SERPENTINE ACCELERATION IN SCALING FFAG

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Abstract

The longitudinal motion with serpentine acceleration in scaling FFAG is examined in this paper. With this acceleration scheme in scaling FFAG, high-energy and high-current beam can be obtained in non-relativistic energy region. In this paper, longitudinal Hamiltonian in scaling FFAG has been derived analytically, and the ring design of proton driver with linear transfer matrix method for ADS is also presented.

INTRODUCTION

High beam power accelerator to produce intense secondary particle beams are desired for ADS (Accelerator Driven System) [1]. Linear accelerators have been considered as a proper candidate so far. An alternative candidate is an FFAG (fixed-field alternating gradient) accelerator [2]. There are two types of FFAG; the non-scaling type and the scaling type. The scaling FFAG ring is composed of non-linear magnetic fields so that the betatron tune is constant for every particle momentum, contrary to the non-scaling FFAG accelerator.

In order to obtain large current beam, In FFAG accelerators, the acceleration scheme with fixed rf frequency has been proposed. In scaling FFAG, the stationary bucket acceleration [3, 4] has been considered. In this scheme, however, the total acceleration energy gain is limited by the bucket height. In order to make a large bucket height, the acceleration in the relativistic energy region is preferable. On the other hand, in non-scaling FFAG, serpentine acceleration [5] has been considered. In order to minimize orbit shifts during acceleration, the parabolic variation in orbit length with energy is created by the appropriate selection of parameters [5]. At the bottom of the parabola, the momentum compaction approaches zero. Since the beam has to cross the transition energy during acceleration, the slippage factor has to change sign. In consequence, only relativistic particles ($\gamma \gg 1$) can be accelerated in this scheme [5]. However, if serpentine acceleration can also be applied to the scaling FFAG, high power beam can be obtained even in the non-relativistic energy region as well.

In this paper, the longitudinal hamiltonian with long drift space in scaling FFAG is derived analytically first. Then the preliminary ring design of proton driver for ADS based on serpentine acceleration is also presented with linear transfer matrix method.

LONGITUDINAL HAMILTONIAN IN SCALING FFAG

In longitudinal particle dynamics with constant rf frequency acceleration, the traveling wave is written as

$$\phi = 2\pi f_{rf} \cdot T - h\Theta, \tag{1}$$

where h is the harmonic number, f_{rf} is the rf frequency, Θ is the azimuthal angle, and T is the revolution period of non-synchronous particle. Using azimuthal angle Θ as an independent variable, the equations of motion and longitudinal Hamiltonian are expressed as follows;

$$\frac{d\phi}{d\Theta} = h\left(\frac{P_s^{1-\alpha}}{E_s} E P^{\alpha-1} - 1\right), \tag{2}$$

$$\frac{dE}{d\Theta} = \frac{eV_{rf}}{2\pi} \sin\phi, \tag{3}$$

$$\begin{aligned} H(E,\phi;\Theta) = h\Big(\frac{1}{\alpha+1} \frac{\sqrt{E^2-m^2}^{\,\alpha+1}}{E_s\sqrt{E_s^2-m^2}^{\,\alpha-1}} - E\Big) \\ + \frac{eV_{rf}}{2\pi}\cos\phi, \end{aligned} \tag{4}$$

where V_{rf} is the rf voltage per turn, R_s, P_s, E_s are the mean radius at arc section, the momentum and the energy of synchronous particle, respectively, α is the momentum compaction and m is the rest mass of the beam particle.

Longitudinal Phase Space in Non-relativistic Energy Region

When the rf frequency is fixed near the transition energy, serpentine channel between the two stationary buckets appears. Serpentine channel can be found in non-relativistic energy region as shown in Fig. 1.

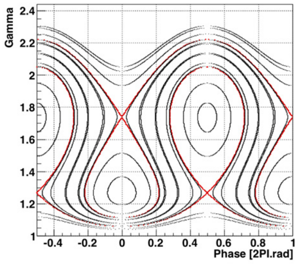

Figure 1: Longitudinal phase space near the transition energy. Red lines are the separatrixes. A serpentine channel is created between two separatrixes.

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PROTON RACE TRACK RING DESIGN FOR ADS

The accelerator for ADS is required to make proton beam up to several tens of MW power with over 1 GeV beam energy. A low-injection energy around 500 MeV is desired. These requirements can be achieved with serpentine acceleration in scaling FFAG. From the study of serpentine acceleration in longitudinal motion so far, in order to make injection energy lower, small k-value is required. Small harmonic number is also desired for decreasing rf voltage per turn. Since the circumference of the ring should be smaller to satisfy these requirements, long drift spaces to put many rf cavities are considered. In this case, focusing elements have to be installed in a long straight section to keep the beam stable [6]. Since the field laws in circular and straight sections are different, discontinuity of reference trajectories can occur at the border between these two sections. In order to combine the reference trajectories at the border, dispersion functions in circular and straight sections must be matched at one momentum P_0 [6].

In order to satisfy the zero-chromaticity in a race track ring, ring tune needs to be constant for different momenta. Once geometrical configuration is given in the straight section, horizontal and vertical tunes are independent of momentum. Then if horizontal and vertical phase-advance are satisfied with $n \times \pi$ (n is integer) in the circular part, ring tunes of horizontal and vertical are also constant.

In this section, preliminary proton race track ring design is presented analytically in longitudinal, and then in transverse with linear transfer matrix method.

Longitudinal Design

The phase equation with long straight sections can be derived as

$$\frac{d\phi}{d\Theta} = h\left[\left(\frac{1}{1+\frac{nL}{2\pi R_s}}\right)\frac{EP_s}{E_s P}\left(\frac{P^\alpha}{P_s^\alpha}+\frac{nL}{2\pi R_s}\right)-1\right], \tag{5}$$

where n is the number of straight sections in a ring and L is the length of a straight section. From Eqs. 3 and 5, the longitudinal Hamiltonian with long straight sections can be derived as

$$\begin{aligned} H(\phi, E, \Theta) = \\ h\Bigg[\frac{1}{(1+\frac{nL}{2\pi R_s})}\Bigg(\frac{1}{1+\alpha}\frac{\sqrt{E^2-m^2}^{1+\alpha}}{E_s\sqrt{E_s^2-m^2}^{\alpha-1}} \\ +\frac{nL}{2\pi R_s}\frac{\sqrt{E_s^2-m^2}}{E_s}\sqrt{E^2-m^2}\Bigg)-E\Bigg]+\frac{eV}{2\pi}\cos\phi. \end{aligned} \tag{6}$$

The longitudinal parameters of proton race track ring for ADS are presented in Table 1. The resulting longitudinal phase space is shown in Fig. 2. The initial energy is 500 MeV, and the final energy is around 2.2 GeV.

Transverse Design

The first step is to determine the characteristics of the closed orbit for given geometrical parameters of the cell. For simplicity reasons we assume that the curvature of the closed orbit is constant in each magnet, and null in the drift spaces between magnets. The determination of the closed orbit parameters in the FDF arc section and the DFD straight section are drawn in Fig. 3 and Fig. 4. A schematic view of half of the ring is also drawn in Fig. 5.

Table 1: Longitudinal Parameters for the Proton Race Track Ring

k at circular section	2.8
Mean radius of the arc part (500 MeV) [m]	10
Long straight section [m]	12
Stationary energy below transition[MeV]	820
rf voltage [MV/turn]	23 (h=1)
rf frequency [MHz]	2.75 (h=1)

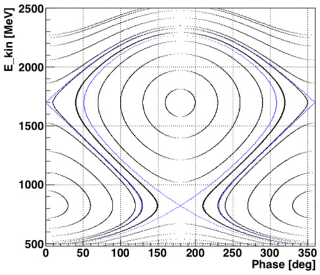

Figure 2: Longitudinal phase space. Blue lines are separatrixes.

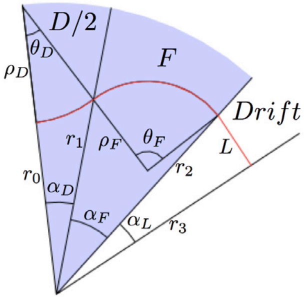

Figure 3: Schematic view of half FDF cell in the arc section.

The relative strength x between F and D magnets is defined as $x = \theta_F/\theta_D$ in the arc section. From the geometrical consideration in Fig. 3, the unknown parameters: θ_F, θ_D, ρ_D, ρ_F, and L can be expressed with x, α_F, α_D, α_L, and r_1. In the straight section, the unique solutions ρ_F, ρ_D, and θ can be obtained from known parameters: l_F, l_D, l_1 defined in Fig. 4.

In the circular section with linear approximation, the field index n can be written with the geometrical filed index k as $n = \pm\frac{\rho}{r}k$ where the $+$ and $-$ signs corresponding

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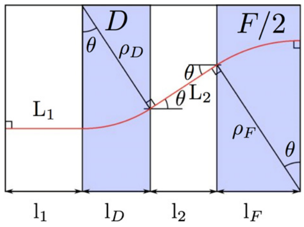

Figure 4: Schematic view of half DFD cell in the straight section.

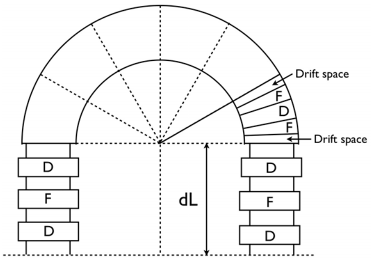

Figure 5: Schematic view of half of the ring. The ring is composed of circular sections FDF cells, and long straight sections DFD cells. dL is a long drift space.

to the case of D and F magnets, respectively. The normalized field gradient m in the straight section is defined as $m = \frac{1}{B}\frac{dB}{d\chi}$ where χ is average abscissa and B is the magnetic field. In the straight section with linear approximation, the field index n can be written with the normalized field gradient m as $n = \pm m\rho$ where the $+$ and $-$ signs corresponding to the case of D and F magnets, respectively.

Horizontal and vertical beta-functions are obtained from linear transfer matrix. Matrix elements are calculated with the closed orbit geometrical parameters and the field index. The resulting horizontal and vertical beta-functions are shown in Fig. 6. Values of parameters represented in Fig. 3 and Fig. 4 used for calculations are presented in Table 2.

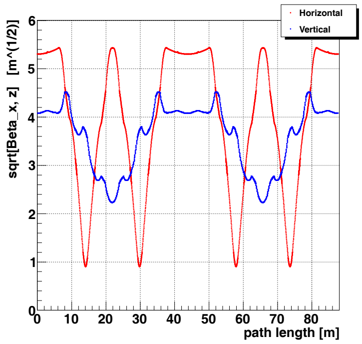

Figure 6: Horizontal (red) and vertical (blue) beta functions for the proton race track ring.

Table 2: Geometrical Parameters for the Proton Race Track Ring

Circular section : FDF scaling FFAG cell	
k	2.8
α_F and α_D [deg]	10.2, 3.58
α_L [deg]	1.22
θ_F and θ_D [deg]	22.7, 7.65
Mean radius (500 MeV) [m]	10
ρ_F and ρ_D (at 500 MeV) [m]	4.58, 4.69
Horizontal phase advance per cell [deg]	60.8
Vertical phase advance per cell [deg]	30.2
Straight section : DFD scaling FFAG cell	
l_F and l_D [m]	0.17, 0.16
l_1 [m]	2.62
θ [deg]	1.67
ρ_F and ρ_D [m]	5.85, 5.50
Horizontal phase advance per cell [deg]	0.75
Vertical phase advance per cell [deg]	20.2
Circumference (at 500 MeV) [m]	87.7
Horizontal ring tune	2.14
Vertical ring tune	1.23

SUMMARY

In order to obtain high power beam in non-relativistic energy region, serpentine acceleration has been proposed for the scaling FFAG. The longitudinal hamiltonian has been derived analytically. Preliminary ring design of proton race track ring for ADS has also been done with linear transfer matrix method. Horizontal and vertical beta-functions of the ring with 5 m of long drift space to put many rf cavities in the straight section have been computed. For the next step, experiments for the demonstration of serpentine acceleration in scaling FFAG have to be taken. Further ring design of proton driver for ADS will also be done.

REFERENCES

[1] K. Mishima et al, *Journal of Nuclear Science and Technology*, vol.44(2007), NO3 Special Issue on GLOBAL 2005 p.499-503.

[2] C. Ohkawa, in *Proc. of JPS*, (1953).

[3] Y. Mori, *International Workshop on FFAG Accelerators(FFAG2006)*, FNAL, Chicago, USA(2006).

[4] T. Planche, *Nuclear Inst. and Methods in Physics Research, A*, vol.622, No.1, p.21-27 (2010).

[5] R. Barlow et al, *Nuclear Inst. and Methods in Physics Research, A*, vol.624, No1, p.1-19 (2010).

[6] J.-B. Lagrange, T. Planche, and Y. Mori, *International Journal of Modern Physics A*, vol. 26, p.1785-1793 (2011).