# EFFECT OF BEAM-BEAM INTERACTIONS ON STABILITY OF COHERENT OSCILLATIONS IN A MUON COLLIDER*

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*Abstract*

In order to achieve peak luminosity of a muon collider in the $10^{34}$/cm$^2$/s range the number of muons per bunch should be of the order of a few units of $10^{12}$ rendering the beam-beam parameter as high as 0.1 per IP. Such strong beam-beam interaction can be a source of instability if the working point is chosen close to a coherent beam-beam resonance. On the other hand, the beam-beam tunespread can provide a mechanism of suppression of the beam-wall driven instabilities. In this report the coherent instabilities driven by beam-beam and beam-wall interactions are studied with the help of BBSS code for the case of 1.5 TeV c.o.m muon collider.

## INTRODUCTION

Muon Collider (MC) - proposed by G.I. Budker and A.N. Skrinsky more than 40 years ago – is now considered as the most exciting option for the energy frontier machine in the post-LHC era. It has a number of important advantages over e+e– colliders: better energy resolution, larger cross-section of scalar particles production etc. [1]. However, taking into account relatively high transverse emittance which can be obtained with ionization cooling, the bunch population should be as high as ~$2\cdot10^{12}$ in order to achieve competitive luminosities. This brings to the forefront the beam-beam effects, coherent instabilities and their interplay.

In this report we present the results of strong-strong beam-beam simulations for the baseline scheme of 1.5TeV c.o.m muon collider.

## MC BASELINE PARAMETERS

An important feature of a muon collider necessary for achieving high luminosity is small beta-function at IP, $\beta^* \leq 1$cm, which is more typical for e+e– factories than for TeV-range circular machines. As a result the final focus quadrupoles excite very strong chromatic beta-wave which should be suppressed with sextupoles as close to the origin as possible. The beam-beam interaction changes phase advances between the sextupoles across the IP making the problem more complicated: it must be taken into account already at the stage of lattice design.

A successful IR lattice design providing sufficiently large momentum acceptance and dynamic aperture of the whole ring with little sensitivity to the beam-beam effect was presented in [2]. Its basic parameters are cited in Table 1. The key to success was to arrange the optics so that the sextupoles correcting chromaticity in one plane were located at minima of beta-function in the other plane with phase advances from IP being multiples of $\pi$. In the result the beam-beam interaction reduces the beta-function values at the minima further suppressing spherical aberrations produced by these sextupoles.

Table 1: Muon Collider Parameters

| ***Parameter*** | ***Unit*** | ***Value*** |
|---|---|---|
| Beam energy | TeV | 0.75 |
| Repetition rate | Hz | 15 |
| Average luminosity / IP | $10^{34}$/cm$^2$/s | 1.1 |
| Number of IPs | - | 2 |
| Number of bunches / beam | - | 1 |
| Circumference, $C$ | km | 2.73 |
| $\beta^*$ | cm | 1 |
| Momentum compaction, $\alpha_p$ | $10^{-5}$ | -1.3 |
| Normalized emittance, $\varepsilon_{\perp N}$ | $\pi$·mm·mrad | 25 |
| Momentum spread | % | 0.1 |
| Bunch length, $\sigma_s$ | cm | 1 |
| Number of muons / bunch | $10^{12}$ | 2 |
| Beam-beam parameter / IP, $\xi$ | - | 0.09 |
| RF voltage at 800 MHz | MV | 16 |
| Betatron tunes | - | 20.56 / 16.58 |
| Synchrotron tune | - | 0.00057 |

## DYNAMIC BETA EFFECT

Conventional wisdom suggests choosing phase advances between IPs to be just above multiples of $\pi$. Then the beam-beam interaction reduces the beta-function values at IPs enhancing the luminosity – the phenomenon known in circular colliders as the “dynamic beta effect”.

However, this effect increases beta-functions at the IR quadrupole locations and may be detrimental in a TeV-range muon collider where the quadrupole aperture is restricted by high gradient requirements and the necessity of protection from the muon decay products [3]. From this point of view the “neutral” phase advances – odd multiples of $\pi/2$ – are preferable. Then with 2 IPs we get

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half-integer tunes which are also beneficial for orbit stability and low detuning with amplitude in a bare lattice.

Still - as R. Palmer pointed out - there will be some luminosity enhancement by the beam-beam interaction due to a large length of bunches ($\sigma_s \sim \beta^*$) which is akin to the "disruption" effect in linear colliders. Strong-strong simulation of this effect in linear approximation for the beam-beam force showed that at given parameters it almost completely compensates for the luminosity reduction due to the "hour-glass" effect [4].

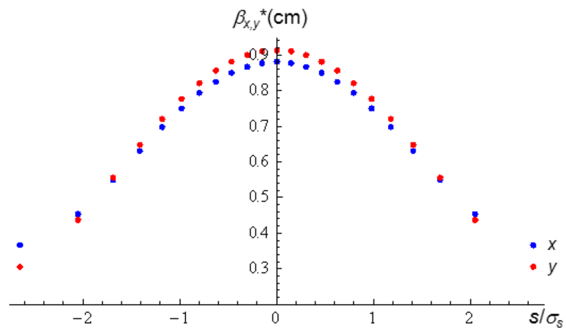


Figure 1: Self-consistent beta-functions at the IP vs. slice position in the bunch.

In these calculations a Gaussian longitudinal profile was assumed and represented by a number of slices (23 per bunch) according to Zholents-Shatilov algorithm [5]. Self-consistent beta-functions seen by all slices at the IP are presented in Fig.1 as functions of their position in the bunch which was considered frozen.

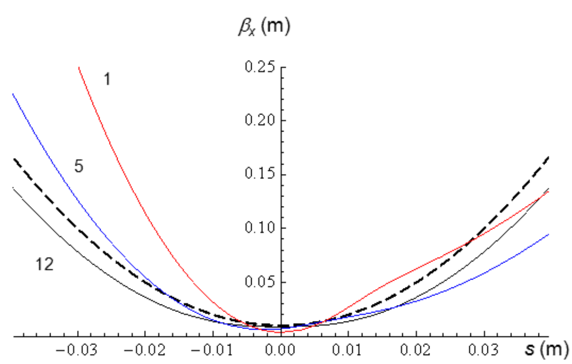


Figure 2: Horizontal beta-function seen around IP by slice #1 (2.65$\sigma_s$ from the bunch center), #5 (1.18$\sigma_s$ from the center) and #12 (central). Dashed line shows bare lattice values.

Even more important is large difference in beta-function values for different slices over the interaction length (Fig. 2) and at large distance from the IP. As a result the beam-beam tuneshifts vary strongly along the bunch (Fig. 3). For the edge slices they exceed by ~50% the cited in Table 1 value which does not take into account the dynamic beta effect.

The found strong enhancement of the beam-beam tuneshifts and beta-functions at large distances from the IP can pose serious problems which should be studied in more detail. As possible cures we may consider reducing ratio $\sigma_s/\beta^*$, truncating the tails of longitudinal distribution and increasing the tunes.

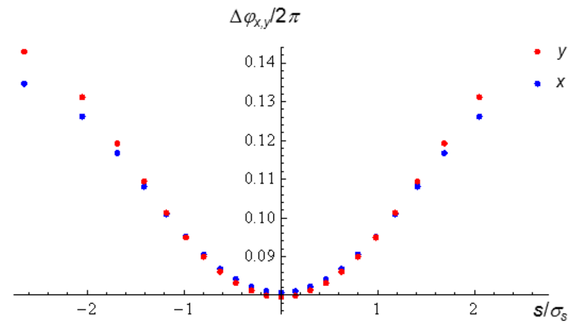


Figure 3: Self-consistent beam-beam tuneshifts per IP vs. slice position in the bunch.

## COHERENT BEAM-BEAM OSCILLATIONS

To study the effect of beam-beam interaction on coherent oscillations we used the BBSS code which employs the extended "synchro-beam" mapping [6, 7] to the arbitrary potential. In simulations below $2{\cdot}10^6$ particles per bunch were used.

### *Beam-Beam Resonances*

With the chosen working point the most danger can present the third-integer resonance or – in the language of coherent modes – resonance of dipole-quadrupole oscillations, $Q_{\text{dipole}} + Q_{\text{quad}} \approx 3Q_x \approx integer$.

For such a resonance to occur there should be an offset at the IP [8]. Though the beams circulate in the same ring an almost constant offset can be produced by a spurious dispersion and difference in energy of the two beams which varies very slowly due to low synchrotron tune (see Table 1).

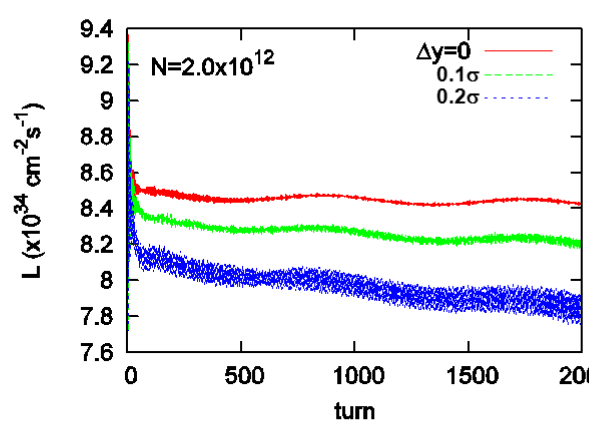


Figure 4: Instantaneous luminosity vs. turn number for three values of constant offset.


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From Fig. 4 it follows that the relative reduction in luminosity due to offset $\Delta$ is rather mild, $\approx 0.35\ \Delta/\sigma$ in 2000 passes through IPs (referred to as "turns"). However, there is slow degradation of luminosity even in the case of $\Delta = 0$, probably by high order resonances. The fast drop in luminosity within first 20-50 turns can be attributed to the initial mismatch: the matching conditions are different for different slices of the bunch due to large variation of the dynamic beta functions (Fig. 2), therefore they were fulfilled only on average.

### *Beam-Breakup Instability*

Since the synchrotron tune in a muon collider is very low and the longitudinal motion is virtually frozen the beams are subject to a transverse BBU-like instability known in linear accelerators. Its rise-time in a single bunch may be as short as a few hundred turns [9]. A possibility was discussed of using RF quadrupoles to taper the tunes along the bunch and provide BNS damping [9].

The beam-beam interaction may render such a complication unnecessary: the instability – which is in essence a single-particle response to the wakefield generated by the head of the bunch – should be strongly suppressed by the beam-beam tunespread. For parameters of Table 1 the decoherence time of the initial (driving) perturbation can be estimated as $1/(N_{IP}\xi) \approx 5$ turns.

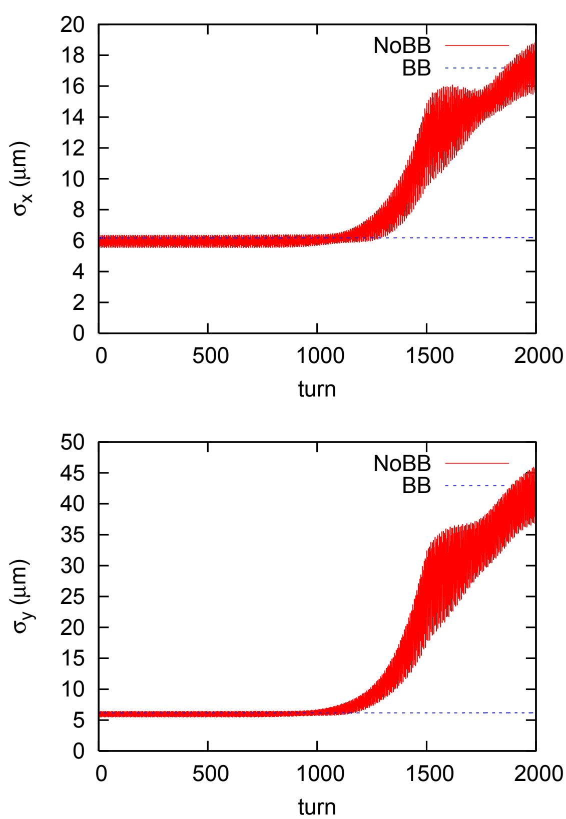

Figure 5: R.m.s. beam sizes vs. turn number with beam-beam interaction on and off.

We studied this instability numerically for a narrow-band wake with frequency 4.8 GHz and magnitude which renders the growth parameter O = 0.01 in both planes. Figure 5 shows the evolution of beam sizes with beam-beam interaction on and off starting from statistical noise.

As expected, the beam-beam tunespread completely suppresses the growth. Figure 6 shows the effect of BBU instability on instantaneous luminosity with and without beam-beam interaction (though of course luminosity without beam-beam interaction has a purely symbolic meaning).

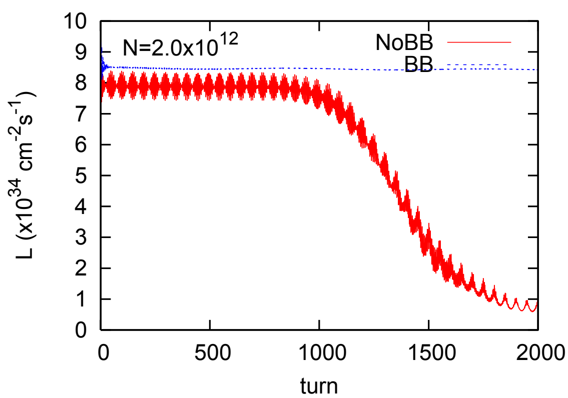

Figure 6: Instantaneous luminosity vs. turn number with beam-beam interaction on and off.

## ACKNOWLEDGEMENTS

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