

# LONGITUDINAL DYNAMICS OF INTENSE HEAVY-ION BUNCHES IN SIS-100\*

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## Abstract

In the SIS-100 highest heavy ion intensities have to be accelerated to deliver beam to the FAIR experiments. In the projected SIS-100 synchrotron the heavy ion bunches will be strongly affected by the longitudinal space charge force. Due to the limited RF bucket area all mechanisms which might cause longitudinal phase space dilution must be understood and controlled. Space charge effects, like the reduction in the RF voltage and the loss of Landau damping, have already been part of elaborate studies. It has been shown that cavity beam loading can deform the flattened bunch shape in the dual RF bucket. Among the different counter measures an inductive insert has been proposed in order to partially compensate the longitudinal space charge impedance. Optimized settings for the difference between the two RF phases in a dual RF bucket might be an option to reduce the effect of beam loading. In this contribution we will analyse the matched bunch distribution for SIS-100 parameters in single and dual RF buckets. Analytical and numerical studies of the interplay of longitudinal space charge, cavity beam loading and an inductive insert will be presented.

## INTRODUCTION

Different waveforms and the stability of coherent synchrotron oscillations of Gaussian heavy ion distributions under space charge (SC) are described in [1] and [2]. There is shown that it reduces the RF voltage amplitude below transition energy and that it leads to loss of Landau damping. It is shown that beam loading (BL) caused by the real part of the impedance of the RF cavities can deform bunch shapes by energy loss.

To compensate the SC effect at the 10th harmonic in SIS-100 an inductive insert has been proposed. It can counterbalance the 20th harmonic partially. This is shown in [3]. The insert has an impedance where the reactive (inductive) part below transition is dimensioned to be equal to the capacitive SC impedance of the beam. The resistive part of the insert, given by the used material, is responsible for the losses leading to phase shift and bunch form deformation by potential-well distortion (PWD) [4] both in single and dual harmonic RF systems. This adds up to the BL of the RF cavities in the synchrotron SIS-100.

The phase shift and beam distribution deformation can be described by the Haissinski equation [4] for Gaussian lon-

gitudinal ion distributions. The phase shift of a single harmonic RF system can be corrected by giving the RF voltage a synchronous phase  $\Phi_{S1}$  as during acceleration. The beam distribution deformation cannot be corrected; it yet increases because of the accelerating bucket form.

This proceeding will concentrate on the description of the correction of phase shift and bunch form deformation caused by PWD in dual harmonic RF systems. The impedance of the insert described in [3] will be used as an example to show the effects in comparison with a  $Q = 0.1$  impedance. It will be described that SC impedance makes the correction easier and that the correction is quality factor  $Q$  dependent and leads to RF voltage amplitude reduction.

## PHASE SHIFT- AND BUNCH FORM DEFORMATION CORRECTION

For correcting phase shift and bunch form deformation a dual harmonic RF system has to be applied as described by Equation 1 where  $\alpha = \frac{V_2}{V_1} = 0.5$  and  $\frac{h_2}{h_1} = 2$ . It reduces the SC effect by bunch flattening and makes it possible to countervail for the beam distribution deformation by adjusting the right phase difference between main and second RF harmonic  $\Delta\Phi_S$ . The phase shift correction is done as in a single harmonic RF system by adjusting the synchronous phase  $\Phi_{S1}$  to the main harmonic.

$$V_{RF} = V_0(\sin\Phi - \sin\Phi_{S1} - \alpha(\sin(\Phi_{S2} + \frac{h_2}{h_1}(\Phi - \Phi_{S1}) + \Delta\Phi_S) - \sin\Phi_{S2})) \quad (1)$$

$$\Delta\Phi_S = \Phi_{S1} - \Phi_{S2}$$

In Figure 1 the black pointed curve shows the dual harmonic beam distribution without PWD and SC impedance  $X_C$  (defined in [3]) with the  $1\sigma$ -bunch length of the main harmonic of 0.7 radat injection energy. The red triangles (number of  $U^{28+}$  ions  $N_b = 4 \cdot 10^{10}$ ), green squares ( $N_b = 9 \cdot 10^{10}$ ), blue stars ( $N_b = 13 \cdot 10^{10}$ ) and pink line ( $N_b = 20 \cdot 10^{10}$ ) give examples for the phase shift and beam distribution deformation for different  $N_b$  with the quality factor  $Q = 0.1$ ,  $f_{RF} = 1.6$  MHz and a shunt impedance  $R_{Sh} = 16$  k $\Omega$  using Equation 2 (top). Only PWD was observed. The beam interacts with the longitudinal part of the resistive impedance. The Haissinski equation only contains this resistive part which leads to the observed bunch form deformation.

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$$Z_{Sh||} = \frac{R_{Sh}}{1 + iQ\left(\frac{\omega}{\omega_{RF}} - \frac{\omega_{RF}}{\omega}\right)} \quad (2)$$

$$Z_{Sum||} = Z_{Sh||} - iX_C$$

The difference between Figure 1 and Figure 2 is that in the second one  $X_{SC}$  was taken into account (Equation 2, bottom). It can be seen that the phase shift, the bunch form deformation, the bunch length and the peak of the bunch form distribution decrease. The amplitude of the right side of the distribution increases. Therefore the SC impedance itself works against PWD.

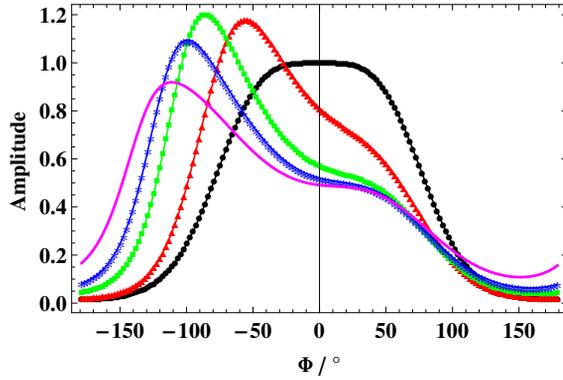


Figure 1: Beam distribution deformation and phase shift by PWD using  $Z_{Sh||}$  in Equation 2. The quality factor  $Q = 0.1$ . Black points: no PWD ( $N_b = 0.0$ ), red triangles:  $N_b = 4 \cdot 10^{10}$ , green squares:  $N_b = 9 \cdot 10^{10}$ , blue stars:  $N_b = 13 \cdot 10^{10}$  and pink line:  $N_b = 20 \cdot 10^{10}$ .

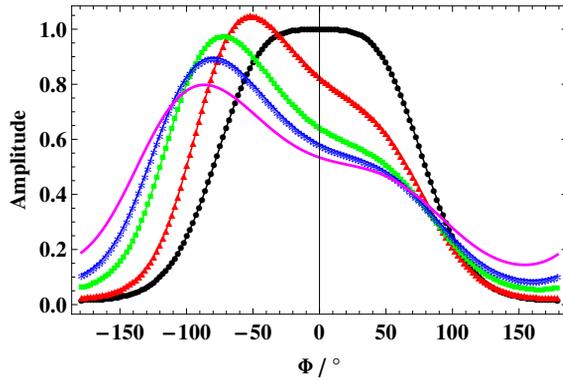


Figure 2: Beam distribution deformation and phase shift by PWD using  $Z_{SC||}$  in Equation 2. The quality factor  $Q = 0.1$ . Black points: no PWD ( $\Sigma = \frac{1}{(\frac{\omega_{RF}}{\omega} - 1)} = 0.0$  as defined in [1], no beam), red triangles:  $N_b = 4 \cdot 10^{10} (\hat{\Sigma} = 0.1)$ , green squares:  $N_b = 9 \cdot 10^{10} (\hat{\Sigma} = 0.3)$ , blue stars:  $N_b = 13 \cdot 10^{10} (\hat{\Sigma} = 0.5)$  and pink line:  $N_b = 20 \cdot 10^{10} (\hat{\Sigma} = 1.0)$ .

For the distributions in Figure (1- 5) a numerical program was used. Beginning with starting values for the potentials  $Y_{BL}(\Phi)$ ,  $Y_{RF}(\Phi)$  for given impedances and voltages determined by FFT the beam line density  $\lambda(\Phi)$  was

evaluated by the Haissinski equation. Multiplication with impedance Equation 2 top or bottom and inverse FFT leads to the new  $Y_{BL}(\Phi)$  that goes back into the Haissinski equation till convergence is reached.

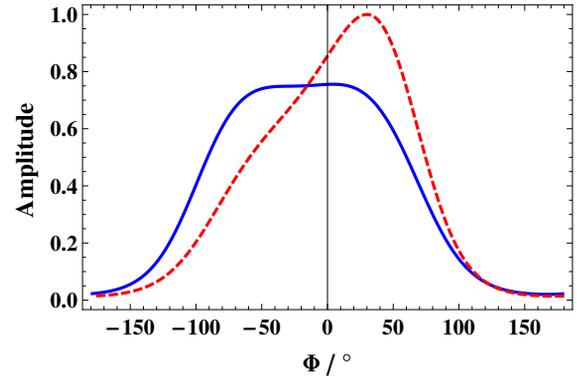


Figure 3: Correction of beam distribution deformation by  $\Delta\Phi_S$  using Equation 2 (top). Red dashed line: bunch form distribution without PWD impedance, blue line: bunch form distribution with PWD after deformation correction.

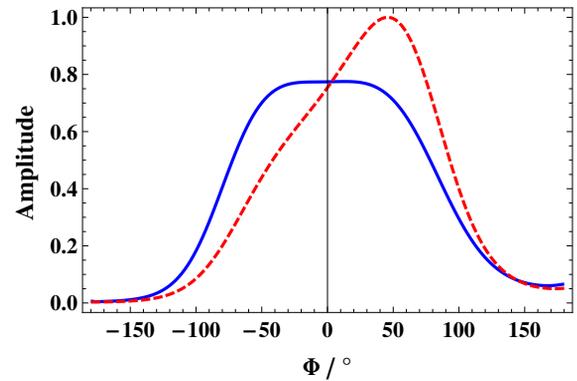


Figure 4: Correction of phase shift by  $\Phi_{S1}$  using Equation 2 (bottom). Red dashed line: bunch form distribution without PWD, blue line: bunch form distribution with PWD after phase shift and deformation correction.

Figure 3 and Figure 4 show an example for the bunch form and phase correction. In Figure 3 using  $\Delta\Phi_S = -0.48$  rad the bunch form deformation is corrected (blue line). The dashed red line shows the bunch form with  $\Delta\Phi_S$  but with  $\Sigma = 0.0$ . In Figure 4 in addition the phase shift was corrected with  $\Phi_{S1} = -0.22$  rad. Comparing the dashed red line of Figure 3 and Figure 4 with the red triangles in Figure 1 and Figure 2 it is obvious that a reflection at the ordinate is necessary for compensation.

For Figure 5 for  $N_b$  and for Figure 6 for  $N_b$  (equivalent to above given  $\Sigma$  with  $X_C$ ) such correction phases  $\Delta\Phi_S$  and  $\Phi_{S1}$  were identified. This has been done for  $Q = 0.45$ ,  $Q = 0.1$ , the measured impedance table of the insert and an impedance table with constant values.  $\Delta\Phi_S$  and  $\Phi_{S1}$  with PWD and with PWD and SC were identified.

To demonstrate the behaviour of the phase correction Fig-

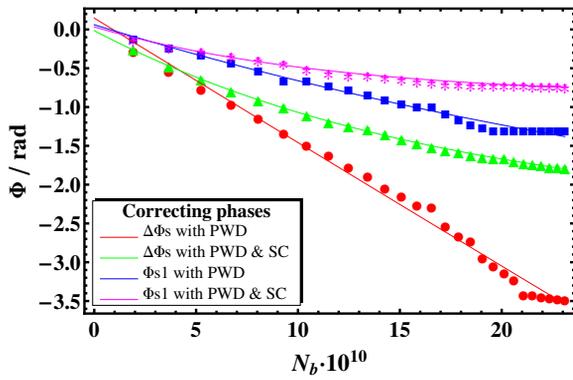


Figure 5: Phase advance over number of ions  $N_b$  for beam distribution deformation- and phase shift correction with  $Q = 0.1$  by using impedance function Equation 2.

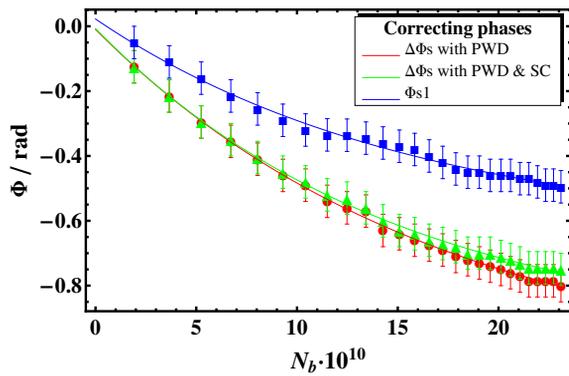


Figure 6: Phase advance over number of ions  $N_b$  for beam distribution deformation- and phase shift correction with measured impedance table.

ure 5 for  $Q = 0.1$  and Figure 6 for the measured insert impedance table has been chosen which is much more broadband. Because of the smaller ordinate value range the error bars in Figure 6 can be seen. These error bars are from the reading error. To make the figures the phase values were variegated till the bunch form was flat and the center of the bunch at the phase  $\Phi = 0$  as is shown with the blue line in Figure 4.

The correction of the beam distribution deformation is SC parameter dependent. To investigate this the phases for different SC parameters  $\Sigma$  using Equation 2 top or bottom were determined. It was found that the correction phases are smaller with SC impedance as can be seen by comparing the blue squares with the pink stars and the red points with the green triangles in Figure 5 and Figure 6. The correction phases are smaller with less higher harmonics and they are smaller the more broadband. For  $Q = 0.45$  it was found that the phase correction was not possible any more. Only the bunch form deformation was. It is important to say that the correction always lead to a RF voltage reduction.

The dependencies of all the phases from  $N_b$  were fitted by Equation 3

$$\Phi(\Sigma) = -a + b \exp[-kN_b] \quad (3)$$

with  $a$ ,  $b$  and  $k$  as the fitting parameters. The fit was done with Mathematica. The results for  $Q = 0.1$  are:

$$\begin{aligned} \Delta\Phi_{S(PWD)} &= -184(1.0 - e^{-0.001N_b}) \\ \Delta\Phi_{S(PWDSC)} &= -(1.0 - e^{-0.06N_b}) \\ \Phi_{S1(PWD)} &= -3(1.0 - e^{-0.02N_b}) \\ \Phi_{S1(PWDSC)} &= -(1.0 - e^{-0.08N_b}) \end{aligned} \quad (4)$$

The results for the measured insert impedance table are:

$$\begin{aligned} \Delta\Phi_{S(PWD)} &= -(1.0 - e^{-0.06N_b}) \\ \Delta\Phi_{S(PWDSC)} &= -(1.0 - e^{-0.07N_b}) \\ \Phi_{S1(PWD)} = \Phi_{S1(PWDSC)} &= -(1.0 - e^{-0.07N_b}) \end{aligned} \quad (5)$$

Here (PWD) labels the values with potential-well distortion and (PWDSC) labels the values with potential-well distortion taking SC impedance into account.

## CONCLUSION

Bunch form deformation and phase shift are correctable below  $Q < 0.45$ . These corrections decrease the RF voltage amplitude as the SC impedance itself. Therefore the necessity for the correction should be avoided which means that the real part impedance has to be as low as possible. SC itself simplifies the corrections but with two RF voltage amplitude decreasing factors, namely SC impedance and deformation- and phase shift correction. Curves for these corrections can be given for control systems but they are SC and quality factor  $Q$  dependent. So both values have to be known to chose the right curve.

For quality factors above about  $Q = 0.4$  it is necessary to find an alternative method like an additional small RF voltage [4] or dual harmonic systems with  $\alpha \neq 0.5$ . This has to be investigated.

## ACKNOWLEDGMENT

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## REFERENCES

- [1] O. Boine-Frankenheim, T. Shukla, "Space charge effects in bunches for different rf wave forms", Phys. Rev. ST Accel. Beams **8**, 034201 (2005)
- [2] O. Boine-Frankenheim, O. Chorniy, "Stability of coherent synchrotron oscillations with space charge", Phys. Rev. ST Accel. Beams **10**, 104202 (2007)
- [3] M. Mehler et al., "A ferrite insert for passive space charge compensation", GSI Scientific Report 2011
- [4] K. Y. Ng, "Physics of Intensity Dependent Beam Instabilities", (World Scientific, Singapore, 2006)