Symplectic Tracking and Compensation of
Dynamic Field Integrals in Complex Undulator Structures

Johannes Bahrdt, HZB / BESSY II, IPAC 2012, New Orleans

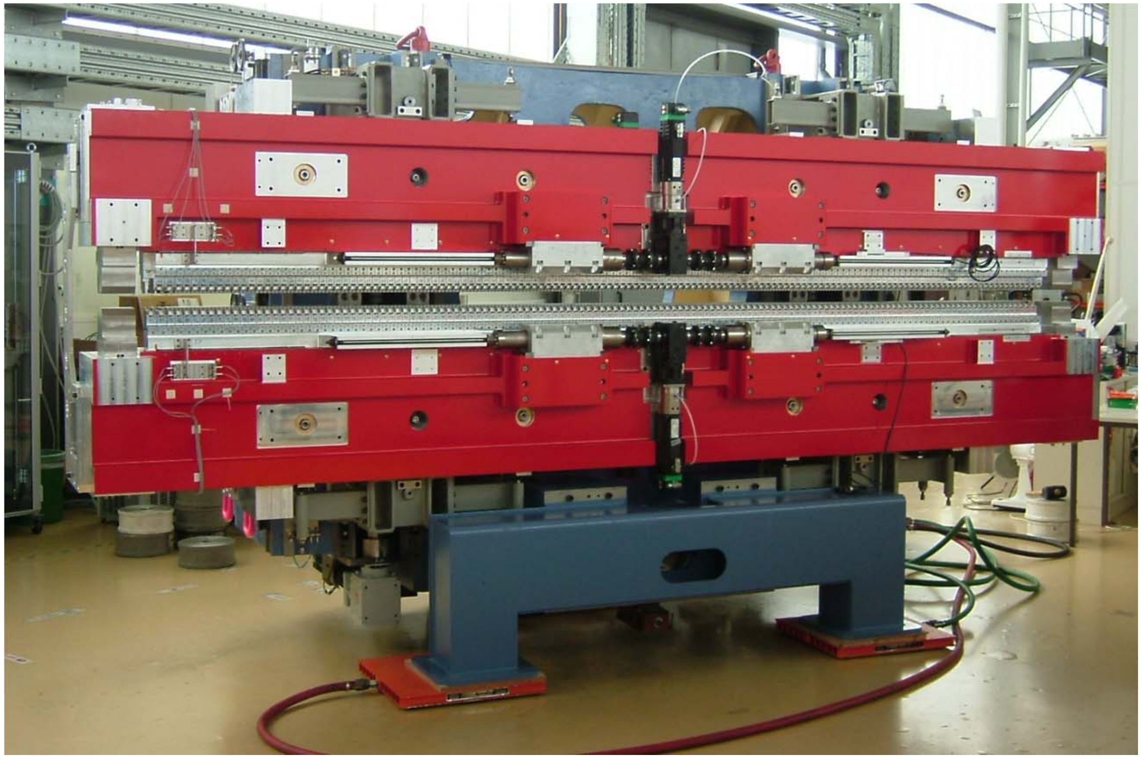
Outline

- Symplectic tracking based on analytic generating functions
- Analytic description of arbitrary undulator fields
- Potential applications to other magnet structures
- Analytic equations for dynamic field integrals
- Benefits and limitations of shimming techniques

J. Bahrdt, G. Wüstefeld, Phys. Rev. ST Accel. Beams 14, 040703 (2011)

Motivation for Symplectic and Fast Tracking Scheme

UE112 APPLE II Undulator at BESSY II



Complicated 3D field
Huge dynamic effects for:
- long period length
- low energy (1.7GeV)

Large operational parameter space:
- energy (gap)
- polarization (phase)
- compensation of beamline effects (universal mode)

Advantages of analytic tracking scheme:

→ extremly fast (CPU time reduction of 1-2 orders of magnitude)

→ full parametrization of 3D fields in all operation modes

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Two approaches for deriving the generating function

Numeric approach

- fast and symplectic, full turn FFAG orbit tracking
  H. Lustfeld, Ph. F. Meads, G. Wüstefeld et al., LINAC 1984
- tracking of superconducting wave length shifter (strong field devices)
  M. Scheer, G. Wüstefeld, EPAC 1992

Analytic approach

- tracking of undulators (nonlinear, weak fields)
  J. Bahrdt, G. Wüstefeld, Proc. of PAC, San Francisco, USA (1991) 266-268
  J. Bahrdt, G. Wüstefeld, Phys. Rev. Special Topics, A & B 14, 040703 (2011)

Very simple interface between field simulation and tracking

e.g. for an APPLE II:

- about 30 Fourier coefficients & transverse expansion width
- phases ψ_1 (elliptical), ψ_2 (inclined), ψ_1 and ψ_2 (universal mode)
- magnetic gap
- distance of magnet rows

Euqivalent Descriptions of Stationaritity of Action

Differential Expressions of Equations of motion

Lagrange Euler equation

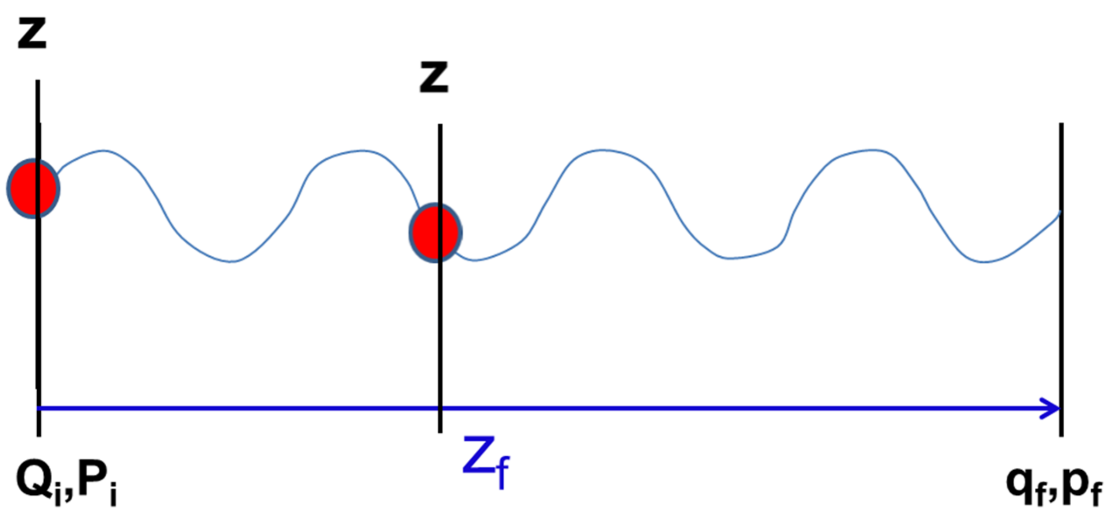
2nd order DEs in N variables:

$$\frac{\partial L}{\partial q} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}} = 0$$

Hamilton's equations of motion

1st order DEs in 2N variables:

$$\frac{\partial H}{\partial x} = -\dot{p}_x \quad \frac{\partial H}{\partial y} = -\dot{p}_y \quad \frac{\partial H}{\partial p_x} = \dot{x} \quad \frac{\partial H}{\partial p_y} = \dot{y}$$



Integral Expression: Hamilton-Jacobi Equation

permits step sizes as long as undulator length

always symplectic

$$\frac{\partial F_3}{\partial z} + H = 0$$

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Hamiltonian of relativistic particle in a magnetic field:

$$\widetilde{H} = \sqrt{(\vec{\tilde{p}} - e\vec{\tilde{A}})^2 c^2 + m_0^2 c^4}$$

Canonical variables: x, y, p_x, p_y, and independent variable t
Hamiltonian is independent upon t.

Change **independent variable:** **time t** ⇒ **distance z**
to enable further transformation to cyclic coordinates; new Hamiltonian:

$$\widehat{H} = -\tilde{p}_z = -\sqrt{\frac{\widetilde{H}^2}{c^2} - m_0^2 c^2 - \left(\tilde{p}_x - e\tilde{A}_x\right)^2 - \left(\tilde{p}_y - e\tilde{A}_y\right)^2} - e\tilde{A}_z$$

Normalization and 2nd order expansion in p_x, p_y, x_3

$$H = -p_z = -1 + (p_x - A_x x_3)^2/2 + (p_y - A_y x_3)^2/2 - A_z x_3$$

$x_3 \sim 1/kB\varrho$ small quantity in undulators; well suited as expansion variable

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Canonical transformation to cyclic coordinates with generating function:

$$F_3(Q_{xi}, Q_{yi}, p_{xf}, p_{yf})$$

Substitution: $P_{xi} = -\dfrac{\partial F_3}{\partial Q_{xi}} = -F_{3x}$ $\qquad P_{yi} = -\dfrac{\partial F_3}{\partial Q_{yi}} = -F_{3y}$

The Hamilton-Jacobi Equation (HJE) has the form:

$$-1+(-F_{3x}-A_x x_3)^2/2+(-F_{3y}-A_y x_3)^2/2-A_z x_3+F_{3z}=0$$

Insert Taylor series expansion Ansatz of generating function in HJE

$$F_3=\sum_{ijk} f_{ijk}\, p_x^i\, p_{yf}^j\, x_3^k$$

expansion variables: $\boldsymbol{p_{xf}, p_{yf}, x_3}$

Each individual expansion term must be zero.
Iterative solution and determination of z-derivatives of f_{ijk}
Integration of $\partial f_{ijk}/\partial z$ along z yields generating function.

Algebraic code (e.g. REDUCE) can be used to derive generating function analytically from analytic vector potential

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In 2nd order expansion (plus f_{003} term)
the generating function (GF) has the form:

$$F_3 = z_f - (p_{xf}x + p_{yf}y) - (p_{xf}^2 + p_{yf}^2)z_f/2 + f_{101}p_{xf}$$
$$+f_{011}p_{yf} + f_{003} + f_{002} + f_{001}$$

Once, the GF in terms of initial coordinates and final momenta is known, this set of 4 equations can be solved

$$x_f = -\partial F_3 / \partial p_{xf}$$
$$y_f = -\partial F_3 / \partial p_{yf}$$
$$P_{xi} = -\partial F_3 / \partial Q_{xi}$$
$$P_{yi} = -\partial F_3 / \partial Q_{yi}$$

In **2nd order** (1st order in momenta) equations can be solved explicitly:

$$p_{xf} = ((1 - f_{011y})(p_x + f_{002x} + f_{001x}) + f_{011x}(p_y + f_{002y} + f_{001y})) / p_n$$
$$x_f = x - f_{101} + p_{xf} z_f$$
$$p_{yf} = ((1 - f_{101x})(p_y + f_{002y} + f_{001y}) + f_{101y}(p_x + f_{002x} + f_{001x})) / p_n$$
$$y_f = y - f_{011} + p_{yf} z_f .$$

with $p_n = (1 - f_{011y})(1 - f_{101x}) - f_{011x} f_{101y}$

Symplectic Map from Generating Function

In 2nd order expansion (plus f_{003} term) the generating function (GF) has the form:

$$F_3 = z_f - (p_{xf}x + p_{yf}y) - (p_{xf}^2 + p_{yf}^2)z_f/2 + f_{101}p_{xf} + f_{011}p_{yf} + f_{003} + f_{002} + f_{001}$$

Once, the GF in terms of initial coordinates and final momenta is known, this set of 4 equations can be solved

$$x_f = -\partial F_3 / \partial p_{xf}$$
$$y_f = -\partial F_3 / \partial p_{yf}$$
$$P_{xi} = -\partial F_3 / \partial Q_{xi}$$
$$P_{yi} = -\partial F_3 / \partial Q_{yi}$$

In **2nd order** (1st order in momenta) equations can be solved explicitly:

$$p_{xf} = ((1 - f_{011y})(p_{x}\ldots$$
$$x_f = x - f_{101} + p_{xf}z\ldots$$
$$p_{yf} = ((1 - f_{101x})(p_{y}\ldots$$
$$y_f = y - f_{011} + p_{yf}z\ldots$$

Note: Procedure is not limited to 2nd order in the momenta, however, iterative techniques (e.g. Newton Raphson Method) are required to solve the set of implicit equations

with $p_n = (1-f_{011y})(1-f_{101x})-f_{011x}f_{101y}$

Analytic expressions of the vector potential are required since GF is derived from analytic integrations over z

2nd order terms:

$$f_{001} = 0, \qquad f_{101} = \int A_x dz, \qquad f_{011} = \int A_y dz,$$

$$f_{002} = -(1/2) \int \left(A_x^2 + A_y^2\right) dz,$$

The following 3rd order term improves accuracy:

$$f_{003} = \frac{1}{2} \int \left(\frac{\partial}{\partial y} \left(\int \left(A_x^2 + A_y^2\right) dz \right) \right) A_y dz'.$$

$$+ \frac{1}{2} \int \left(\frac{\partial}{\partial x} \left(\int \left(A_x^2 + A_y^2\right) dz \right) \right) A_x dz'$$

Scalar potential of a Halbach type undulator:

$$V = -(B_0 / k_y)\cos(k_x x)\sinh(k_y y)\cos(kz + \varphi).$$

Derivation of vector potential from scalar potential

$$A_x = -\int (\partial V / \partial y) dz + C_1$$

$$A_y = \int (\partial V / \partial x) dz + C_2$$

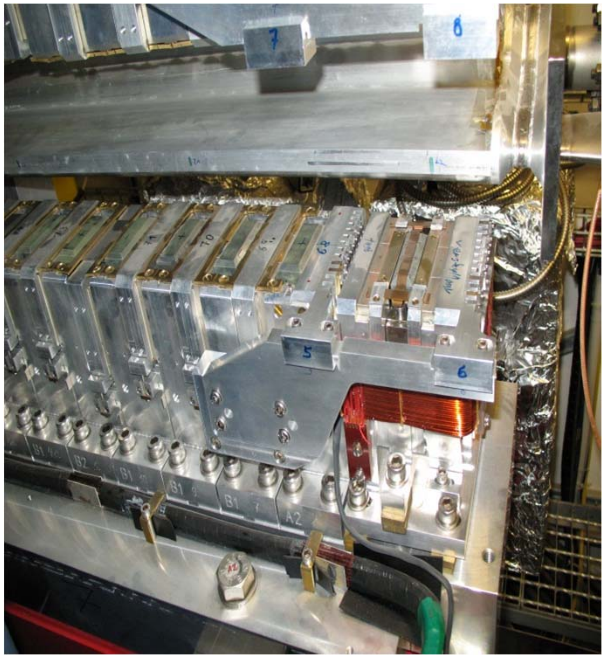
$$A_z = 0.$$

which is

$$A_x = (B_0 / k_z)\cos(k_x x)\cosh(k_y y)\sin(kz + \varphi)$$

$$A_y = ((B_0 k_x)/(k_y k_z))\sin(k_x x)\sinh(k_y y)\sin(kz + \varphi)$$

$$A_z = 0.$$



Magnet structure of BESSY II U125

$\Rightarrow$ Generating Function

Method can easily be extended to a sum over Fourier component

Assumption of linear superposition of fields (permeability = 1) justified for ppm

BESSY II U125 Wiggler I

Fourier expansion of fields
- longitudinally
- transversally

$$k_{yi,\tilde{j}} = \sqrt{k_{\tilde{j}}^2 + k_{xi}^2}$$

$$\tilde{j} = 1,3,5\ldots$$

(1,5,9… for ppm structure)

$$\hat{B}_y(x,y,z) = \sum_{i=0}^{n}\sum_{j=1}^{m} c_{i,\tilde{j}} \cos(k_{xi}x)\cosh(k_{yi,\tilde{j}}y)\cos(k_{\tilde{j}}z)$$

Derivation of Fourier coeffifficients
- undulator:
  single trans. field distribution
- wiggler:
  seveveral trans. field distributions

$$C_{0,1} = \sum_{j=1}^{m} c_{0,\tilde{j}}$$

…

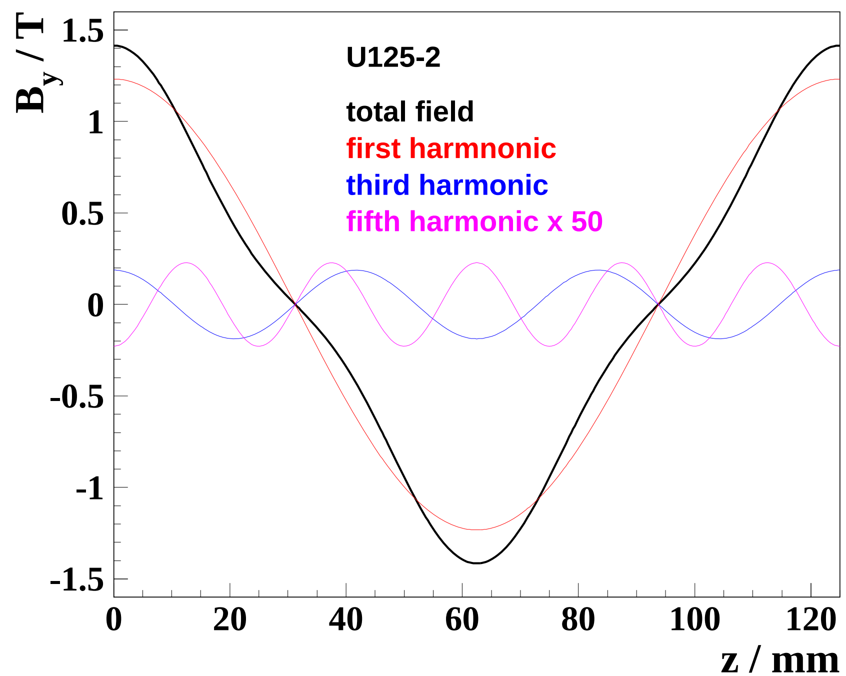
$$C_{n,1} = \sum_{j=1}^{m} c_{n,\tilde{j}}$$

…

$$C_{n,m} = \sum_{j=1}^{m} c_{n,\tilde{j}} \cos(k_{\tilde{j}} z_m)$$

solve system of linear equations

J. Bahrdt et al., IPAC 2011



Field reconstruction using different numbers of harmonics

Example II: APPLE II Undulator

Scalar potential of APPLE II field
Contributions from 4 magnet rows:

$$\vec{B} = -\vec{\nabla}(V)$$

$$V = \sum_{i=1}^{n} (V_{1i} + V_{2i} + V_{3i} + V_{4i})$$

$$V_{1i} = +((e^{+k_{yi}y} / k_{yi}) \cdot (B_{cyi}c_{xi-} + B_{syi}s_{xi-}) + B_0 e^{+k_z y} / nk_z) \cdot c_{z+}$$

$$V_{2i} = +((e^{+k_{yi}y} / k_{yi}) \cdot (B_{cyi}c_{xi+} + B_{syi}s_{xi+}) + B_0 e^{+k_z y} / nk_z) \cdot c_{z-}$$

$$V_{3i} = -((e^{-k_{yi}y} / k_{yi}) \cdot (B_{cyi}c_{xi+} + B_{syi}s_{xi+}) + B_0 e^{-k_z y} / nk_z) \cdot c_{z+}$$

$$V_{4i} = -((e^{-k_{yi}y} / k_{yi}) \cdot (B_{cyi}c_{xi-} + B_{syi}s_{xi-}) + B_0 e^{-k_z y} / nk_z) \cdot c_{z-}$$

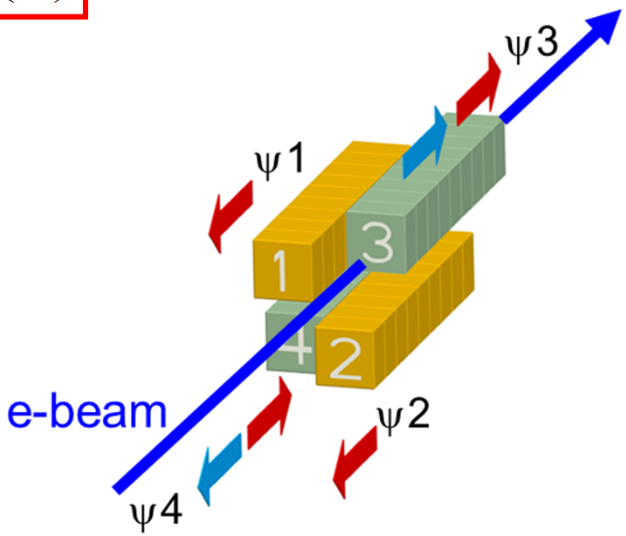
$$c_{xi\pm} = \cos(k_{xi}(x \pm x_0))$$

$$s_{xi\pm} = \sin(k_{xi}(x \pm x_0))$$

$$c_{z\pm} = \cos(k_z z \pm \psi / 2)$$

$$k_{xi} = i \cdot k_{x0}$$

$$k_{yi} = \sqrt{k_{xi}^{\ 2} + k_z^2}$$



helical undulator
shifting of the poles

$$A_x = -\int (\partial V / \partial y) dz + C_1$$

$$A_y = \int (\partial V / \partial x) dz + C_2$$

$$A_z = 0.$$

implemented in
Elegant (A. Xiao)

Example III: Asymmetric Figure 8 Undulator

Asymmetric Figure 8 undulator for linear / helical polarization and reduced on-axis power density

T. Tanaka, H. Kitamura, Nucl. Instr. and Meth. in Phys. Res. A 449 (2000) 629-637

$$A_x = \sum_{i=0}^{\infty} \frac{c_i}{k}((cos(k_{xi}\,(x - x_0\,/2)\,) \cdot sin(kz + \zeta_1) +$$

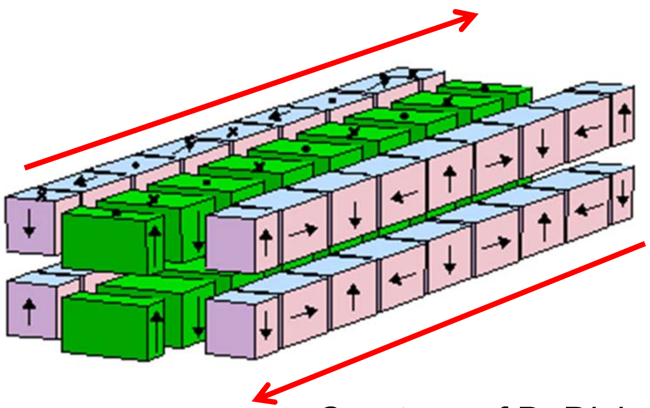
$$cos(k_{xi}(x + x_0/2)) \cdot sin(kz + \zeta_2)) \cdot exp(-k_{yi}\Delta g/2) +$$

$$((cos(k_{xi}\,(x - x_0/2\,)\,) \cdot sin(kz + \zeta_3) +$$

$$cos(k_{xi}(x + x_0/2)) \cdot sin(kz + \zeta_4)) \cdot exp(+\Delta g/2)) +$$

$$\sum_{i=0}^{\infty} \frac{d_i}{\tilde{k}}(\cos(\tilde{\mathrm{k}}_{\mathrm{xi}}\,\mathrm{x}\,) \cdot \cos\left(\tilde{k}z + \zeta_5\right) \cdot exp(-\tilde{k}_{yi}\Delta g/2) +$$

$$(\cos(\tilde{\mathrm{k}}_{\mathrm{xi}}\,\mathrm{x}\,) \cdot \cos\left(\tilde{k}z + \zeta_6\right) \cdot \exp(\tilde{k}_{yi}\Delta g/2)$$



Courtesy of B. Diviacco

$$k_{yi} = \sqrt{k_{xi}^2 + k^2},\ \tilde{k}_{yi} = \sqrt{k_{xi}^2 + \tilde{k}^2},\ \tilde{k} = k/2 = 2\pi/\lambda_0,$$

Different period lengths of inner and outer arrays

Similar expression for A_y, and A_z=0

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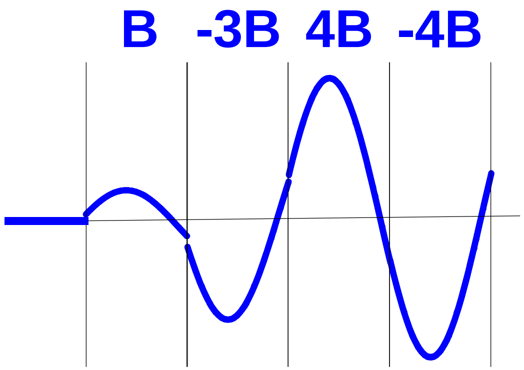
Two methods:

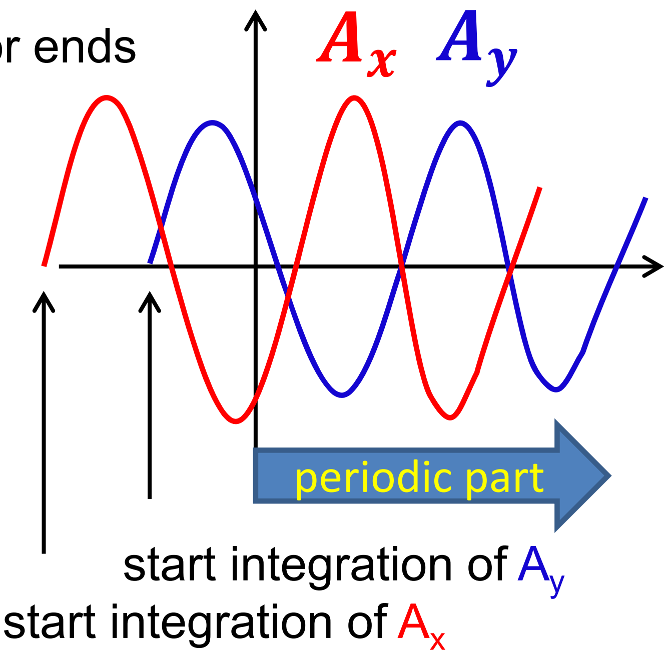
A) Each end consists of 2 half periods
- transverse and longitudinal features similar to periodic part but
- scaled with ¼ and -¾



B) Extrapolation of vector potential at undulator ends
vector potential of each magnet row:

$$A_{x,i} = A_{x0,i}\sin(k_z z + \zeta_i)$$
$$A_{y,i} = A_{y0,i}\sin(k_z z + \zeta_i)$$

$$A_x = \sum_{i=1}^{4} A_{x,i} = A_{x0}\sin(k_z z + \zeta_x)$$
$$A_y = \sum_{i=1}^{4} A_{y,i} = A_{y0}\sin(k_z z + \zeta_y)$$



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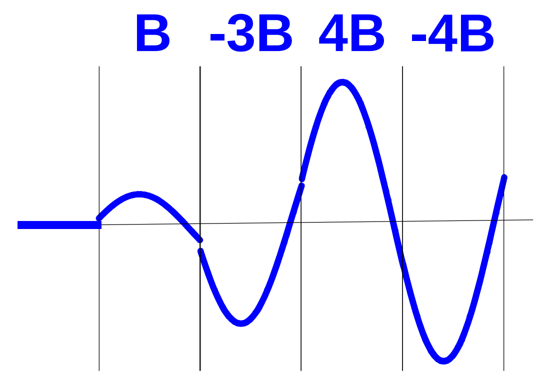

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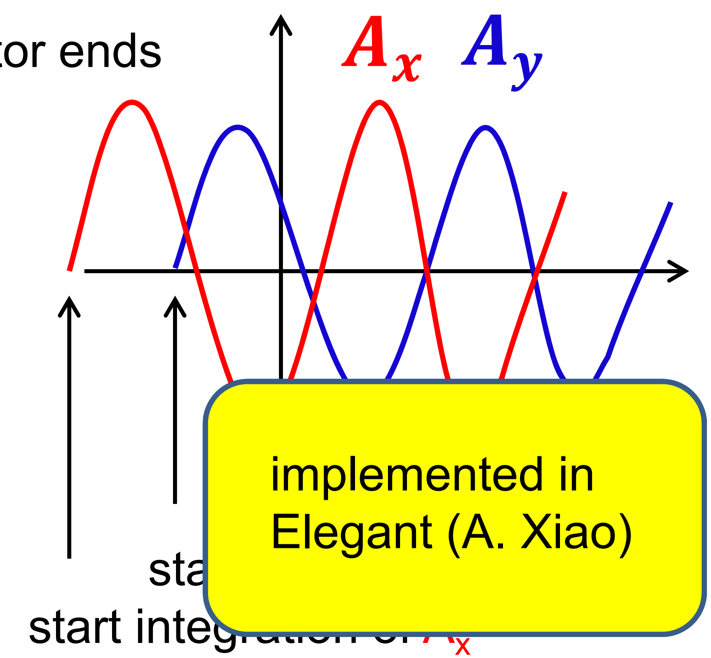
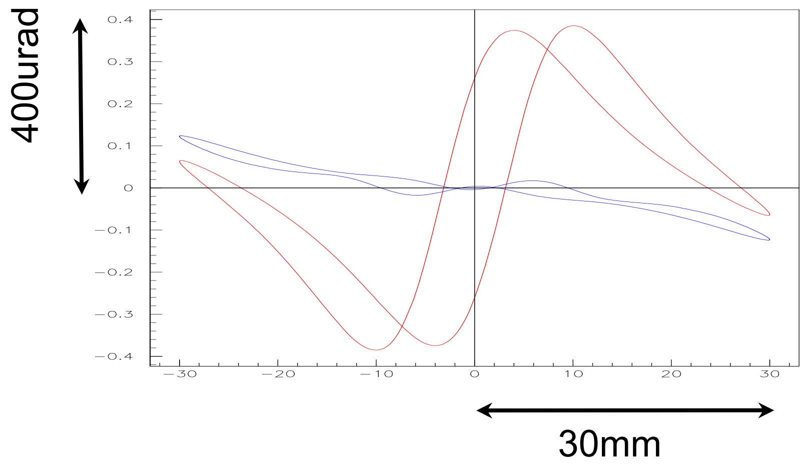


implemented in Elegant (A. Xiao)

UE112 shimming effect



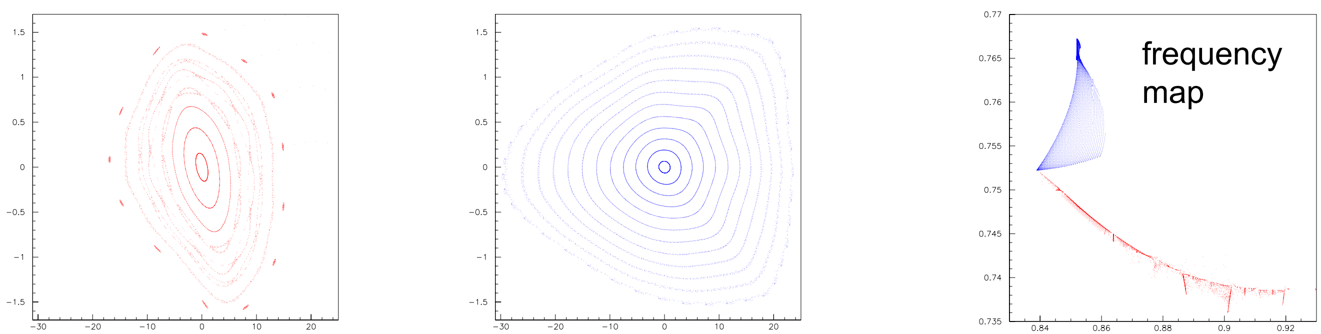
x-kick per ID passage (vertical linear mode)

particles distributed on horizontal phase space ellipse, semi axes: 30mm / 1.87mrad

red: no shims

blue: wire shims powered

BESSY II: 1000-turns tracking (x-x'-plane)



Bahrdt, Scheer& Wüstefeld
Proc. of EPAC (2006) 3562-3564

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Rewriting the formalism in cylindrical coordinates and follow same procedure:

$$H = \frac{1}{2m}\left((p_r - eA_r)^2 + \frac{1}{r^2}\left(p_\varphi - erA_\varphi\right)^2 + p_z^2\right)$$

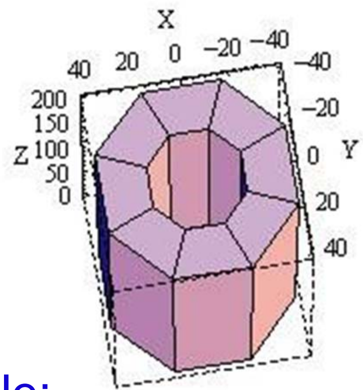
$$H = -p_z = -1 + (p_r - A_r)^2/2 + \left(\frac{p_\varphi}{r} - A_\varphi\right)^2/2 - A_z$$

The generating function in cylindrical coordinates is:

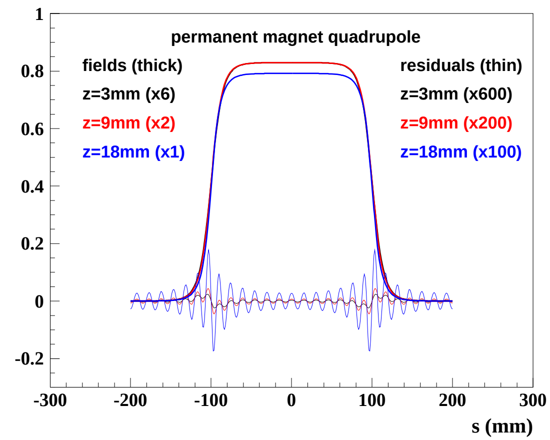
$$F_3 = z_f - \left(p_{rf} r + p_{\varphi f}\varphi\right) - \left(p_{rf}^2 + p_{\varphi f}^2/r^2\right) z_f/2 + f_{101}p_{rf} + f_{011}p_{\varphi f} + f_{003} + f_{002} + f_{001}$$

where f_{ijk} include analytic expressions of integrals and partial derivatives of the vector potential in cylindrical coordinates

Analytic Expressions of the Vector Potential of Multipole Magnets



Example:
Halbach type quadrupole



Longitudinal Fourier decomposition

Bassetti and Biscari

$$B_{\varphi}(r,0,z) = \sum_{p=0}^{\infty} G_{02p}(z) r^{2p+1}$$

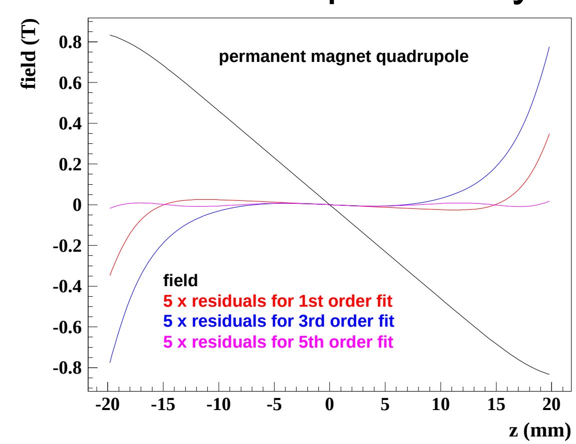
$$G_{m0}(z) = \sum_{i=0}^{\infty} a_i \cos(k_i z)$$

$$A_r(r,\varphi,z) = -\frac{\cos(m\varphi)}{(m-1)!}\left(A_{r0} + \sum_{p=1}^{\infty} \frac{1}{4(m+p)p} \cdot G_{m2p-1}(z) r^{2p+m-1}\right)$$

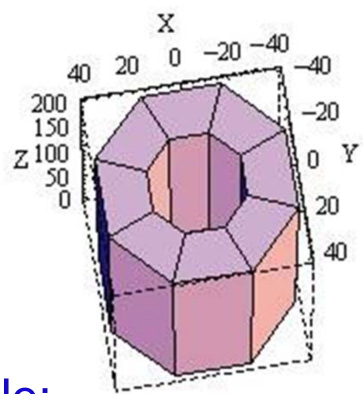
$$A_{\varphi}(r,\varphi,z) = \frac{\sin(m\varphi)}{m!}\left(A_{\varphi 0} + \sum_{p=1}^{\infty} \frac{m+2p}{4(m+p)p} \cdot G_{m2p-1}(z) r^{2p+m-1}\right)$$

$$A_{r0} = A_{\varphi 0} = G_{m-1}(z) r^{m-1}$$

Radial dependency



Analytic Expressions of the Vector Potential of Multipole Magnets



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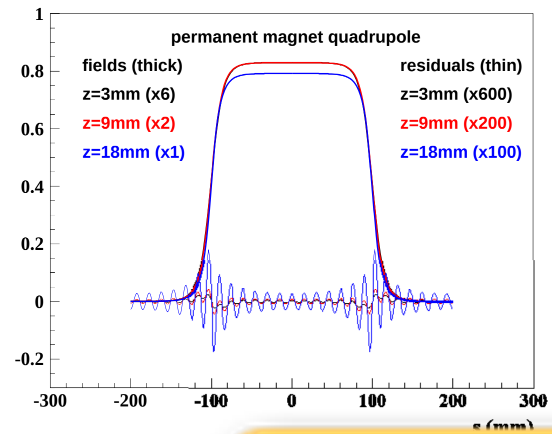
Halbach type quadrupole



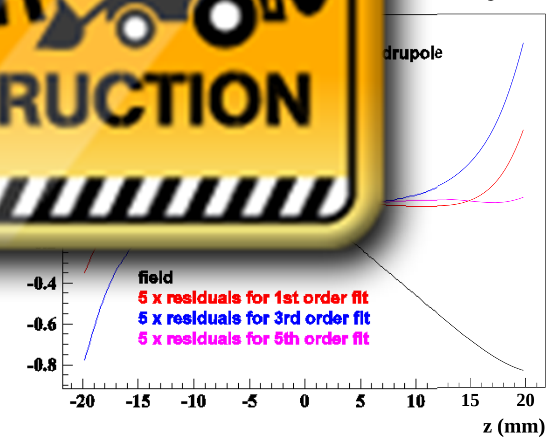
Longitudi

Bassetti and Biscari

$$B_\varphi(r,0,z) = \sum_{p=0}^{\infty} G_{02p}(z) r^{2p+1}$$

$$G_{m0}(z) = \sum_{i=0}^{\infty} a_i \cos(k_i z)$$

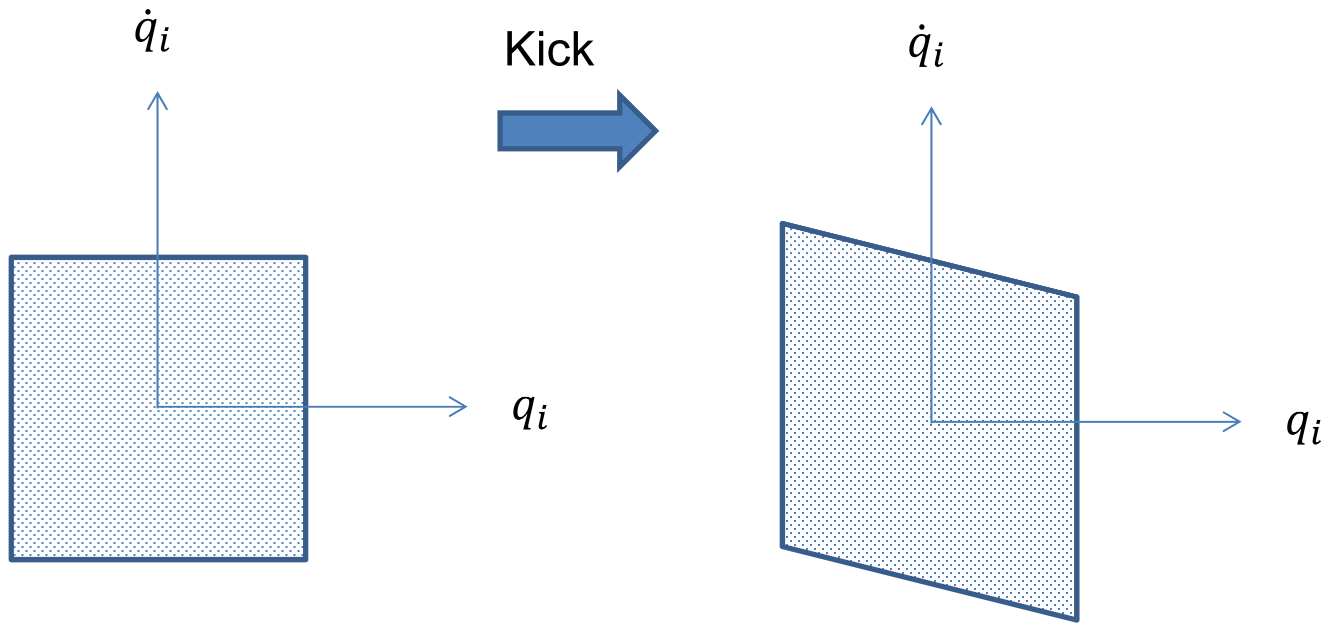
dency



$$A_r(r,\varphi,z) = -\frac{\cos(m\varphi)}{(m-1)!}\left(A_{r0} + \sum_{p=1}^{\infty} \frac{1}{4(m+p)p} \cdot G_{m}\right.$$

$$A_\varphi(r,\varphi,z) = \frac{\sin(m\varphi)}{m!}\left(A_{\varphi 0} + \sum_{p=1}^{\infty} \frac{m+2p}{4(m+p)p} \cdot G_{m2p-1}(z) \cdot \right)$$

$$A_{r0} = A_{\varphi 0} = G_{m-1}(z) r^{m-1}$$



$$\dot{q}_i^{\,new} = \dot{q}_i^{\,old} + W(q_i)$$

The kickmap is easily derived from the generating function coefficient f_{002}

$$\theta_x = \partial f_{002} / \partial x \qquad \theta_y = \partial f_{002} / \partial y$$

where f_{002} depends upon a set of Fourier coefficients

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The integrated dynamic kicks due to the wiggling motion in undulators follow directly from generating function:

transverse Fourier coefficients

$$\theta_x = -\frac{z_f}{2(B\rho)^2}\sum_{i1=0}^{n}\sum_{i2=0}^{n}\sum_{j=1}^{m} c_{i1,\tilde{j}}c_{i2,\tilde{j}}\frac{k_{xi1}}{k_{\tilde{j}}^2}\sin(k_{xi1}x)\cos(k_{xi2}x)$$

$$\times\left(\frac{1}{k_{yi1,\tilde{j}}}\frac{k_{xi2}^2}{k_{yi2,\tilde{j}}}\sinh(k_{i1,\tilde{j}}y)\sinh(k_{i2,\tilde{j}}y)-\cosh(k_{i1,\tilde{j}}y)\cosh(k_{i2,\tilde{j}}y)\right)$$

$$\theta_y = -\frac{z_f}{2(B\rho)^2}\sum_{i1=0}^{n}\sum_{i2=0}^{n}\sum_{j=1}^{m} c_{i1,\tilde{j}}c_{i2,\tilde{j}}\frac{1}{k_{\tilde{j}}^2}\cosh(k_{yi1,\tilde{j}}y)\sinh(k_{yi2,\tilde{j}}y)$$

$$\times\left(k_{yi2,\tilde{j}}\cos(k_{xi1}x)\cos(k_{xi2}x)+k_{xi1}\frac{k_{xi2}}{k_{yi2,\tilde{j}}}\sin(k_{xi1}x)\sin(k_{xi2}x)\right)$$

➡ **analytic kick map**

- for undulators: m=1 is sufficient
- high field wigglers require inclusion of higher field harmonics (m >/= 3)

BESSY II U125 Wiggler II

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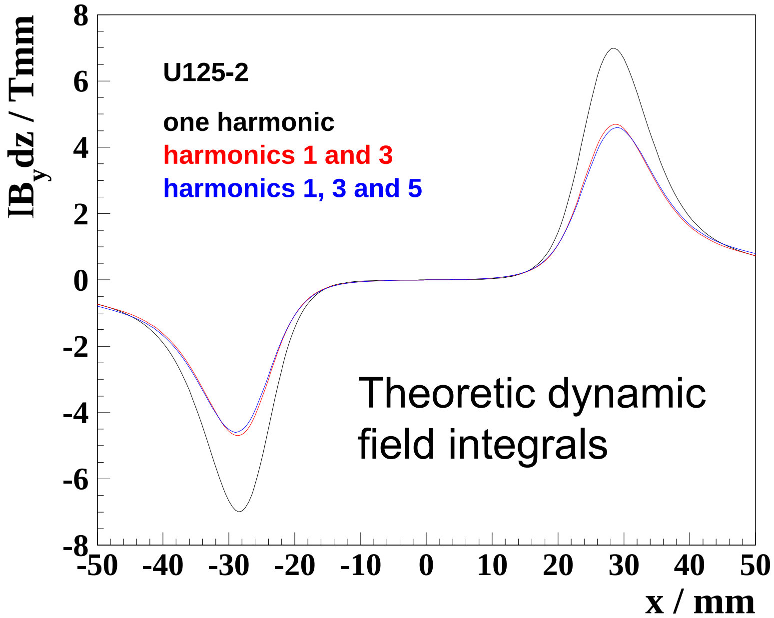
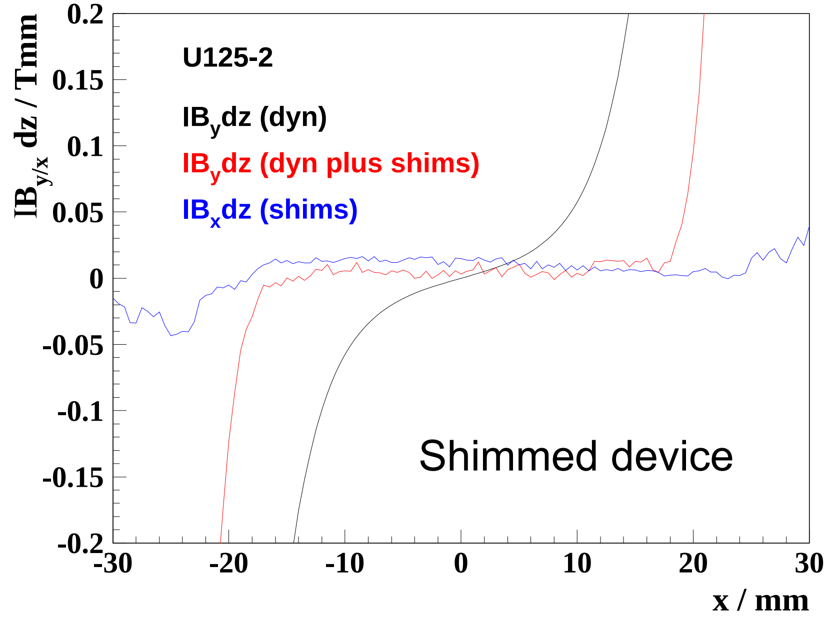
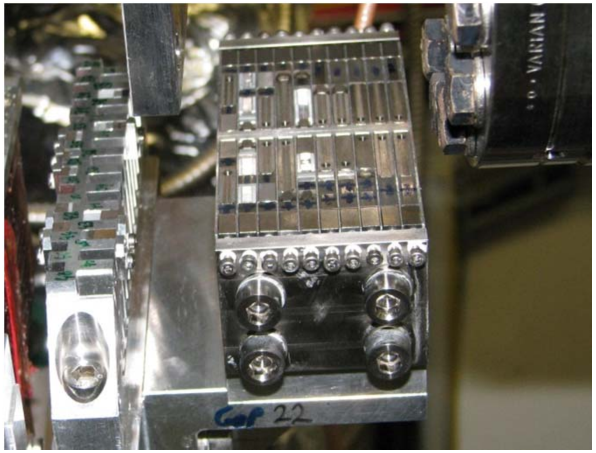




The 3rd field harmonic has to be included for a a field reconstruction on the percent level

For undulators usually 1st field harmonic is sufficient



U125-2 Magic fingers

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Elliptical mode

analytic functions of row phases

$$\theta_x = f_0 \cdot c_{pp}^2 + f_\pi \cdot s_{pp}^2$$

transverse Fourier coefficients

$$\theta_y = 0$$

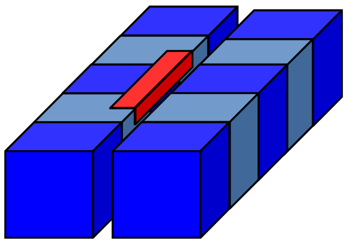
terms include magnetic gap

$$f_0 = \frac{L \cdot 8}{k^2 (B\rho)^2} \sum_{i=0}^{n} \sum_{j=0}^{n} c_i c_j e_i e_j k_{xj} c_{xi0} c_{xi} c_{xj0} s_{xj}$$

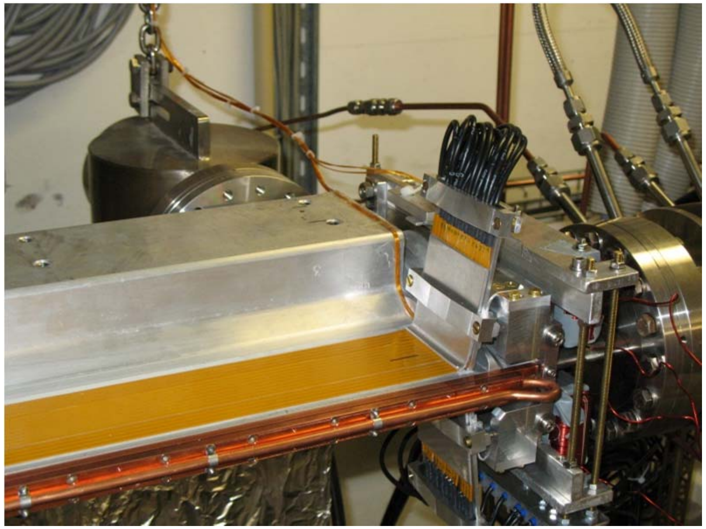
$$f_\pi = \frac{L \cdot 8}{k^2 (B\rho)^2} \sum_{i=0}^{n} \sum_{j=0}^{n} c_i c_j e_i e_j \frac{k_{xi}}{k_{yi}} \frac{k_{xj}^2}{k_{yj}} s_{xi0} c_{xi} s_{xj0} s_{xj}.$$

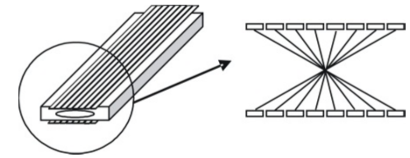
Similar analytic expressions for **inclined** and **universal mode**, for details see PRST

Successful shimming of weak devices with L-shims ☺
based on analytic kick maps

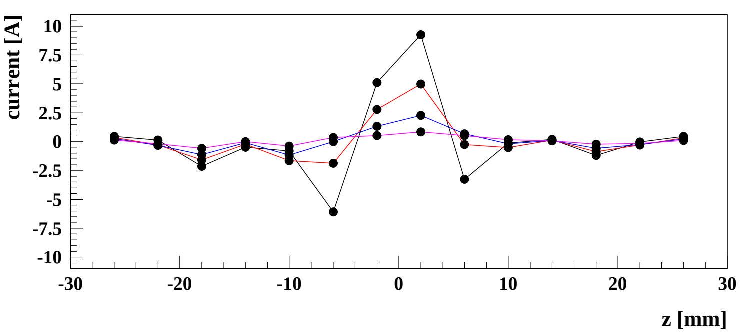


Active Compensation of Dynamic Kicks with Flat Wires: BESSY II UE112 APPLE II



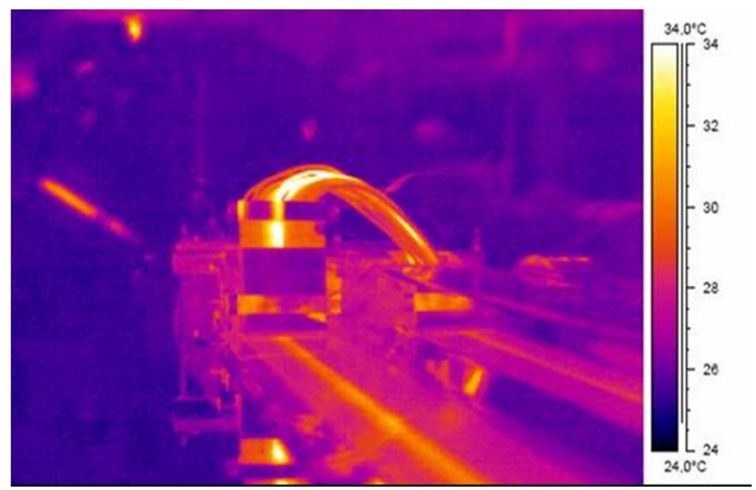


water cooled undulator vacuum chamber



current settings for gaps of 20mm 24mm, 30mm and 40mm

J. Bahrdt et. al., Proc. of EPAC, Genoa, Italy (2008) 2222-2224



UE112: Tune Shift and Beam Size Variation

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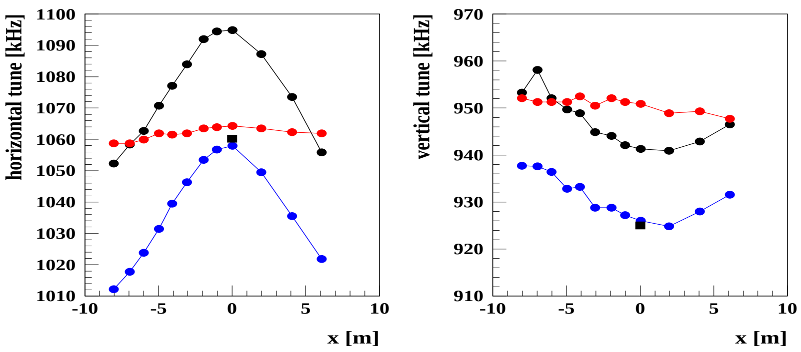
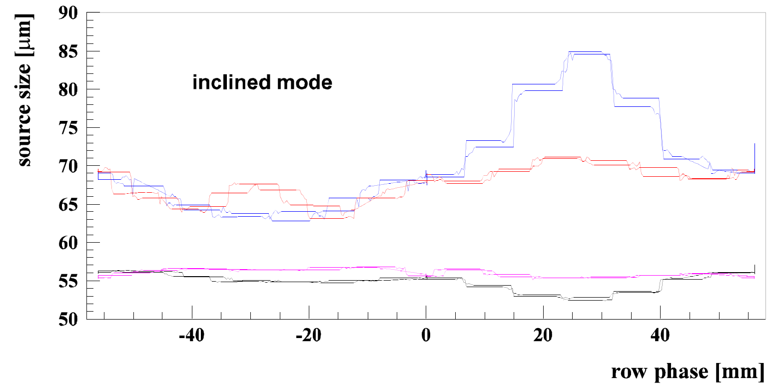
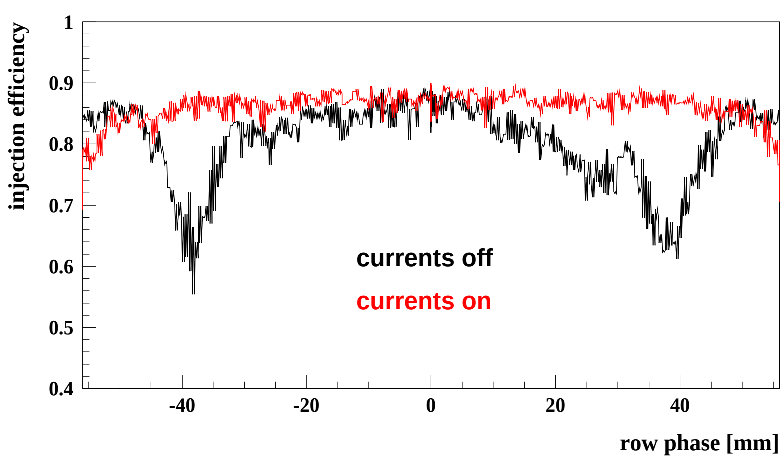


Horizontal and vertical tunes
vs horizontal displacement:
black: tune correction off, wires off
blue: quad correction on, wires off
red: no quad correction, wires on



Source size variation with row phase of the UE112 at gap = 24mm in the inclined mode. Black, blue: currents switched off; red, magenta: currents switched on.



Recovery of injection efficiency
in inclined mode

Results for UE112 are achieved using analytic kick maps without further iteration.

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Static multipoles:

Complete description of two dimensional straight line field integral distributions on a source free circular disc

$\overline{F} = \overline{B}_x - i\overline{B}_y$ is an analytic function; the bar indicates a straight line integration

Cauchy Riemann relations:

$$\frac{\partial \overline{B}_x}{\partial x} = \frac{\partial(-\overline{B}_y)}{\partial y}$$

$$\frac{\partial(-\overline{B}_y)}{\partial x} = -\frac{\partial \overline{B}_x}{\partial y}$$

are equivalent to the 2D-Maxwell equations

“Dynamic multipoles” (DM):

$\widetilde{F} = \widetilde{B}_x - i\widetilde{B}_y$ is not an analytic function; the tilde indicates an integration along a wiggling trajectory

Cauchy Riemann relations are not fullfilled:

$$\vec{\nabla}\cdot(\widetilde{B}_x, \widetilde{B}_y) = \partial\widetilde{B}_x / \partial x + \partial\widetilde{B}_y / \partial y \propto -f_{002yx} + f_{002xy} = 0$$

$$\left(\vec{\nabla}\times(\widetilde{B}_x, \widetilde{B}_y)\right)_z = \partial\widetilde{B}_y / \partial x - \partial\widetilde{B}_x / \partial y \propto f_{002xx} + f_{002yy} \neq 0$$

Note: By principle “dynamic multipoles” can not be compensated with shims which are usually described by static field integrals

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Why does shimming of DM work at all?

Shimming of DM in the midplane has no principle limitations, but vertical off-axis effects are enhanced; this is acceptable because:

- usually, vertical beta-function smaller than horizontal beta functions
- usually vertical emittance smaller than horizontal emittance
- large particle amplitudes occur during horizontal injection

What about gap dependency?

DM are expected to drop off much faster than shim field integrals due B^2 dependency, but:

- detailed considerations show similar gap dependence of dynamic multipoles and static multipoles for long period lengths
- DM scale with square of period length; Murphy’s Law does not apply ☺

Summary and Outlook

Fast analytic, symplectic GF-based tracking scheme
    one step per undulator is possible

Analytic description of undulator fields and shim field integrals
    simplifies interface

APPLE undulator implemented in Elegant, other devices straight forward

Multipoles with fringes fields will be implemented soon
    dipole with fringe fields needs specific Hamiltonian along bent orbit

Analytic kick maps derived from analytic generating function
    used for evaluation of shim strength