# ANALYSIS OF NUMERICAL NOISE IN PARTICLE-IN-CELL SIMULATIONS OF SINGLE-BUNCH TRANSVERSE INSTABILITIES AND FEEDBACK IN THE CERN SPS*

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## INTRODUCTION

The operation at high energy of the SPS is limited by fast growing transverse instabilities caused by the interaction of the beam with the electron cloud [1]. This effect represents a limitation for future intensity upgrades of the LHC injection complex at CERN. A high-bandwidth feedback control system is currently under study as a possible solution to actively damp e-cloud induced instabilities [2]. We used the multi-particle code WARP to model the bunch interaction with the e-cloud together with an ideal transverse feedback system based on a 5 taps bandpass FIR filter [3]. We analyzed effects of noise and their implication on feedback performances, focusing in particular on statistical noise related to the numerical simulation and introducing a white gaussian noise source in the model. We ran simulations using a fixed set of parameters to evaluate beam dynamics in closed loop (feedback active) analyzing beam vertical instabilities and relative emittance growths, with the purpose to evaluate system requirements of bandwidth, gain and minimum power required to efficiently damp the beam. We report on simulation results and finally we discuss conclusions and future developments of the model.

## FEEDBACK MODEL

The feedback system is characterized by several elements connected together in a loop, as showed in Fig. 1. The pick-up acquires the vertical position of each slice of the bunch with sampling frequency $f_s = 10.435$ GHz. The loop in Fig. 1 includes 'Statistical Noise', representing the effect of modeling the proton beam using a limited number of macroparticles, 'White Gaussian Noise' representing other sources of noise ($e.g.$ from electronics) and a low-pass filter limiting the bandwidth of the signal. The processing channel is an FIR filter that damps the beam vertical displacement while limiting the bandwidth around the nominal fractional tune $[Q_y] = 0.185$ and advancing the phase by 90 degrees at the tune frequency. The output $z_i(k)$ of the FIR is multiplied by a gain $G$ and finally the control signal is applied to the beam by a kicker [4].

The output $z_i(k)$ of the filter is calculated on 5 previous measurements of the bunch vertical displacement $y_i(k)$ as

$$z_i(k) = a_1 y_i(k-1) + a_2 y_i(k-2) + ... + a_n y_i(k-n), \quad (1)$$

* Work supported by the US-DOE and the US-LHC Accelerator Research Program LARP under Contract DE-AC02-05CH11231. Used resources of NERSC and the Lawrencium cluster at LBNL

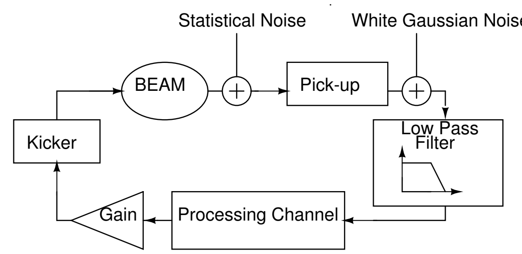


Figure 1: Complete feedback loop.

where $i = 1, \cdots, N_{\text{slices}}$ identifies the bunch slice, $k$ is the machine turn number, $n = 5$ is the # of taps and the set of coefficients $a_1, a_2...a_n$ defines the impulse response of the filter. The receiver and kicker are ideal and have no orbit offset and no longitudinal motion appearing as a vertical signal. The action of the feedback system can be expressed in terms of the following simplified linearized model of bunch dynamics

$$y'' + \omega^2 y = K(y_e - y) + \Delta_{p_\perp}, \quad (2)$$

where $y$ is the amplitude of the vertical oscillation of a slice and $y_e$ the transverse offset of the electron cloud baricenter corresponding to that slice; the constant $K$ is a measure of the interaction beam-ecloud and $\Delta_{p_\perp}$ the signal applied by the kicker. When the feedback is active the vertical displacement of each slice is driven towards zero, $y \simeq 0$, reducing (2) to

$$|K y_e| \simeq |\Delta_{p_\perp}|, \quad (3)$$

suggesting the analysis of $\Delta_{p_\perp}$ will give a measure of $K$.

## SIMULATION RESULTS

All the simulations share the same set of initial parameters reported in Table 1. Single-Bunch runs in closed loop were performed at the SPS injection energy $E_b = 26$ GeV, with 20 stations around the ring and a density of electrons $n_e = 10^{12} m^{-3}$ at each station. The bunch were injected with no transverse offset. The macro-electrons $N_e = 10^6$ are placed in the vacuum chamber on a grid with $256^2$ cells size with a random uniform distribution updated every turn.

The statistical noise intrinsic of the numerical simulation depends on the number of macro-protons $N_i$ present in the bunch and it is inversely proportional to the square root of the number of particles per slice. Since the feedback control loop is based on the estimation of the vertical position

Table 1: WARP Parameters Used in the SPS Simulations

| Parameter | Symbol | Value |
|---|---|---|
| beam energy | $E_b$ | 26 GeV |
| bunch population | $N_b$ | 1.1 x $10^{11}$ |
| rms bunch length | $\sigma_z$ | 0.229 m |
| rms transv. sizes | $\sigma_y$, $\sigma_x$ | 2.6948, 1.849 mm |
| rms transv. emittance | $\epsilon_{x,y}$ | 2.8, 2.8 mm·mrad |
| rms longit. emittance | $\epsilon_z$ | 0.0397 eV·s |
| rms momentum spread | $\delta_{rms}$ | 1.9 x $10^{-3}$ |
| beta functions | $\beta_{x,y}$ | 33.85, 71.87 m |
| betatron tunes | $Q_{x,y}$ | 26.13, 26.185 |
| chromaticities | $Q'_{x,y}$ | 0, 0 |
| mom. compact. factor | $\alpha$ | 1.92 x $10^{-3}$ |
| # of beam slices | $N_{\text{slices}}$ | 64 |

of each slice, this signal must not be dominated by statistical noise in order to study the minimum kick required to damp e-cloud induced transverse instabilities. Several simulations have been run for 1000 turns with gain $G = 0.2$, keeping every other parameter fixed and varying the number of macro-protons $N_i$ in the bunch while forcing the kicker field $\Delta_{p_\perp}$ to saturate at different levels. For each value of $N_i$ it was possible to find a kicker saturation level around which the relative vertical emittance starts growing above 0.1% of its initial value in a 1000 turns span. Figure 2 shows the saturation levels of kick signal momentum $\Delta_{p_\perp}$ around which the beam undergoes instabilities for each value of $N_i$. Increasing Ni decreases the statistical noise experienced by the pick-up, thus raising the signal to noise ratio and resulting in lower saturation values. From Fig. 2 we can observe that the minimum kicker signal required to stabilize the beam converges to the value of $\approx 5 \cdot 10^{-6}$ eV·sec/m and becomes about independent of $N_i$ for $N_i > 2 \cdot 10^7$. As a result, we used this value of $N_i$ in the rest of our simulations. This value corresponds to an rms error on the first and last slices of the pick-up signal of $\sigma_{stnoise} = 124\mu m$, where fewer particles are distributed and noise level is high, and $\sigma_{stnoise} \approx 5.7\mu m$ on +/-2$\sigma_z$.

Another set of simulations has been run with the purpose to investigate the minimum bandwidth required by the feedback system to effectively damp the beam. A low pass filter with linear phase has been used to limit the bandwidth of the pick-up signal, the filter kernel is a sinc windowed with a Blackman window. The frequency response of the filter for different cut-off frequencies is reported in Fig. 3.

Figure 4 shows the rms of the beam vertical position and relative average vertical emittance growth for different cases: for 800 MHz and 1 GHz there is little emittance growth, when a cut-off frequency at 500 MHz is set, both vertical position and emittance show fast growths. Moreover simulations have been run using a low pass filter with 1GHz cut-off, the target frequency chosen for the study of the transverse kicker [5], adding to the pick-up signal a white gaussian noise with $\sigma_{noise}$ equal to a fraction of the nominal $\sigma_y$. It has been chosen $\sigma_{noise}$ as 1%, 2.5%, 5% and 10% of $\sigma_y$, this noise is additional to the statistical one. Simulation results are reported in Fig. 5: the bunch undergoes large vertical motion and emittance growths for $\sigma_{noise} >= 0.025\sigma_y \approx 67\mu m$, while little emittance growth is present in the case of $\sigma_{noise} = 0.01\sigma_y \approx 26.9\mu m$.

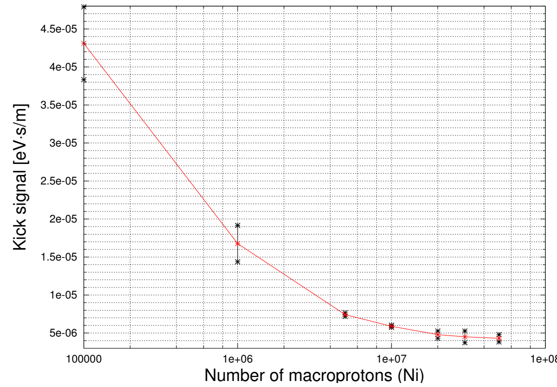


Figure 2: Minimum kicker signal required to stabilize the beam (i.e. for the vertical emittance to grow less than 0.1% over 1000 turns) as a function of $N_i$. The black vertical bars correspond to the error intervals, the amplitude of each interval is related to the number of simulations performed for each $N_i$ value.

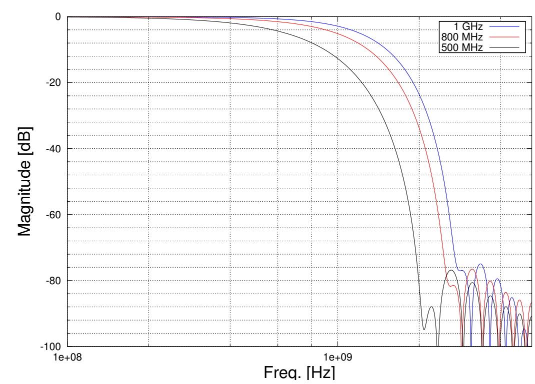


Figure 3: Frequency response of the low pass filter for different cut-off frequencies at -3dB.

All simulations ran so far had a fixed gain $G = 0.2$, which proved to give a constant kick signal and damped motion for long runs [3]. For the efficiency of the feedback system it is fundamental to use an optimal gain in relation to the noise sources introduced in the feedback. Runs for 4000 turns were performed using gain $G = 0.2$, 0.5 and 1, with no bandwidth limitation and no white gaussian noise addition. Figure 6 shows that gain $G = 0.5$ yields lower emittance growth respect to $G = 0.2$ and the rms of the beam vertical position stays flat. However when $G = 1$ is set, the beam eventually starts growing exponentially and the feedback action is triggering vertical instabilities rather than damping them.




ISBN 978-3-95450-115-1

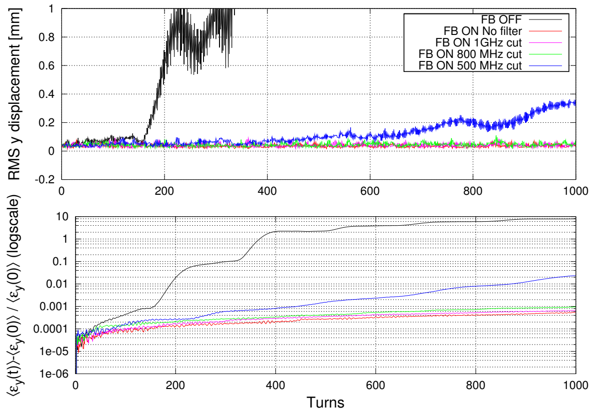


Figure 4: Rms of vertical position (top) and relative average vertical emittance growth (bottom) of the beam in the case of FB OFF (black) and FB ON with no filter (red), with filter cut off at 1GHz (magenta), 800 MHz (green) and 500 MHz (blue).

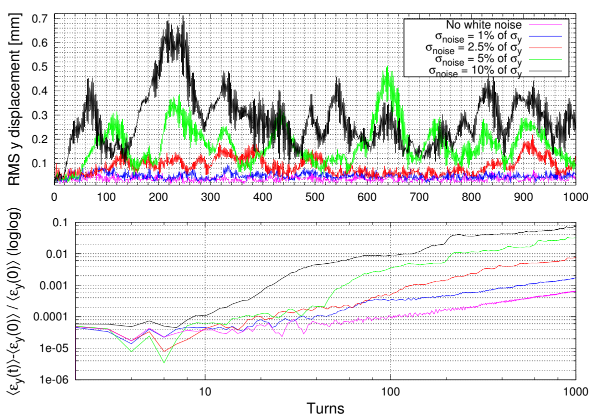


Figure 5: Rms of vertical position (top) and relative average vertical emittance growth (bottom) of the beam in the case of no white noise applied (magenta), $\sigma_{noise} = 0.01\sigma_y$ (blue), $\sigma_{noise} = 0.025\sigma_y$ (red), $\sigma_{noise} = 0.05\sigma_y$ (green) and $\sigma_{noise} = 0.1\sigma_y$ (black).

## CONCLUSIONS

A high bandwidth feedback control system represents a possible solution to damp electron cloud driven instabilities limiting high intensity proton beams in the SPS at CERN. Several simulations has been performed using the macro-particle code WARP including in the model a simple feedback control system. Statistical noise in the numeric simulation depends on the number of macro-protons $N_i$ in the bunch and strongly affects the evaluation of the vertical position of the beam. Simulation results show that for $N_i > 10^7$ the kicker momentum signal saturates at $\approx 5 \cdot 10^{-6}$ eV·sec/m, therefore $N_i = 2 \cdot 10^7$ has been chosen as fixed parameter for all simulations, resulting in a worst case rms error on the pick-up signal of $\sigma_{stnoise} = 124\mu m$ on +/-4$\sigma_z$ and $\sigma_{stnoise} = 5.7\mu m$ on +/-2$\sigma_z$. Moreover little vertical emittance growth was observed in simulations limiting the bandwidth of the pick-up with a low pass filter with 800 MHz and 1GHz cut-off, while for 500 MHz transverse instabilities showed up. A white gaussian noise has been added to the pick-up signal with bandwidth limited at 1GHz and runs with $\sigma_{noise} > 0.01\sigma_y$ were characterized by large emittance growths. Finally simulations using different gains were performed: $G = 0.5$ produced a better performance than $G = 0.2$, while setting $G = 1$ resulted in a feedback action driving the beam unstable. We plan to continue to study bunch dynamics in closed loop for different electron densities, improving the model adding a more realistic noise source similar to the one present in the real SPS pick-ups and a decimation filter to reduce the number of samples. We will carry on simulations using different gains and we look forward to benchmark our results with other multi-particle codes [6].

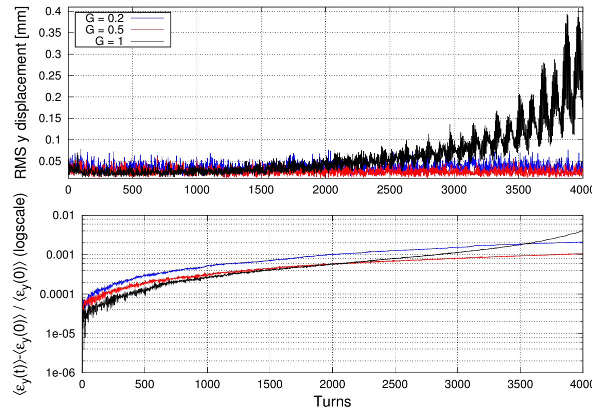


Figure 6: Rms of vertical position (top) and relative average vertical emittance growth (bottom) of the beam in the case of gain $G = 0.2$ (blue), $G = 0.5$ (red) and $G = 1$ (black).

## ACKNOWLEDGEMENTS

We would like to thank M. Furman, M. Placidi, A. Ratti and the CBP of LBNL for the constant support and M. Pivi, J.D. Fox, C.H. Rivetta of the ARD team of SLAC as well as W. Höfle and G. Rumolo of CERN for fruitful discussions.

## REFERENCES

[1] G. Arduini et al., CERN-2005-001, (2005) 31-47
[2] J. D. Fox et al. MOEPPB015, Proc. of IPAC 2012, New Orleans, USA.
[3] R. Secondo et al. Proc. of PAC 2011, New York, NY, USA, 1773-1775.
[4] J.-L. Vay et al., Proc. of IPAC 2010, Kyoto, Japan, 2438-2440.
[5] S. De Santis et al. WEPPP074, Proc. of IPAC 2012, New Orleans, USA.
[6] C. H. Rivetta et al. WEPPP079, Proc. of IPAC 2012, New Orleans, USA.