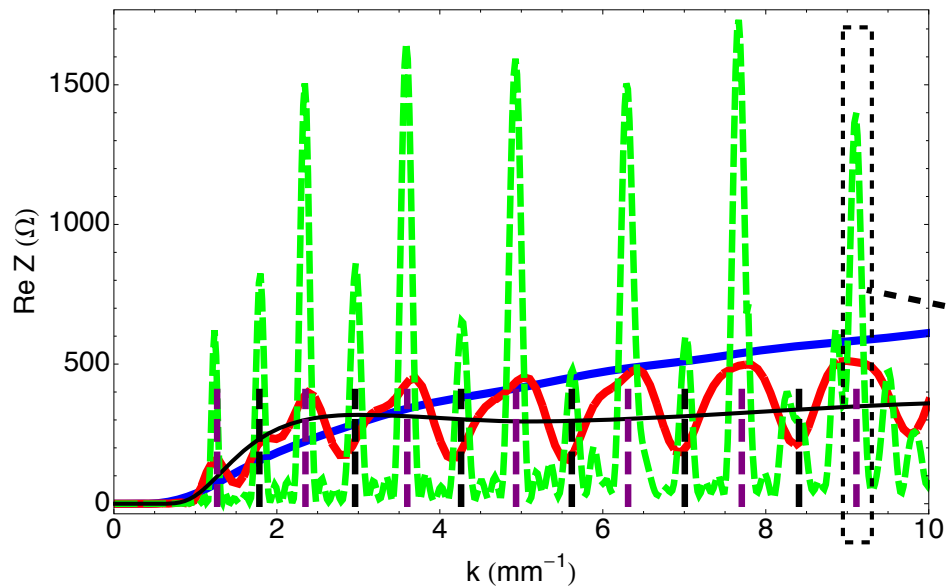


$$\mathbf{E}_x \text{ mode: } E_x(x, y) = E_{x0} \text{Ai} \left(k_y^2 \kappa^2 - x/\kappa \right) \sin \left[k_y \left(y + b/2 \right) \right]$$

2. Numerical calculation of CSR

Resonance poles = Eigen modes

$a/b=60/30$ mm, $L_{\text{bend}}=8$ m, $R=5$ m

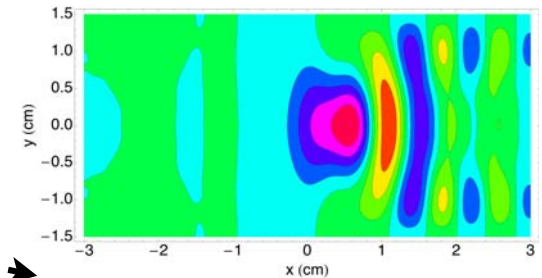


Freq. & index

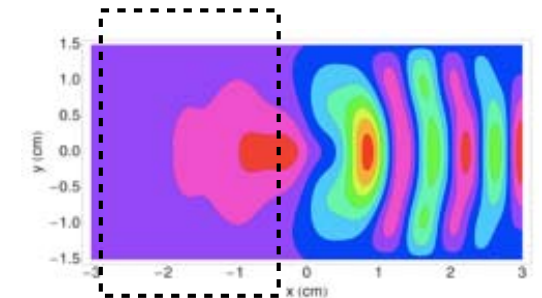
$$k = 9100 \text{ m}^{-1}$$

$$(m, p) = (6, 1)$$

Im. E_x^r :

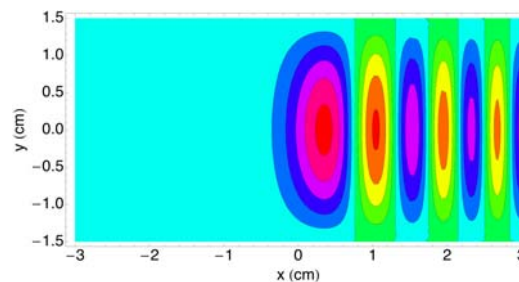


Re. E_x^r :



Overtaking fields

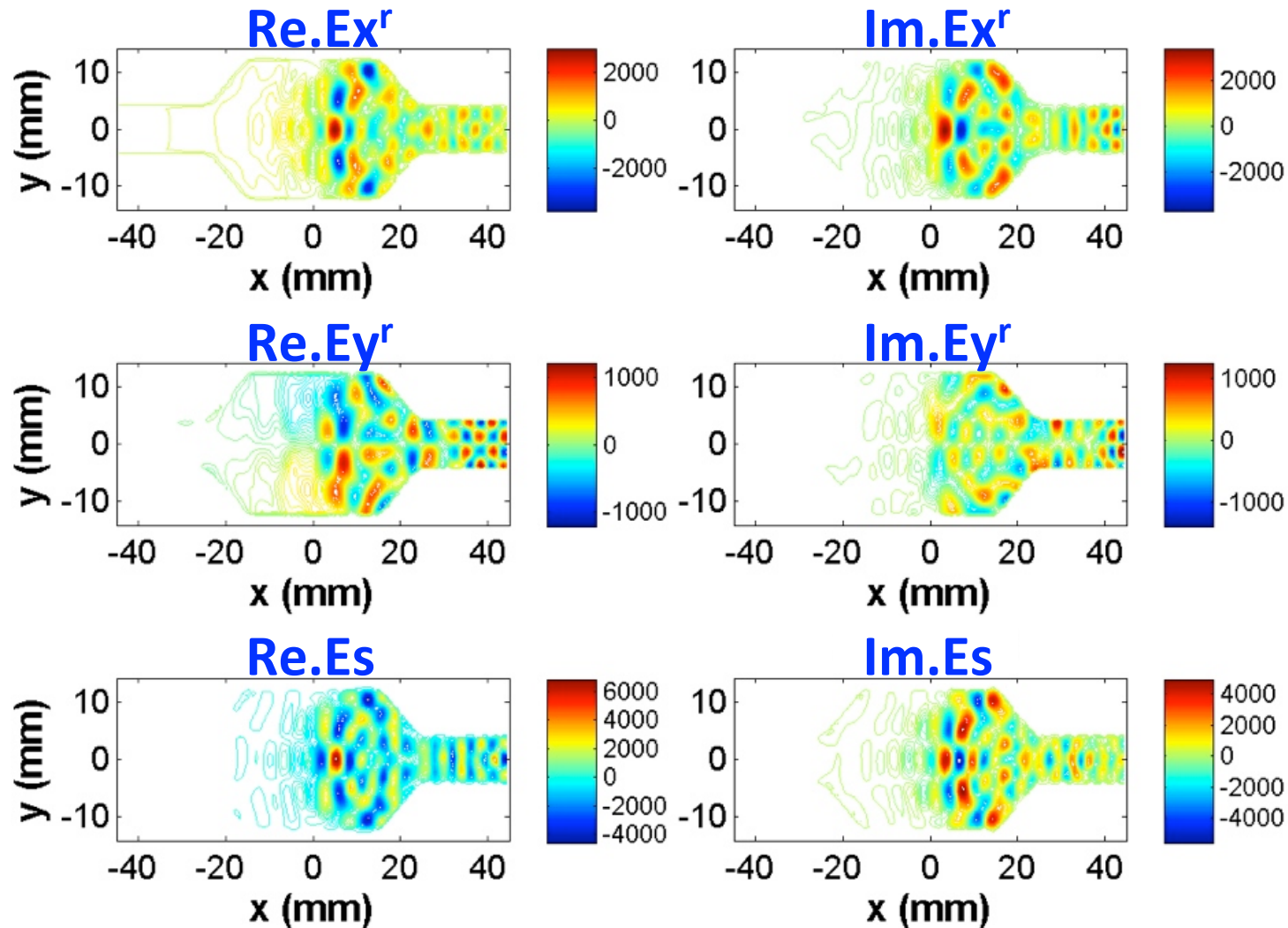
Mode pattern:



$$\mathbf{E}_x \text{ mode: } E_x(x, y) = E_{x0} \text{Ai} \left(k_y^2 \kappa^2 - x/\kappa \right) \sin \left[k_y \left(y + b/2 \right) \right]$$

2. Numerical calculation of CSR

Arbitrary cross-section: Finite element technique + parabolic equation (L. Wang@SLAC)

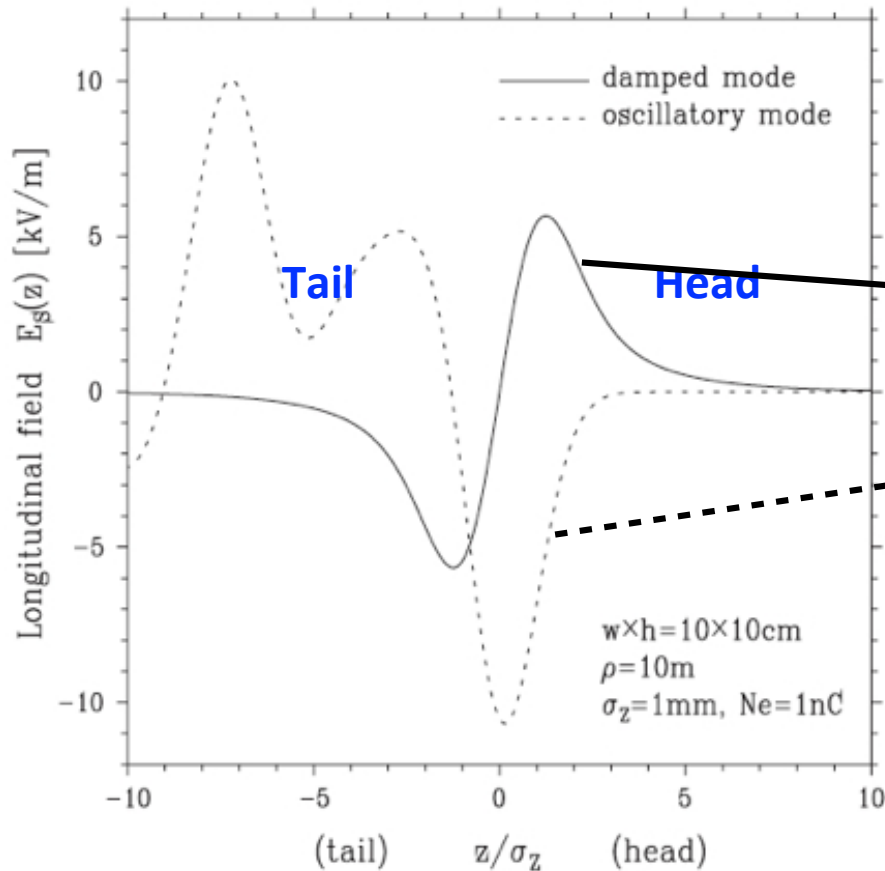


Courtesy of L. Wang

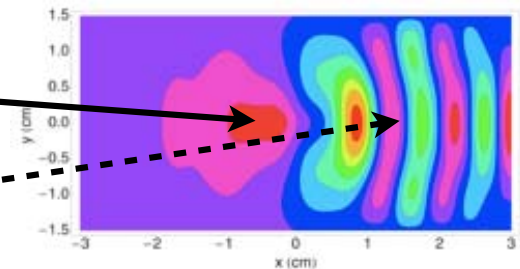
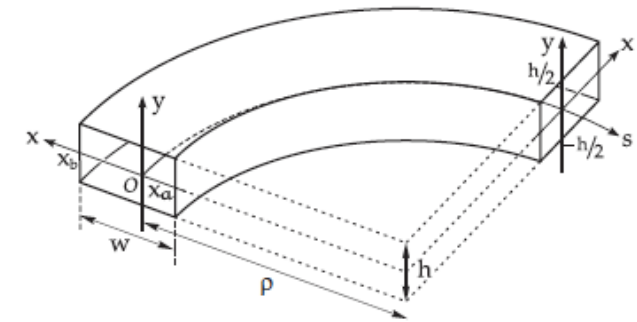
2. Numerical calculation of CSR

CSR fields can be decomposed to a sum of propagating (**oscillatory and trailing**) and decaying (**damped and overtaking**) waves in a toroidal waveguide [Agoh (2009)].

Long. wakefields for a Gaussian bunch

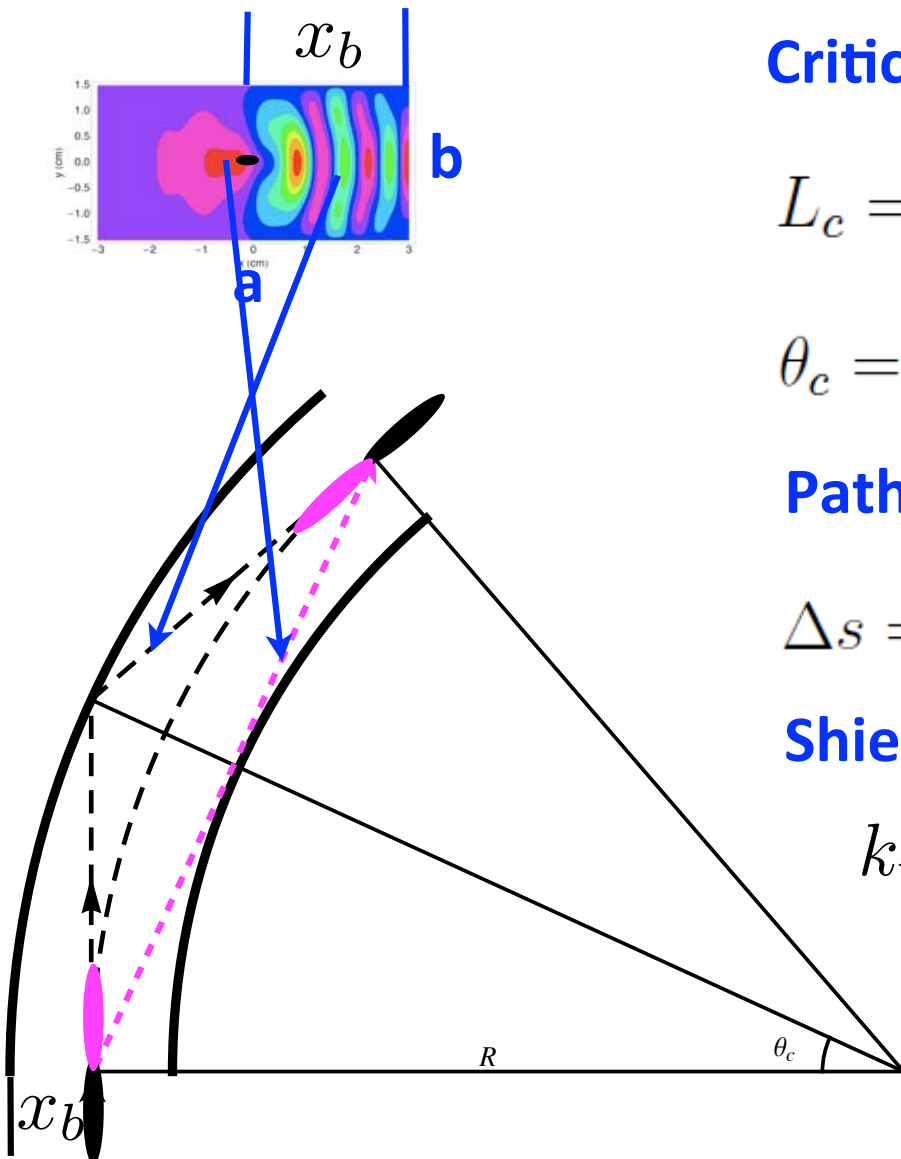


Geometry



2. Numerical calculation of CSR

Outer-wall reflection can be well approximated by a geometric model [Derbenev (1995), Carr (2001), Sagan (2009), Oide (2010)]



Critical length (Catch-up distance):

$$L_c = 2R\theta_c \approx 2\sqrt{2Rx_b} \quad x_b \ll R$$

$$\theta_c = \text{ArcCos}(R/(R+x_b)) \approx \sqrt{2x_b/R}$$

Path difference:

$$\Delta s = 2R(\text{Tan}(\theta_c) - \theta_c) \approx \frac{4}{3}\sqrt{\frac{2x_b^3}{R}}$$

Shielding threshold:

$$k_{th} = \pi\sqrt{R/b^3}$$

Y. S. Derbenev, et al., TESLA FEL-Report 1995-05 (1995).

G. L. Carr, et al., PAC'01, p. 377 (2001).

D. Sagan, et al., PRST-AB 12, 040703 (2009).

K. Oide, Talk at CSR mini-workshop, Nov. 08, 2010.

|| D. Zhou, et al., Jpn. J. Appl. Phys. 51 (2012) 016401.

3. "1D" model for CSR power spectrum

CSR measurements at NSLS VUV ring



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Nuclear Instruments and Methods in Physics Research A 463 (2001) 387–392

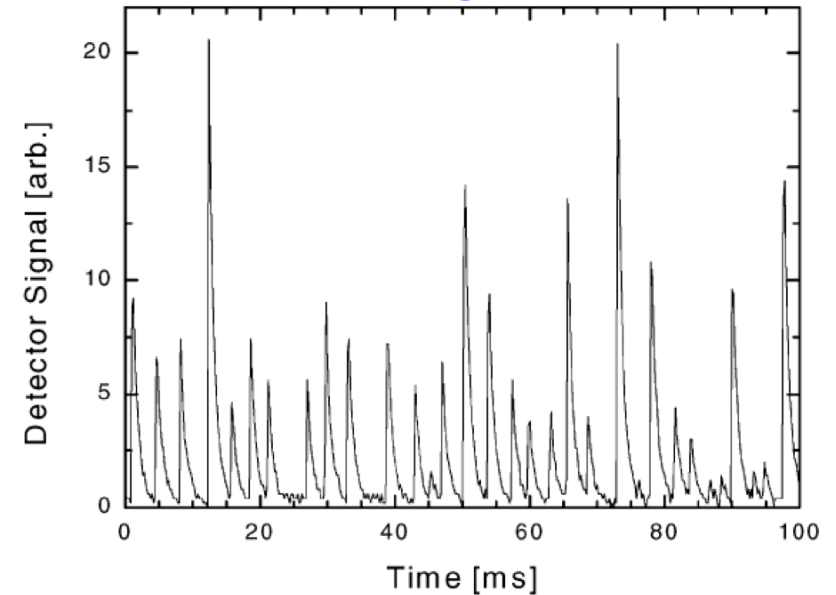


www.elsevier.nl/locate/nima

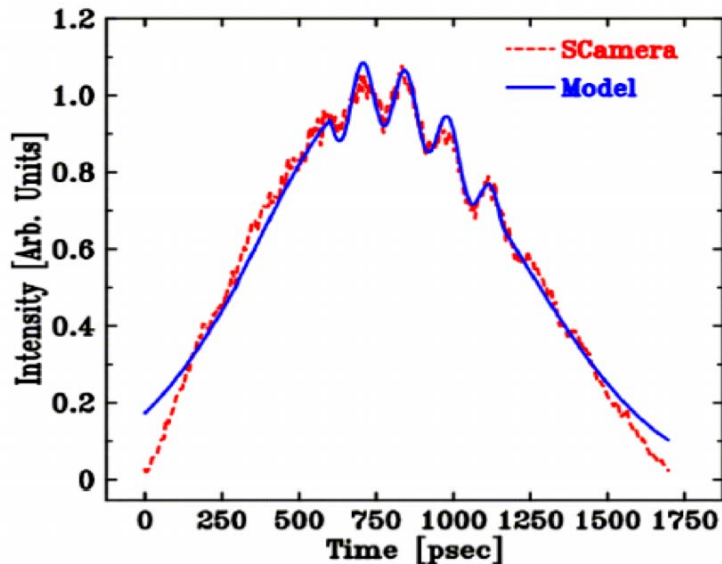
Observation of coherent synchrotron radiation from the NSLS VUV ring

G.L. Carr^{a,*}, S.L. Kramer^a, J.B. Murphy^a, R.P.S.M. Lobo^b, D.B. Tanner^b

FIR signal

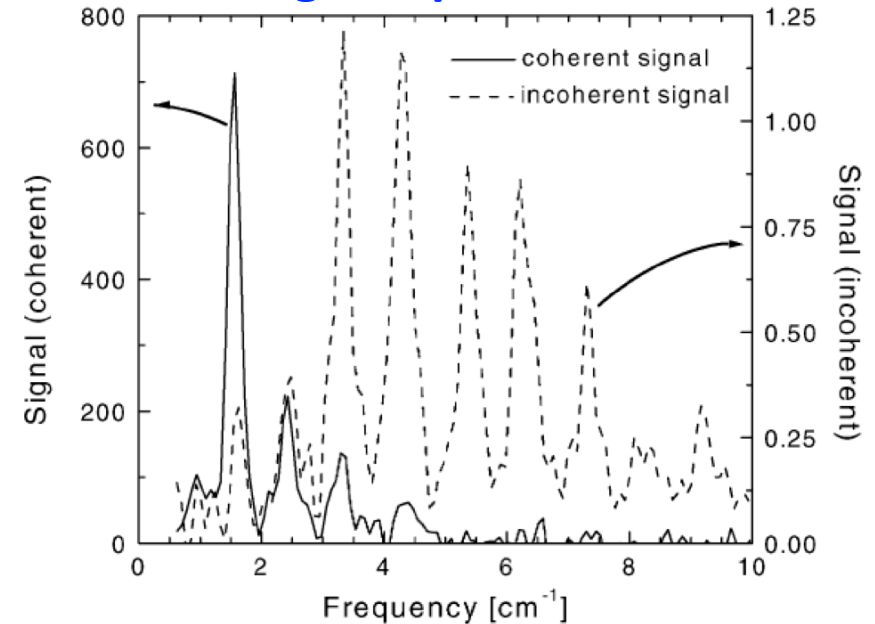


Microbunching@Streak Camera



S. Kramer, EPAC 2002

Signal spectrum



3. “1D” model for CSR power spectrum

Energetic spectrum of Coherent SR:

$$\left. \frac{dU(k)}{dk} \right|_{\text{bunch}} = \left[N + N(N-1) \left| \tilde{\lambda}(k) \right|^2 \right] \left. \frac{dU(k)}{dk} \right|_{\text{single}}$$

Steady-state free-space model:

$$\left. \frac{dU(k)}{dk} \right|_{\text{single}} = \frac{e^2}{2\pi\epsilon_0} 3^{\frac{1}{6}} \Gamma(2/3) (kR)^{\frac{1}{3}} \quad \begin{aligned} k &\equiv \frac{\omega}{c} = \frac{2\pi}{\lambda} \\ k &\ll k_c = 3\gamma^3 / (2R) \end{aligned}$$

Generic relation between SR impedance and energetic spectrum:

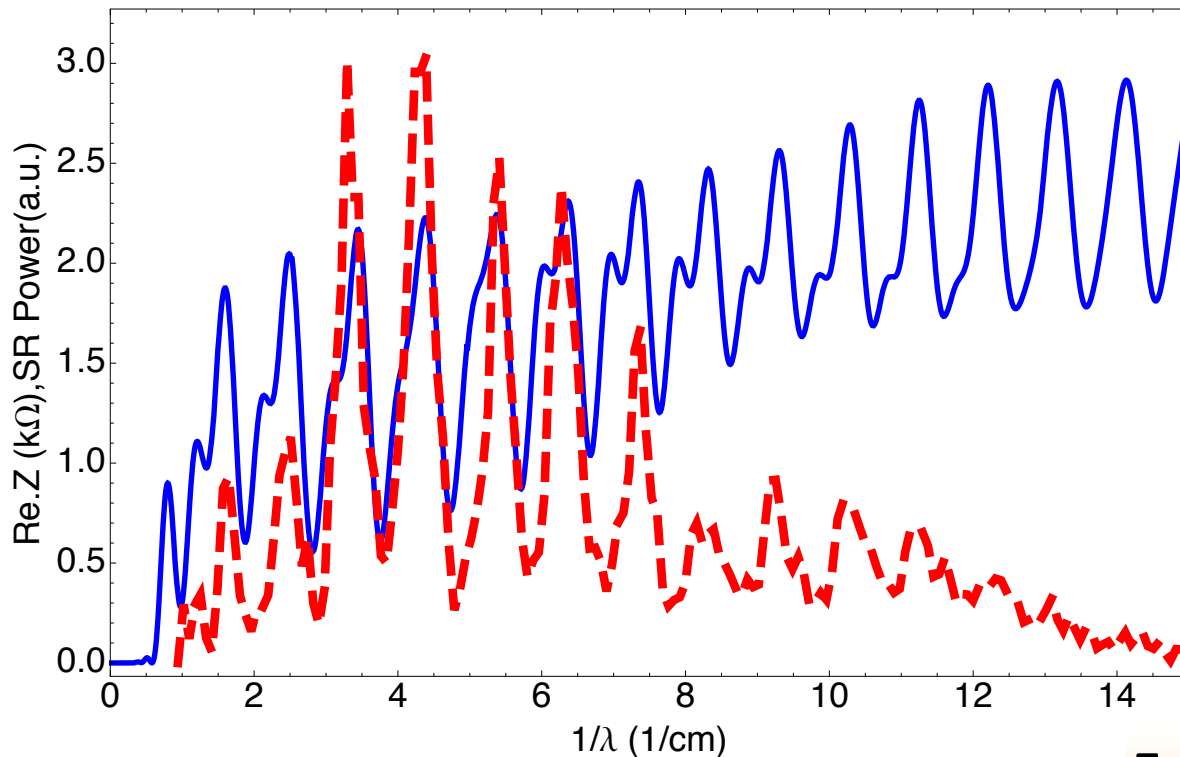
$$\left. \frac{dU(k)}{dk} \right|_{\text{single}} = \frac{e^2 c}{\pi} \text{Re}. Z_{\parallel SR}(k)$$

$Z_{\parallel SR}(k)$ is generic, representing impedance of a single or a series of bends. It depends on orbit curvature, magnet length, and chamber dimensions.

3. "1D" model for CSR power spectrum

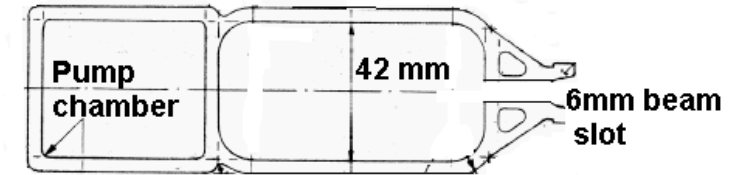
CSR measurements at NSLS VUV ring

$a/b=80/42$ mm, $L_{\text{bend}}=1.5$ m, $R=1.91$ m

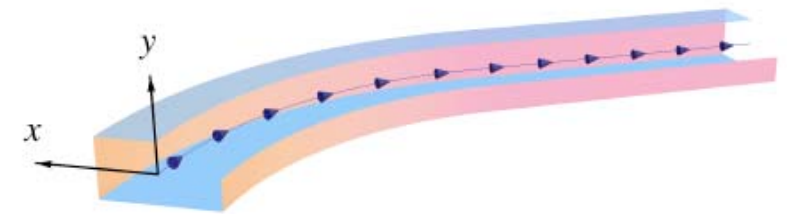


$$L_{\text{bend}} > L_c = 0.8 \text{ m}$$

Chamber cross section



Model for calculation



Blue solid: SR impedance

Red dashed: Measured ISR spectrum
(Data provided by S.L. Kramer)

Excellent agreements in peak positions and widths!

The discrepancy in amplitude at low- and high-frequency parts is attributed to the transfer function of the detection system. See next page.

3. “1D” model for CSR power spectrum

Relation between predicted radiation spectrum and measured signal spectrum:

$$\left. \frac{dU(k)}{dk} \right|_{\text{experiment}} = T(k) \left. \frac{dU(k)}{dk} \right|_{\text{bunch}}$$

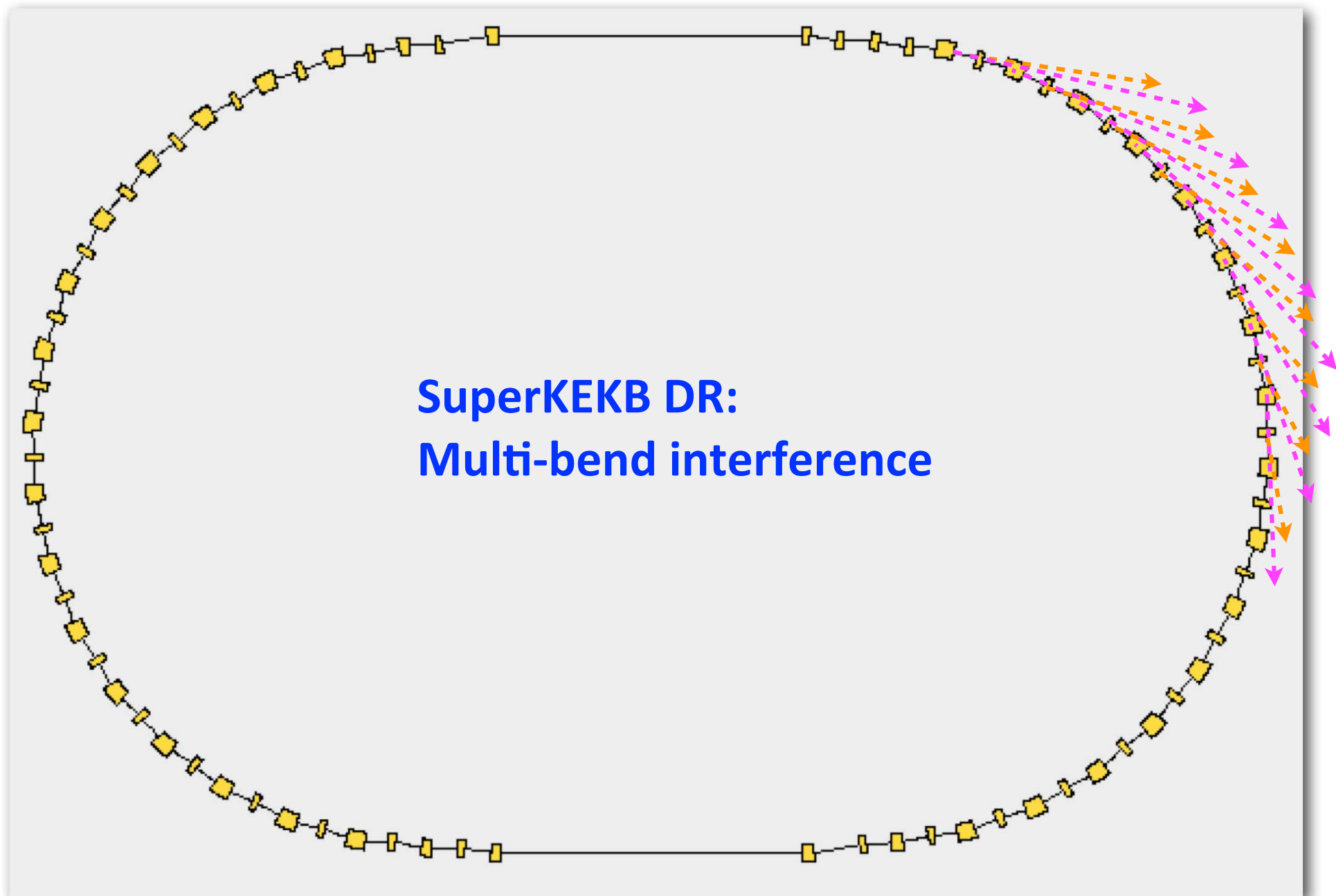
$T(k)$ is a transfer function characterizing the response of the beamline and detection system (Optical efficiency, low-pass filter, etc.).

Two directions of studies on CSR:

1) Assume that $dU(k)/dk$ for single particle and $T(k)$ are known, measuring $dU(k)/dk$ for a bunch give the information of the bunch's profiles → **Beam diagnostics**;

2) Assume that $dU(k)/dk$ for single particle, $T(k)$, and bunch profile are known, one can **design/optimize a THz light source**.

3. “1D” model for CSR power spectrum

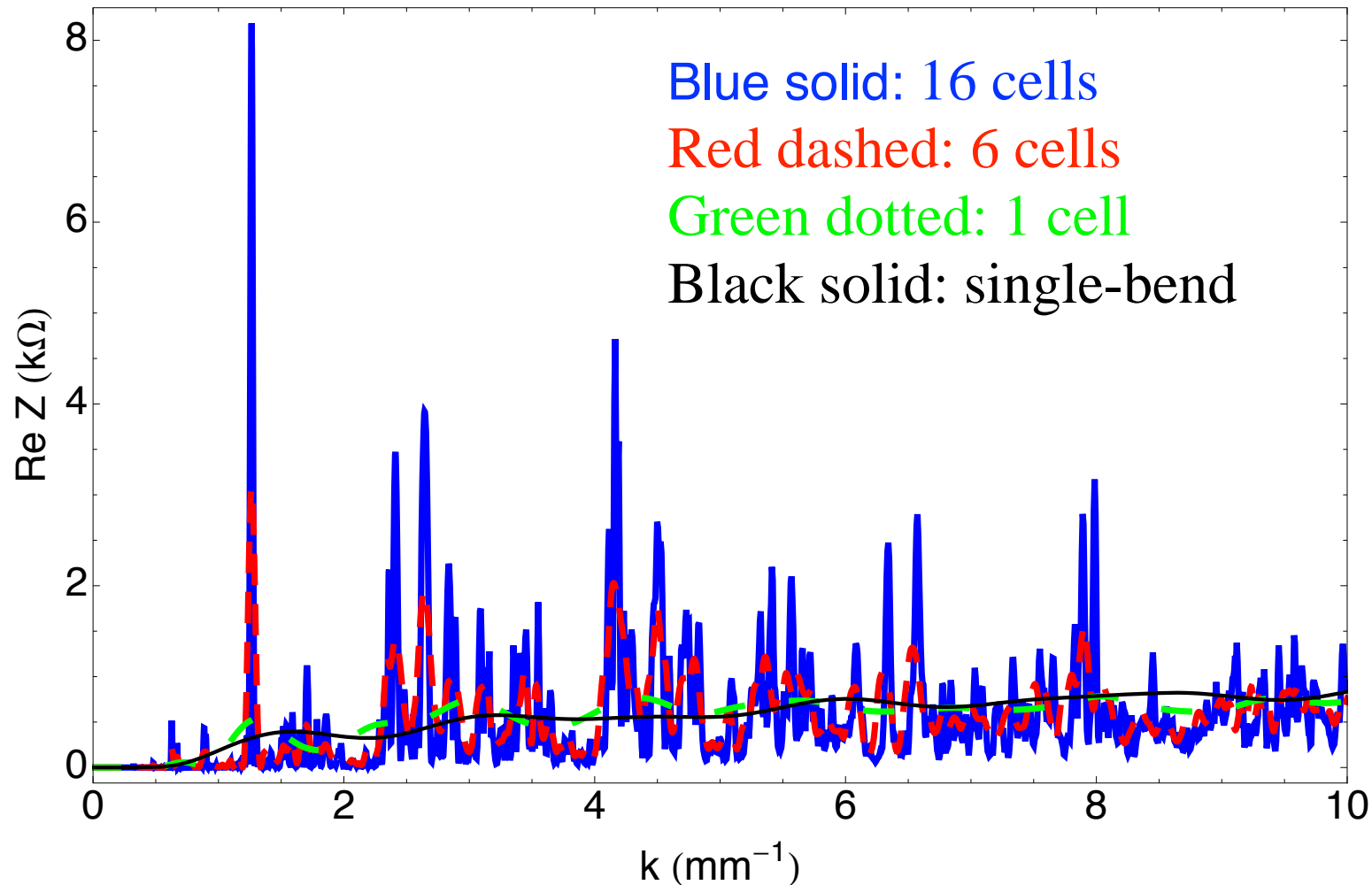


3. “1D” model for CSR power spectrum

Expected SR spectrum at SuperKEKB damping ring

$a/b=34/34$ mm, $L_{\text{bend}}=0.74/0.29$ m, $R=2.7/-3$ m (reverse bends)

$L_{\text{drift}}=0.9$ m, $N_{\text{cell}}=1/6/16$



4. 1D model for simulation of CSR effects

A popular 1D model for CSR w/o shielding :

$$\frac{\partial W_{\parallel b}(z, s)}{\partial s} = T_1(z, R, s) + T_2(z, R, s)$$

$$T_1(z, R, s) = \kappa \int_{z-z_L}^z \frac{d\lambda(z')}{dz'} \left(\frac{1}{z-z'} \right)^{1/3} dz'$$

$$T_2(z, R, s) = \kappa \frac{\lambda(z-z_L) - \lambda(z-4z_L)}{z_L^{1/3}} \quad z_L(R, s) = s^3 / (24R^2)$$
$$\kappa = -\frac{1}{4\pi\epsilon_0} \frac{2}{(3R^2)^{1/3}}$$

Challenges when applied to numerical simulations:

- Derivative of line density
- Singularity at $z-z'=0$

4. 1D model for simulation of CSR effects

Assume a point-charge distribution of $\lambda(z) = \delta(z)$

$$\frac{\partial w_{\parallel b}(z, s)}{\partial s} = -\kappa \left[\frac{1}{3} \left(\frac{1}{z} \right)^{4/3} H(z) H(z_L - z) + \frac{\delta(z - 4z_L)}{z_L^{1/3}} \right]$$

$$\frac{\partial Z_{\parallel b}(k, s)}{\partial s} = \kappa \left\{ \frac{e^{-4ikz_L}}{z_L^{1/3}} - (ik)^{1/3} \left[\Gamma\left(\frac{2}{3}\right) + \frac{1}{3} \Gamma\left(-\frac{1}{3}, ikz_L\right) \right] \right\}$$

$$\Gamma(a) = \int_0^{\infty} t^{a-1} e^{-t} dt \quad \Gamma(a, z) = \int_z^{\infty} t^{a-1} e^{-t} dt \quad H(z) = \int_{-\infty}^z \delta(t) dt$$

Compare with numerically obtained SR impedance:

$$Z(k) = -\frac{1}{q} \int_0^{\infty} E_s(x_c, y_c) ds \quad \rightarrow \quad \frac{\partial Z(k, s)}{\partial s} = -\frac{1}{q} E_s(x_c, y_c, s)$$

4. 1D model for simulation of CSR effects

An alternative 1D model:

$$F(z) = \frac{c}{2\pi} \int_{-k_N}^{k_N} Z_{\parallel SR}(k) \tilde{\lambda}(k) e^{-ikz} dk$$

$$\frac{\partial F(z, s)}{\partial s} = \frac{c}{2\pi} \int_{-k_N}^{k_N} \frac{\partial Z_{\parallel SR}(k, s)}{\partial s} \tilde{\lambda}(k) e^{-ikz} dk$$

k_N : Nyquist frequency

Features:

- Consider transient effect and chamber shielding
- Natural cutoff at extremely high-frequency part
- Do not need derivative of linear charge density
- Space-charge impedance can be simply added
- Incoherent SR is modeled separately
- Numerical challenges are translated to accurate estimate of beam spectrum

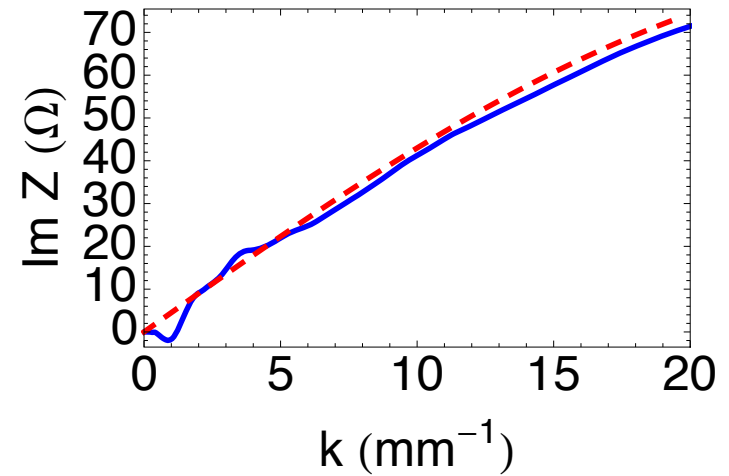
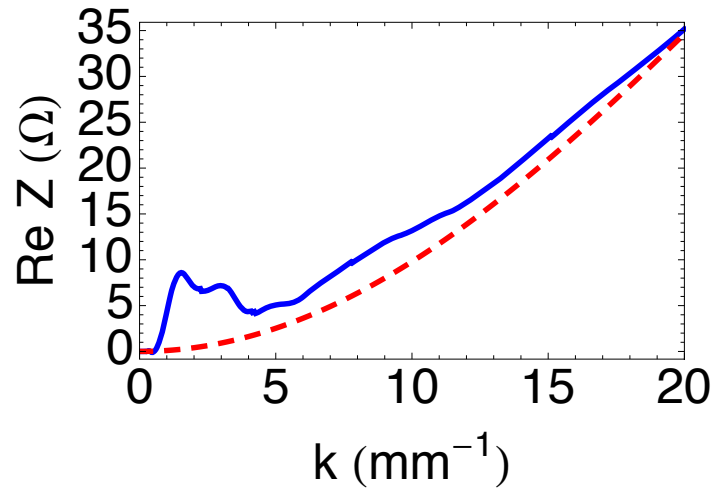
4. 1D model for simulation of CSR effects

A single bend: s -dependent SR impedance

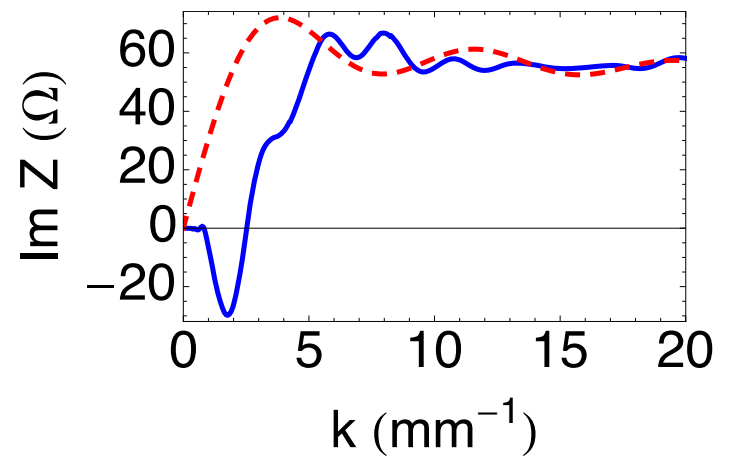
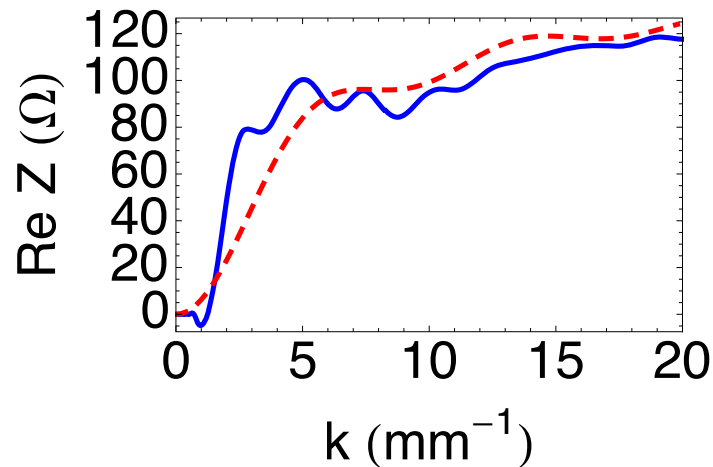
$a/b=33/18$ mm, $R=3.6$ m

Blue solid: Numerical
Red dashed: Analytic

$s=0-0.2$ m



$s=0.2-0.4$ m



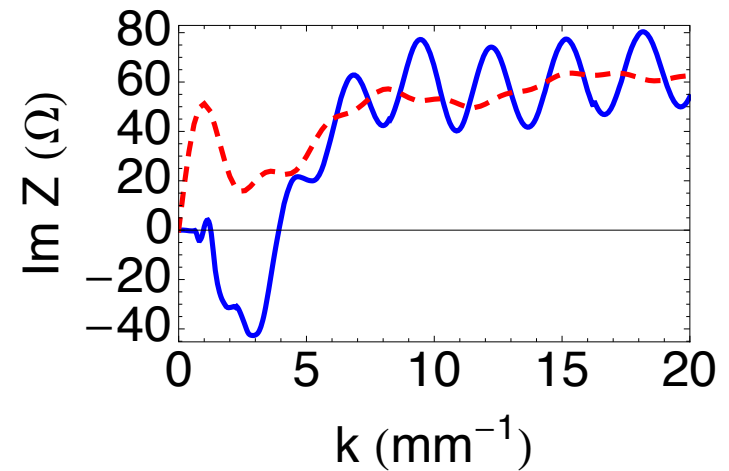
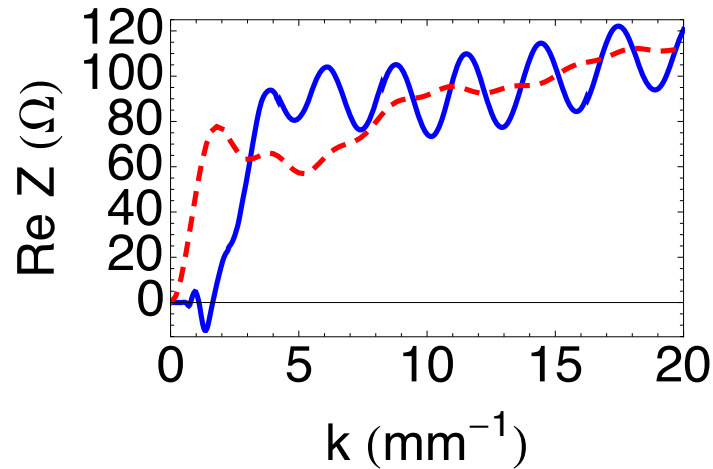
4. 1D model for simulation of CSR effects

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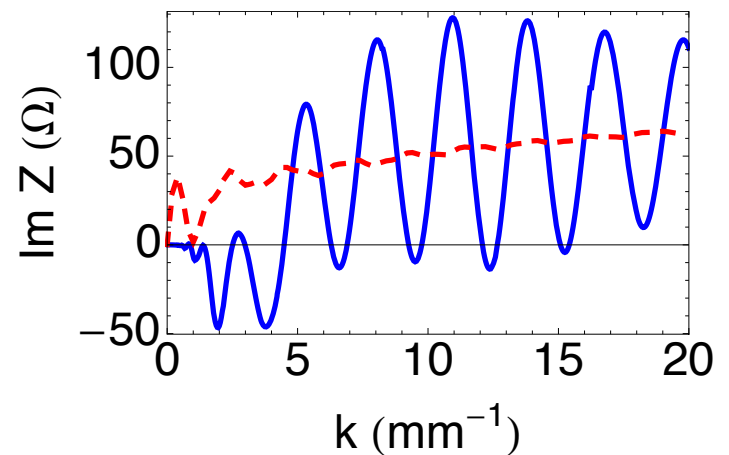
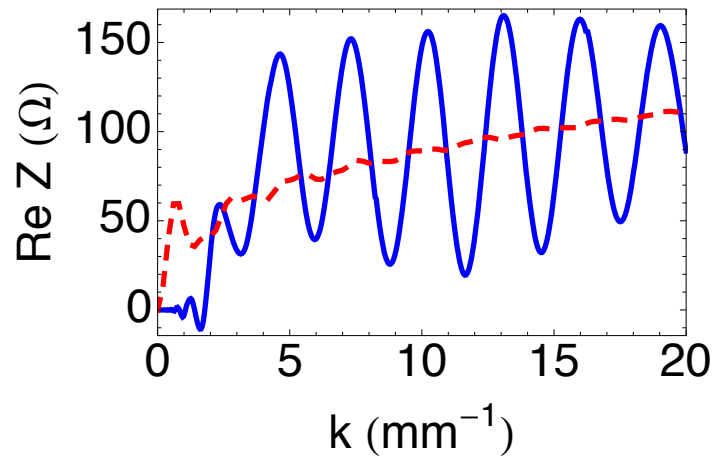
$a/b=33/18$ mm, $R=3.6$ m

Blue solid: Numerical
Red dashed: Analytic

$s=0.4-0.6$ m



$s=0.6-0.8$ m



5. Summary

- **Progresses have been achieved in solving the PE for CSR with chamber shielding. But the power of PE is far from being fully investigated?**
- **In addition to calculation of SR impedance, codes can also be applied to predict CSR power spectrum. Including the beamline of the detection system is also feasible.**
- **It is possible to directly utilize SR impedance in simulation of CSR effects, considering that SR impedance is a distributed impedance along the beam orbit.**

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BNL: S.L. Kramer

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