



Marrying Lasers and Particle Beams

Luca Serafini – INFN Milan

- Common Aspects of Laser and Particle Beams: Brilliance, Brightness, Phase space density, e.m. field intensity
- Interactions: to exchange informations (diagnostics), to exchange energy/momentum (laser based accelerators, vs. RF), to exchange order (coherence) or disorder (heating)
- 3 Examples of interactions in vacuum: Seeded FEL, Inverse FEL, Compton Sources
- Future directions







Lasers are Beams and propagate in free space, under the effect of diffraction, just like particle beams under the effect of their emittance





TEM₀₀ Gaussian Laser mode (circular polarization M²=1 diffraction limited)



$$E_0(x, y, z, t) = A_0 e^{i\omega t} e^{-ikz} \frac{Z_0}{Z_0 - iz} \exp \left[-\frac{k(x^2 + y^2)}{2} \frac{1}{Z_0 - iz} \right] \quad k = 2\pi/\lambda$$

$$|E_0(x,y,z,t)| = E_0 \frac{W_0}{W} e^{-\frac{x^2 + y^2}{w^2}}$$

$$w = w_0 \sqrt{1 + \frac{z^2}{Z_0^2}} \qquad \qquad Z_0 = \frac{\pi w_0^2}{\lambda} \qquad \qquad \vartheta = \frac{w_0}{Z_0} = \frac{\lambda}{\pi w_0}$$

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 $I \propto \left| E_0(x, y, z, t) \right|^2$



PARTICLE BEAM

 $eta^* = rac{\sigma_0^2}{arepsilon_n/\gamma}$









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The Figure of Merits for Lasers and Beams

Brilliance of Lasers and X-ray sources









The 6D Brilliance Chart





e.m. field carried by the electron bunch \oplus With 2 fs LCLS beam we have $E_r^{edge} = \frac{\eta_0 I}{2\pi\sigma}$ \oplus For 2-20 pC beam, we have 1 TV/m fields (!) PWFA Plasma Acceleration



1 TV/m accelerating field: a dream for a table-top TeV-class e⁻e⁺ collider?

e.m. field carried by the electron bunch \oplus With 2 fs LCLS beam we have $E_r^{edge} = \frac{\eta_0 I}{2\pi\sigma}$ \oplus For 2-20 pC beam, we have 1 TV/m fields (!) PWFA Plasma Acceleration

Teravolt-per-meter plasma wakefields from low-charge, femtosecond electron beams

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Recent initiatives in ultra-short, GeV electron beam generation have focused on achieving sub-fs pulses for driving X-ray free-electron lasers (FELs) in single-spike mode. This scheme employs very low charge beams, which may allow existing FEL injectors to produce few-100 as pulses, with high brightness. Towards this end, recent experiments at SLAC have produced ~ 2 fs rms, low transverse emittance, 20 pC electron pulses. Here we examine use of such pulses to excite plasma wakefields exceeding 1 TV/m. We present a focusing scheme capable of producing <200 nm beam sizes, where the surface Coulomb fields are also \sim TV/m. These conditions access a new regime for high field atomic physics, allowing frontier experiments, including sub-fs plasma formation for wake excitation.

PACS numbers: 41.60.Cr, 41.75.-i, 41.85.Gy, 42.60.Jf

case

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Ultrahigh brightness electron beams by plasma-based injectors for driving all-optical free-electron lasers

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$$E_r^{edge} = \frac{\eta_0 I}{2\pi\sigma} \quad I = 20 \quad kA; \quad \sigma = 1 \quad \mu m \quad E_r^{edge} = 1 \quad TV/m$$

Longitudinal Phase Space distributions show violent blow-up of uncorrelated energy spread due to transverse space charge field (electrons are fermions...)









Marrying Lasers and Particle Beams

Seeding a High Gain Free Electron Laser: transfer coherence from the Laser to the FEL radiation through the Electron Beam













High-Gain Harmonic-Generation Free-Electron Laser Seeded by Harmonics Generated in Gas

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- Seed an FEL cascade with harmonics generated in gas
- Study the evolution of the signal while varying the number of radiators/modulators



Seeding with harmonics generated in gas (Ar)

Seeding @ SPARC

Hangaldan XYE

SULEIL

Ministero dell'Università e della Ricerca

* *

EURO FEL



Varying the number of radiators



Transition from CHG to HGHG Transfer the coherence of the seed to higher harmonics



High-Order-Harmonic Generation and Superradiance in a Seeded Free-Electron Laser

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 All the UM tuned at the same resonant wavelength (400nm)

 Induce superradiance by seeding the FEL amplifier at saturation intensity











High harmonics down to 37 nm





Measured energy per pulse, spot size & and bandwidth of the first 11° harmonics





Other example of Coherence Transfer

Extreme ultraviolet free electron laser seeded with high-order harmonic of Ti:sapphire laser

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SPring8 Compact Sase Source (SCSS)



Fig. 1. Experimental setup. HH radiation is generated by loosely focusing a Ti:S laser (800 nm, 100 mJ, 160 fs FWHM, 30 Hz) in a Xe gas cell (focal length f = 4000 mm), and is separated from the fundamental beam by a SiC mirror. Pt-coated concave mirrors with 8000-mm radius of curvature are used for collimating and focusing HH radiation. The HH radiation is transported by the SiC mirror to the undulator section, overlapped with an electron beam (250 MeV, 300 fs, 30 Hz) spatially and temporally. The seeded FEL is observed by the spectrometer at the end of the beamline. The inset shows the beam profile of HH radiation at the undulator entrance. The spatial profile was measured by a phosphor screen coupled with a multichannel plate (MCP) and CCD camera.

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Fig. 3. Spectra of seeded (red lines) and unseeded (blue lines) conditions, as well as that of HH radiation (green line), given by experiment (a) and simulation (b). The inset of (b) shows intensity growths along the undulator for seeded (red line) and unseeded (blue line) conditions.







Fig. 4. The intensity threshold of the seeding source (green line) and output intensity after FEL amplification (red line), as a function of photon energy (lower axis) and wavelength (upper axis). The inset shows the simulated spectra of the seeded (red line) and the unseeded (blue line) conditions at a wavelength of 4 nm. The intensity of HH radiation, 2.6 nJ/pulse, corresponds to the threshold level required for operating seeded FEL.





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Inverse Free Electron Laser: transfer energy from the Laser to the Electron Beam through the FEL radiation

IFEL Interaction

In an FEL energy in the e-beam is transferred to a radiation field



In an IFEL the electron beam absorbs energy from a radiation field.



Undulator magnetic field to couple high power radiation with relativistic electrons

$$K_{l} = \frac{eE_{0}}{mc^{2}k} \qquad \qquad K = \frac{eB}{mck_{w}}$$



Significant energy exchange between the particles and the wave happens when the resonance condition is satisfied.

Useful scalings for IFEL accelerator

Assuming no guiding and a single stage helical undulator

The ideal relationship between the Rayleigh range and the total undulator length is

 $L_{u} \approx 6 \ z_{r} \stackrel{\blacksquare}{\frown} \text{A tight focus increases the intensity, but only in one spot.} \\ \frac{M^{2}\lambda}{4\pi} \approx 8.*10^{-8} \ vs. \ \frac{\varepsilon_{n}}{\gamma_{in}} \approx 1.*10^{-8} \ and \ \frac{\varepsilon_{n}}{\gamma_{fin}} \approx 5.*10^{-10}$

Final energy (assuming a const. K, or taking the average value for K) will be given by

$$\Delta \gamma^2 \cong 12\sqrt{2} \, \frac{e_0 \sqrt{Z_0}}{mc^2} \sqrt{\frac{z_r}{\lambda}} \sqrt{P} \,\overline{K} \cong 6 \cdot 10^{-4} \sqrt{\frac{z_r}{\lambda}} \sqrt{P(W)} \,\overline{K}$$

In order to have the final energy 500 MeV ($\gamma_f^2 = 10^6$) with a 1 (0.8)um laser, $z_r = 15$ cm and K ~ 4 The laser power P needs to be 5 TW or higher

Beam loading

Take as an example the case of a 1 GeV acceleratorThe laser power P needs to be 20 TW or higher4 J in 200 fs20 J in 1 ps100 J in 4 ps

Slippage problem.

Pulse length / optical period has to be larger than number of periods in undulator.

Energy is linear with accelerated charge 1000 MeV x 100 pC = 0.1 J 1000 MeV x 1 nC = 1 J

Need to choose laser pulse length/energy based on these considerations.

From proof-of-principle to 2nd generation IFEL experiments

- Proof-of-principle experiments successful
- Upgrade to significant gradient and energy gain
 - Technical challenges:
 - staging
 - very high power radiation
 - strong undulator tapering
 - Physics problems:
 - microbunching preservation
 - include diffraction effects in the theory
 - beyond validity of periodaveraged classical FEL equation



Cryogenic undulator + 20 TW laser power "green-field" design

If gap is maintained large one can adopt a fully permanent magnet design (no iron poles) Initial energy

Optimization keeping resonant phase at 45 degrees Compromise between capture and gradient 6 mm gap



Initial energy	50 MeV
Final energy	1200 MeV
Avg gradient	1.1 GV/m
Final energy spread	1 %
Laser wavelength	800 nm
Laser power	20 TW
Laser spot size (w ₀)	0.2 mm

High Gradient High Energy Gain Inverse FreeElectron LaserP. Musumeci et al.

- Renovated interest in IFEL acceleration scheme
- Applications as compact scheme to obtain 1-2 GeV electron beam for gamma ray (ICS) or soft x-ray (FEL) generation.

Radiabeam UCLA BNL IFEL Collaboration

Strongly tapered optimized helical permanent magnet undulator

ATF @ BNL 0.5 TW CO2 laser 50 MeV -> 180 MeV in 60 cm 130 MeV energy gain 220 MV/m gradient



LLNL-UCLA	IFEL
experiment	

Reuse UCLA- Kurchatov undulator

Use 5 TW 10 Hz Ti:Sa 50 MeV -> 150 MeV in 50 cm

High rep rate allows beam quality measurement



GeV IFEL experiment	t
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If current experiments succesfull

Looking for access to facility with 50 MeV beam+20 TW laser (BNL, LLNL, LNF-Italy)

Praesodymium based cryogenic undulator

different	
Initial energy	50 MeV
Final energy	1200 MeV
Avg gradient	1.1 GV/m
Final energy spread	1 %
Laser wavelength	800 nm
Laser power	20 TW
Laser spot size (w ₀)	0.2 mm





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Inverse Compton Scattering: the Photon Accelerator frequency Doppler/Lorentz upconversion by the Electron Beam of the Laser Photons

where the phase space densities of the two colliding beams are mapped into the gamma photon beam

Beams of γ , not just one photon per shot





Relativistic electron





Relativistic electron





Relativistic electron

From the double Doppler effect :

$$\mathbf{v} = \mathbf{v}_{\mathrm{L}} \frac{1 - \underline{\mathbf{e}}_{\mathrm{k}} \cdot \underline{\boldsymbol{\beta}}_{0}}{1 - \underline{\mathbf{n}} \cdot \underline{\boldsymbol{\beta}}_{0}} \approx 4\gamma^{2} \mathbf{v}_{\mathrm{L}}$$

 $\lambda = \lambda_{L} \frac{1 - \underline{n} \cdot \underline{\beta}_{0}}{1 - \underline{e}_{k} \cdot \underline{\beta}_{0}} \qquad \text{Head-on scattering } \lambda = \lambda_{L} \frac{1}{4\gamma_{0}^{2}}$ Radiation on axis $\frac{M^{2}\lambda}{4\pi} \approx 8.*10^{-8} \text{ vs. } \frac{\varepsilon_{n}}{\gamma} [Thomson] \approx 5.*10^{-8} \text{ and } \frac{\varepsilon_{n}}{\gamma} [Compton] \approx 5.*10^{-10}$





Classical double differential spectrum

From the electron orbits and the Liénard-Wiechert far zone the expression to calculate the radiation field [Jackson..] for **one electron** is:

$$\frac{d^2 W_i}{d\omega d\Omega} = \frac{e^2}{4\pi^2 c} \left| \int_{-\infty}^{+\infty} dt e^{i\omega t} \frac{\mathbf{n} \times \left[(\mathbf{n} - \beta(t') \times \dot{\beta}(t') \right]}{(1 - \mathbf{n} \cdot \beta(t'))^3} \right|^2 = -\hbar \omega \frac{d^2 N_i}{d\omega d\Omega}$$

And for all the beam:

$$\hbar\omega \frac{\mathrm{d}^2 \mathrm{N}}{\mathrm{d}\omega \mathrm{d}\Omega} = \hbar\omega \sum_{i} \frac{\mathrm{d}^2 \mathrm{N}_{i}}{\mathrm{d}\omega \mathrm{d}\Omega}$$

The previous expressions are at the basis of the semi-analytical classical non linear code **TSST**

P. Tomassini et al., IEEE TRANSACTIONS ON PLASMA SCIENCE, VOL. 36, NO. 4, AUGUST 2008





 $mc^{2}(\gamma - \gamma_{0}) = -h(\nu - \nu_{L})$ $mc(\beta\gamma - \beta_{0}\gamma_{0}) = -h(\underline{k} - \underline{k}_{L})/2\pi$

Energy and momentum conservation laws

 γ_0 :initial Lorentz factor

$$\nu = \nu_{L} \frac{1 - \underline{\mathbf{e}}_{k} \cdot \underline{\boldsymbol{\beta}}_{0}}{1 - \underline{\mathbf{n}} \cdot \underline{\boldsymbol{\beta}}_{0}} + \frac{h\nu_{L}}{mc^{2}\gamma_{0}}(1 - \underline{\mathbf{e}}_{k} \cdot \underline{\mathbf{n}})$$

$$\lambda = \lambda_{L} \frac{1 - \underline{\mathbf{n}} \cdot \underline{\boldsymbol{\beta}}_{0}}{1 - \underline{\mathbf{e}}_{k} \cdot \underline{\boldsymbol{\beta}}_{0}} + \frac{\mathbf{h}}{\mathbf{mc}\gamma_{0}} \frac{1 - \underline{\mathbf{e}}_{k} \cdot \underline{\mathbf{n}}}{1 - \underline{\mathbf{e}}_{k} \cdot \underline{\boldsymbol{\beta}}_{0}}$$

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Angle-frequency correlation

Collecting the radiation in an acceptance angle $\theta >> 1/\gamma$, the spectrum is white.



CAIN results



The most energetic photons are emitted on or close to the axis

White spectrum I.Chaikovska and A. Variola



I.Chaikovska and A. Variola



By selecting the photons on axis with a system of collimators one can reduce conveniently the bandwidth.

White spectrum I.Chaikovska and A. Variola



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White spectrum I.Chaikovska and A. Variola

Λν



By selecting the photons on axis with a system of collimators one can reduce conveniently the bandwidth.

White spectrum I.Chaikovska and A. Variola

 Δv

Quantum cross-section for electron-photon interaction (Klein and Nishina)

Dirac Equation:

$$i\frac{\partial \Psi}{\partial t} = (\hat{H}_0 + \hat{H}_{int})\Psi$$

Radiation potential:

$$\hat{A} = hc(\underline{e}_{L}\sqrt{\frac{1}{\omega_{L}}}(e^{i(\underline{k}_{L}}\cdot\underline{r}-\omega_{L}t)}\hat{a}_{L} + e^{-i(\underline{k}_{L}}\cdot\underline{r}-\omega_{L}t)}\hat{a}_{L}^{+}) + \underline{e}\sqrt{\frac{1}{\omega}}(e^{i(\underline{k}}\cdot\underline{r}-\omega t)}\hat{a} + e^{-i(\underline{k}}\cdot\underline{r}-\omega t)}\hat{a}^{+}))$$

Transition probability (perturbation theory or Feyman-Dyson graphs):

$$w_{n,m} = \frac{2\pi}{\hbar} \rho \left| \sum_{n'} \frac{H_{m,n'}H_{n',n}}{E_m - E_{n'}} + \sum_{n'} \frac{H_{m,n''}H_{n'',n}}{E_m - E_{n''}} \right|^2$$

COMPARISON between classical (TSST), quantum semianalytical (Comp_cross) and quantum MonteCarlo (CAIN)



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Quantum shift ΔE



(a)CAIN (b)Comp_Cross (c)TSST

A part from the quantum shift, the spectra are very similar



7.410 9 E_L (J)Q(pC) ψ^{2}

Analytical Model based on Luminosity

 $N = \frac{1}{hv_{L} (eV)(w_{0}^{2} + 2\sigma_{x}^{2})} \sqrt{1 + \frac{\delta^{2} (\sigma_{x}^{2} + \sigma_{x,e}^{2})}{(w_{0}^{2} + 2\sigma_{x}^{2})}}$ Number of photons/shot within the rms bandwith

Bandwidth







Gamma-ray Source Specifications

Gamma-ray Energy: 1-20 MeV

Bandwidth : $0.3\% \rightarrow 0.1\%$

Spectral Density: $10^4 \rightarrow 10^6$ photons/sec/eV

rms divergence < 200 µrad
(spot size < 5 mm at target)
controllable > 98% polarization

1-2 orders of magn. better than state of the art HiGS (bdw 3%, sp. dens. 10², E < 8 MeV)





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Marrying Lasers and Particle Beams

In Vacuum Acceleration



The Far Future? The Perfect Beam Marriage in Vacuum

VACUUM LASER ACCELERATION EXPERIMENT PERSPECTIVE AT BROOKHAVEN NATIONAL LAB-ACCELERATOR TEST FACILITY

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Figure 1: The distribution of the minimum phase velocity v_* in the plane y=0.

Challenge for Phase Space Density of Injected Beam at low energy (time jitter)



Figure 3: Final energy as a function of laser intensity a₀.





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Angular and Frequency Spectrum (560 MeV electrons)

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Figure 18: Spectrum versus photon energy and collection angle

Sparce Scattered photons in collision



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Scattering angle in Thomson limit (no recoil) is small, i.e. $< 1/\gamma$





Spectral broadening due to ultra-focused beams: Thomson Source classically described as a **Laser Syncrotron Light Source** $\theta(hv_{50\%}) = \frac{1}{2}$









Courtesy P. Tomassini







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Courtesy V. Petrillo





$$\lambda_{R} = \lambda_{w} \frac{\left(1 + a_{w}^{2}\right)}{2\gamma^{2}}$$

(magnetostatic undulator)

Example : for
$$\lambda_R = 1A$$
, $\lambda_w = 2cm$, $E = 7 GeV$
 $a_w = 0.93\lambda_w [cm]B_w[T]$

$$\lambda_{R} = \lambda \frac{\left(1 + a_{0}^{2}/2\right)}{4\gamma^{2}}$$

(electromagnetic undulator)

Example : for $\lambda_R = 1A$, $\lambda = 0.8 \mu m$, E = 25 MeV

 $a_0 = 4.8 \frac{\lambda [\mu m] \sqrt{P[TW]}}{R_0 [\mu m]} \rightarrow laser power$