COMPARING NEW MODELS OF TRANSVERSE INSTABILITY WITH SIMULATIONS*

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Abstract

Recently, Balbekov and Burov have produced an ordinary integro-differential equation that approximates the Vlasov equation for beams with wakefields and large space charge tune shift. The present work compares this model with simulations. In particular, the claim that certain types of transverse wakes cannot lead to mode coupling instabilities is explored

INTRODUCTION

In most low energy ion accelerators the transverse impedance is dominated by space charge, which derives from the mutual electrostatic repulsion of particles within the beam. Space charge alone is purely reactive and does not cause instabilities under conditions typical for a synchrotron [1]. The effect of space charge in conjuction with other sources of impedance is unclear. In coasting beam theory space charge creates a large real coherent tune shift which overcomes collisionless damping and only a small amount of transverse resisitance is needed to cause instability. In bunched beams, with wake potentials of one sign only, the space charge force tends to stabilize the beam [2, 3, 4, 5, 6]. Single sign wake potientials, such as those created by wall resistivity and matched stripline beam position monitors always have the same sign, consistent with energy absorbtion by a passive device. The standard convention is to take this sign as negative, [2] uses the opposite convention, but this work will use the standard convention.

SIMULATION MODEL AND RESULTS

Take the azimuth θ as the timelike variable. Consider a bunched beam with matched longitudinal phase space distribution $\Psi(\tau, v)$ where τ is arrival time with respect to the synchronous particle and $v = (1/Q_s)d\tau/d\theta$. Set $\rho(\tau) = \int dv\Psi(\tau, v)$ and normalize so that $\int d\tau\rho = 1$. Assume a bare tune Q_0 and take the time dependence of the dipole density to be $D(\tau, v, \theta) = \exp[-i(Q_0 + \Delta Q)\theta]D(\tau, v)$, with tune shift small enough to neglect coupling between betatron sidebands. Then,

$$-i\Delta Q D(\tau, v) = Q_s\left\{v\frac{\partial D}{\partial \tau} - \frac{dU}{d\tau}\frac{\partial D}{\partial v}\right\}$$
$$- \frac{i\Psi(\tau, v)}{2\kappa Q}\int_{\tau_b}^{\tau} d\tau_1 W(\tau - \tau_1)\int dv_1 D(\tau_1, v_1)$$

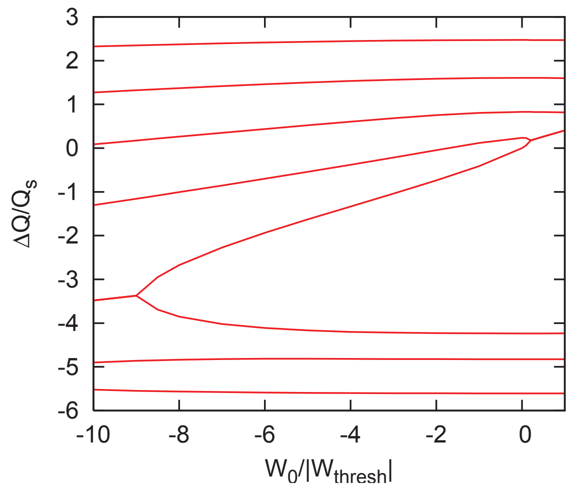

Figure 1: Real part of eigenvalues for the hollow bunch in a square longitudinal well versus step wake divided by absolute value of wake at threshold without space charge. The space charge tune shift is $\Delta Q_{sc} = 4Q_s$. For negative (passive) wakes space charge stabilizes the beam.

$$+ \frac{i\Delta Q_{sc}}{\rho(0)}\rho(\tau)D(\tau, v)$$
$$- \frac{i\Delta Q_{sc}}{\rho(0)}\Psi(\tau, v)\int dv_1 D(\tau, v_1), \qquad (1)$$

where Q_s is the synchrotron tune, ΔQ_{sc} is the space charge tune shift in the center of the bunch, $W(\tau)$ is the wake potential, and κ is a positive constant that depends on particle energy, charge etc. The longitudinal potential $U(\tau)$ is taken to be parabolic, with $U = \tau^2/2$, or a square well. Dividing through by Q_s one sees that $W(\tau)/Q_s$ and $\Delta Q_{sc}/Q_s$ determine system stability.

The origin of bunched beam stabilization from space charge can be seen most easily in the air-bag square well model [7, 2]. In this model the RF is approximated by a square potential well. All the particles have the same synchrotron amplitude so there is a well defined synchrotron tune, Q_s. The chromaticity ξ is zero and the wakefield is a step function. With these assumptions equation (1) reduces to a set of linear, ordinary first order differential equations with constant coefficients. The problem requires numerical solution, but the solutions are exact for practical purposes. The coherent tune shifts ΔQ are real below the instability threshold and come in complex conjugate pairs above threshold. Figure 1 show the real tune shifts, in units of the synchrotron tune versus the wake strength in units of the magnitude of the threshold value for no space charge.

From the figure it is clear that the threshold wake po-

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tential with this amount of space charge is about 9 times higher than the threshold without space charge. The behavior for positive wakes is similar to that shown in Fig 2 of [6] whereas that reference did not consider the threshold for negative wakes. Figure 4 of [3] considers negative wakes, though there is a sign difference in the axes between his plot an mine. Also, my plot shows coupling between the 0 and -1 synchrotron modes while his, for a gaussian bunch, starts with coupling between the +2 and +3 synchrotron modes. It is difficult to come up with a numerical comparison of Burov's thresholds and mine, but some stabilization due to space charge is present in both works. The question is whether this behavior is generic, extending to other bunch shapes and wake potentials. Given the importance of this problem the author chose to undertake a fairly systematic simulation campaign.

The simulations used the code TRANFT [8]. Only single bunch phenomena are considered so the transverse voltage kick is given by

$$V_x(x,t) = \int_{-\tau_b}^{\tau_b} \left[xW_d(\tau)I_b(t-\tau) - W_x(\tau)D_x(t-\tau)\right] d\tau, \tag{2}$$

where τ_b is the full bunch length, $W_d(\tau)$ is the detuning wake [9, 10, 11], $W_x(\tau)$ is the usual transverse wake potential, and $D_x(t)$ is the instantaneous dipole density. Note that $D_x(t)$ is the product of the instantaneous current and the instantaneous value of x. For space charge

$$W_d(\tau) = \frac{Z_0 \ell}{2\pi\beta^2\gamma^2 a^2}\delta(\tau)$$

where $Z_0 = 377\Omega$, ℓ is the distance the beam travels between updates, $\beta = v/c$, $\gamma = 1/\sqrt{1-\beta^2}$, $a = 2\sigma_x$ is the radius of a uniform equivalent beam. The wake potential from space charge is $W_x(\tau) = W_d(\tau)(1 - a^2/b^2)$ where b is the radius of the beam pipe. For the work here I approximate $a/b = 0$. For numerical implementation the delta functions are replaced by gaussians of width σ_τ and the calculations are done on grids with spacing $\lesssim \sigma_\tau/5$. Two different bunch shapes were used, a square bunch of full width 12 ns at base and a smooth bunch with current profile $I(t) = I_{peak}(1 - t^2/\tau^2)^{3.5}$ with $\tau = 6$ ns. Three different wake potentials were studied and are shown in Figure 2.

Several simulations were run using $\sim 10^5$ turns, $\sim 10^5$ macroparticles and 19 space charge kicks per betatron oscillation. The dependence of the wake strength versus space charge tune shift is summarized in Figure 3. The two red curves show the threshold for the step wake (red curve in Fig 2) versus space charge tune shift in units of synchrotron tune. The curve starting at +1 in Figure 3 corresponds to a non-passive wake that requires an amplifier. The red curve starting at −1 corresponds to the wake potential of a long matched stripline. For $\Delta Q_{sc} < 3Q_s$ space charge stabilizes the beam. Figure 4 shows the growth rate versus wake strength for the step wake with a smooth bunch for $\Delta Q_{sc} = 6Q_s$. The threshold wake potential is larger

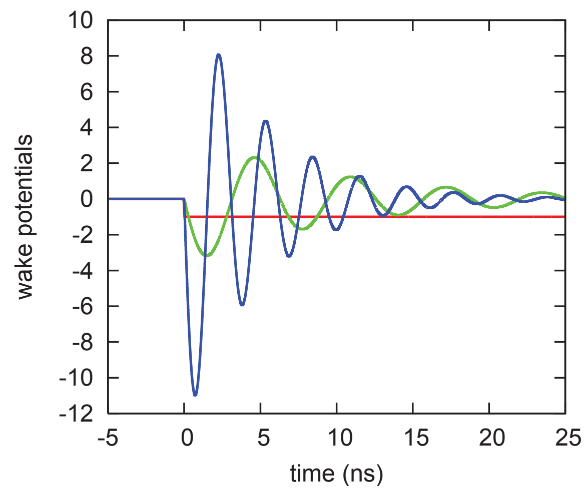

Figure 2: Wake potentials used in the simulations. The magnitudes are in the same ratio as the threshold values for the smooth bunch

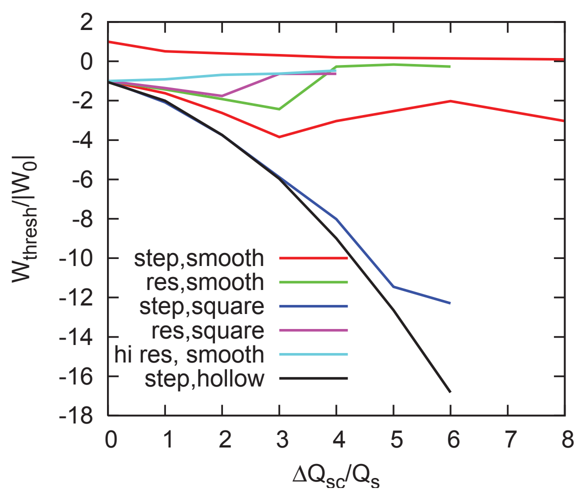

Figure 3: Threshold values of the wake potential scaled by the magnitude at threshold without space charge versus space charge tune shift.

than for no space charge, but significanctly smaller than the threshold for $\Delta Q_{sc} = 3Q_s$. Note that doubling the number of macroparticles has little effect. The dark blue and black curves in Figure 3 show thresholds for square bunches in linear RF and square well, respectively. For both these space charge stabilizes the beam, but the improvement for the square bunch in the linear RF show signs of turning over. The green and magenta curves show the thresholds for smooth and square bunches with the green resonator wake in Figure 2. There is some small improvement for small space charge tune shifts but things turn over quickly and thresholds are soon smaller than without space charge. The light blue curve is for the smooth bunch with the high frequency wake; the dark blue curve in Figure 2. For this wakefield space charge always reduces stability.

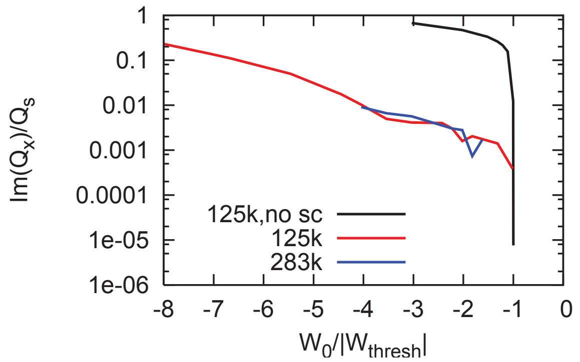

Figure 4: Growth rate versus wake strength for a step wake with a smooth bunch for $\Delta Q_{sc} = 6Q_s$. Results for 125,000 and 283,000 macroparticles are shown. Growth rates for 125k particles and no space charge are shown for comparison.

Some of the results in Figure 3 are in qualitative agreement with Figure 7 in [3]. For space charge tune shifts that are not too big, space charge stabilizes the fast head tail instability. However for larger tune shifts space charge destabilizes the beam. The earlier work [2] stressed single sign wakes like the red, blue and black curves in Figure 3. For these parameter regimes space charge is more consistently stabilizing. However, the growth rate measurements in Figure 4 are compelling. The stabilizing effect of space charge is not monotonic with the size of ΔQ_{sc}, even for single sign wakes. Also, from Figure 1 it is clear that the coupling leading to instability can occur between the 0 and -1 synchrobetatron sidebands. The coupling is not limited to the positive sidebands as considered in [3] and [6]. To explore which modes couple more fully consider Figure 5. For a resonator wake (green curve in Fig 2) and a 12ns hollow bunch, the mode coupling occurs between the $m = -3$ and $m = -4$ synchrobetatron sidebands. Figure 6 shows the comparable set of curves for the high frequency resonator wake. In this case the coupling occurs between the $m = -7$ and $m = -8$ synchrobetatron sidebands. The threshold wake for the high frequency resonater is increased for the hollow bunch, which is opposite to the behavior of the light blue curve in Figure 3. This means that the hollow bunch model overestimates the benefits of space charge and this model cannot be relied on for design estimates. At this point it appears there is no substitute for extensive simulations.

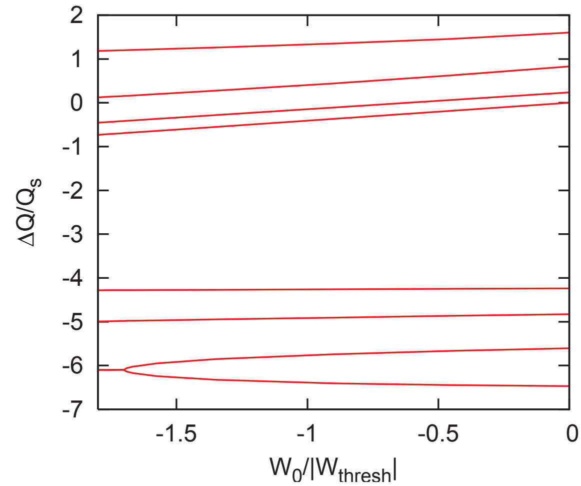

Figure 5: Real part of eigenvalues for the hollow bunch in a square longitudinal well versus resonator wake (green curve in Figure 2) divided by absolute value of wake at threshold without space charge. The space charge tune shift is $\Delta Q_{sc} = 4Q_s$.

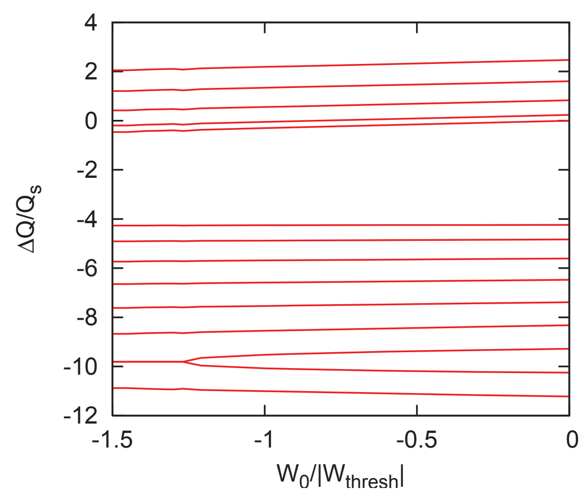

Figure 6: Real part of eigenvalues for the hollow bunch in a square longitudinal well versus high frequency resonator wake (blue curve in Figure 2) divided by absolute value of wake at threshold without space charge. The space charge tune shift is $\Delta Q_{sc} = 4Q_s$.

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