A ROBUST TRANSVERSE FEEDBACK SYSTEM

M. Alhumaidi* and A.M. Zoubir†, TU Darmstadt, Darmstadt, Germany

Abstract

Transverse feedback systems use pickups signals to measure the beam instabilities and kickers to correct the beam. The correction signal is calculated according to the transfer matrices between the pickups and the kickers. However, errors due to magnetic field imperfections and magnets misalignments lead to deviations in the transfer matrices from their nominal values, which affects the feedback quality in a negative manner. In this work we address a new concept for robust feedback system against optics errors or uncertainties. A kicker and multiple pickups are used for each transversal direction. We introduce perturbation terms to the transfer matrices between the kicker and the pickups. Subsequently, the Extended Kalman Filter is used to estimate the feedback signal and the perturbation terms using the measurements from the pickups. Results for the heavy ions synchrotron SIS 18 at the GSI are shown.

INTRODUCTION

Transversal beam oscillations can occur in synchrotrons directly after injection due to injection errors, which can be in position and angle. Furthermore, higher beam intensities are planed for the FAIR project at the GSI in Darmstadt. This leads to a stronger interaction between the travelling beam and accelerator objects, which increases coherent instabilities. Therefore, beam oscillations will occur when the natural damping becomes not enough to attenuate these oscillations and suppress the potential instabilities.

Beam transversal oscillations lead to emittance blow up caused by the decoherence of the oscillating beam. This decoherence comes from the tune spread of the bam particles. The emittance blow up deteriorates the beam quality since it reduces the luminosity [2, 1]. Therefore, beam oscillations must be mitigated actively by employing a feedback system. The Transversal Feedback System (TFS) senses instabilities or oscillations of the beam by means of Pickups (PUs) and acts back on the beam by means of actuators called Kickers.

In [3], an approach has been applied for the heavy ions synchrotron SIS 18 at the GSI, where the horizontal and vertical beam angles at the place of the Kicker along the accelerator ring get estimated using PUs at two different places for each of the transversal directions. The reason we need PUs at two different places is that only beam displacements from the ideal trajectory but not the angles can be measured by PUs.

The TFS itself can lead to beam quality deterioration through its noise generated at the PUs, especially for lower currents. Correcting the beam with a big noise portion at the output of the feedback system will just lead to beam heating [4]. To reduce noise contribution in the feedback signal a new approach of using multiple PUs for noise minimization has been addressed in [5].

The feedback signal is calculated based on the accelerator optics, i.e., the transfer matrices between the PUs and the Kicker. Thus, any deviations in the optics parameters from the known nominal values lead to disturbances in the calculated feedback signal. Therefore, the beam will be disturbed and gets worse. There are many reasons for optics errors in particle accelerator, e.g., magnetic field imperfections and magnets misalignments. A new concept for robust feedback system against optics errors or uncertainties is addressed in this work. We introduce perturbation terms to the transfer matrices between the PUs and the Kicker for each of the transversal directions. Subsequently, the Extended Kalman Filter is used to estimate the feedback signal and the perturbation terms using the measurements from the PUs.

SYSTEM MODEL

We address a bunch-by-bunch feedback system, which deals with the signals of different bunches as parallel channels. The TFS is copmosed of multiple PUs at different places and one Kicker for each transversal direction. The signals from the PUs, which correspond to the transversal beam displacements, are delayed differently, such that they correspond to the same bunch at every time. The driving signal at the kicker is the output of the digital processing of the delayed PUs signals. Figure 1 shows a block diagram of the TFS.

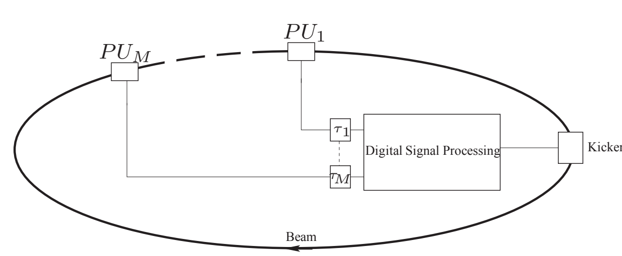

Figure 1: Block diagram of the TFS.

Let $\mathbf{x_P}(n) = [x_1(n), \cdots, x_M(n)]^T$ be the vector of the beam positions (horizontal or vertical) at the M PUs at the n^{th} turn for one of the bunches. Therefore, the measurement vector $\mathbf{x_{MS}}(n)$ will be this positions vector distorted by the noise vector $\mathbf{z}(n) \sim N(\mathbf{0},\ \mathbf{R_{zz}})$ of the PUs, i.e.,

* malhumai@spg.tu-darmstadt.de
† zoubir@spg.tu-darmstadt.de

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$$\begin{aligned}\mathbf{x_{MS}}(n) &= \mathbf{x_P}(n) + [z_1(n), \cdots, z_M(n)]^T \\ &= \mathbf{x_P}(n) + \mathbf{z}(n).\end{aligned} \tag{1}$$

Define

$$\mathbf{x}(n) = [x_{DK}(n),\ x'_{DK}(n)]^T \tag{2}$$

to be the beam status vector at the n^{th} turn, where $x_{DK}(n)$ is the beam position at the Kicker place, and $x'_{DK}(n)$ is the beam angle at this place.

The beam status vector at turn $n+1$ can be found in dependence of the beam status vector at turn n according to the accelerator optics model. It will be the multiplication of the complete turn transfer matrix $\mathbf{M_{KK}}(n)$ by the kicked beam status vector at turn n. Furthermore, small disturbances can be added to this model. Thus,

$$\mathbf{x}(n+1) = \mathbf{M_{KK}}(n) \cdot \mathbf{x}^k(n) + \mathbf{n_p}(n), \tag{3}$$

where

$$\mathbf{x}^k(n) = \mathbf{x}(n) + [0,\ \Delta x'_{DK}(n)]^T, \tag{4}$$

is the kicked beam status vector by $[0,\ \Delta x'_{DK}(n)]^T$, $\Delta x'_{DK}(n)$ denotes the kick at turn n, and $\mathbf{n_p}(n) = [n_{p1}(n),\ n_{p2}(n)]^T \sim N(\mathbf{0},\ \mathbf{R_{nn}})$ denotes the disturbances in the beam position and angle due to nonlinearities and external sources.

The measurement vector $\mathbf{x_{MS}}(n)$ in Eq. (1) can be written in dependence of the beam status vector at the n^{th} turn as follows

$$\mathbf{x_{MS}}(n) = \mathbf{M_{MS}}(n) \cdot \mathbf{x}(n) + \mathbf{z}(n), \tag{5}$$

where the measurement matrix $\mathbf{M_{MS}}(n)$ is given by

$$\mathbf{M_{MS}}(n) = \mathbf{I}_M \otimes [0,\ 1] \cdot \begin{bmatrix} \mathbf{M_{PK1}}(n) \\ \vdots \\ \mathbf{M_{PKM}}(n) \end{bmatrix} \mathbf{x}(n) + \mathbf{z}(n), \tag{6}$$

where $\mathbf{I}_M$ denotes the identity matrix of dimension M, and $\mathbf{M_{PK1}}(n), \cdots, \mathbf{M_{PKM}}(n)$ dentote the transfer matrices from the Kicker to the M PUs respectively.

KALMAN FILTERING

The Kalman Filter is an estimator for the so called linear-quadratic problem, which is the problem of estimating the instantaneous state of a linear dynamic system perturbed by white noise using measurements linearly related to the state but corrupted by white noise. The resulting estimator is statistically optimal with respect to any quadratic function of estimation error [6].

If the accelerator optics are known for every turn, the transfer matrix $\mathbf{M_{KK}}(n)$ and the measurement matrix $\mathbf{M_{MS}}(n)$ will be known exactly. Since the kick in Eq. (4) is the input of the TFS, and therefore known, the beam status vector $\mathbf{x}(n)$ is the only one to be estimated for complete identification of the system. Therefore, the Kalman Filter will be the optimal estimator of the beam status vector under the assumption of white noises, since the system dynamic model in Eq. (3) and the measurement model in Eq. (9) are linear.

However, deviations in the optics from the nominal driven values exist always due to many reasons as mentioned before. This leads to the fact that the complete turn transfer matrix and the measurement matrix are not known exactly. Therefore, complete system identification for more precise estimation of the beam status vector and more effecient feedback requires the estimation of these matrices as well.

Our approach is to consider the matrices as superposition of known parts, which are the nominal values, and unkown parts to be estimated, i.e.,

$$\mathbf{M_{KK}}(n) = \mathbf{M_{KK}^{nom}}(n) + \mathbf{M_{KK}^{P}}(n) \tag{7}$$

$$\mathbf{M_{MS}}(n) = \mathbf{M_{MS}^{nom}}(n) + \mathbf{M_{MS}^{P}}(n). \tag{8}$$

The uncertainty matrices $\mathbf{M_{KK}^{P}}(n)$ and $\mathbf{M_{MS}^{P}}(n)$ will be then considered as part of the system status variables to be estimated. Therefore, the system status vector can be written as

$$\mathbf{X}(n) = [\mathbf{x}(n),\ Vec(\mathbf{M_{KK}^{P}}(n))^T,\ Vec(\mathbf{M_{MS}^{P}}(n))^T]^T, \tag{9}$$

where $Vec(\cdot)$ denotes the vectorization operator of a matrix, which stacks the columns of the matrix into a vector

For constant optics and errors over the time, and therefore constant complete turn transfer matrix and measurement matrix, the observability of the whole system has been shown according to the criterion in [7].

The evolution of the system state vector will be thus according to a nonlinear function, i.e.,

$$\mathbf{X}(n+1) = f(\mathbf{X}(n)) + [0,\ \Delta x'_{DK}(n)]^T + \mathbf{N_P}, \tag{10}$$

where the beam status propagates like in Eq. (3), and the uncertainty matrices propagate without any changes.

The Extended Kalman Filter is a well known approach for solving such a system. It works iteratively as follows [7]:

- Computing the predicted state estimate

$$\hat{\mathbf{X}}^-(n+1) = f(\hat{\mathbf{X}}^+(n)) + [0,\ \Delta x'_{DK}(n)]^T, \tag{11}$$

- Computing the predicted measurement

$$\hat{\mathbf{x}}_{\mathbf{MS}}(n+1) = \underbrace{\mathbf{M_{MS}^{-}}(n+1) \cdot \mathbf{x}^-(n+1)}_{h(\hat{\mathbf{X}}^-(n+1))} \tag{12}$$

- Conditioning the predicted estimate on the measurement

$$\begin{aligned}\hat{\mathbf{X}}^+(n+1) = \hat{\mathbf{X}}^-(n+1) + \mathbf{K}_{(n+1)} \cdot (&\mathbf{x_{MS}}(n+1) \\ &- \hat{\mathbf{x}}_{\mathbf{MS}}(n+1))\end{aligned} \tag{13}$$

The Kalman gain at turn $n+1$ is given by

$$\mathbf{K}_{(n+1)} = \mathbf{P}^{-}_{(n+1)} \cdot \mathbf{H}^{T}_{(n+1)} \cdot [\mathbf{H}_{(n+1)}\mathbf{P}^{-}_{(n+1)}\mathbf{H}^{T}_{(n+1)} + \mathbf{R_{ZZ}}]^{-1}, \tag{14}$$

the a priori covariance matrix can be written as

$$\mathbf{P}^{-}_{(n+1)} = \mathbf{\Phi}_n \mathbf{P}^{+}_{(n)} \mathbf{\Phi}^{T}_{n} + \mathbf{R_{NN}}, \tag{15}$$

and the a posteriori covariance matrix

$$\mathbf{P}^{+}_{(n+1)} = [\mathbf{I} - \mathbf{K}_{(n+1)}\mathbf{H}_{(n+1)}]\mathbf{P}^{-}_{(n+1)}, \tag{16}$$

$\mathbf{\Phi}_n$ and $\mathbf{H}_{(n+1)}$ denote the linear approximation matrices of the status evolution function $f(\mathbf{X}(n))$ and the measurement function $h(\hat{\mathbf{X}}(n+1))$ respectively.

RESULTS

In this section we show simulation results of the above addressed approach for the Synchrotron SIS 18 at the GSI. In the SIS 18 there are 12 beam position PUs for the horizontal and the vertical directions, which are located periodically along the synchrotron ring. There is also one feedback Kicker for each transversal direction.

During acceleration, focusing changes continuously from the so called doublet mode to the triplet mode, which changes the betatron functions during opreation. We show in the following simulation results for the triplet mode in the horizontal direction for using the two closest PUs to the Kicker. The nominal tune for the horizontal direction is $Q_x^{nom} = 4.31$, where we assume an acutal tune of $Q_x = 4.15$. This corresponds to about $60°$ deviation in the tune value. The phase difference between the Kicker and the first PU has a nominal value $\Delta\phi_1 = 103.7°$, and the phase difference between the two PUs is $\Delta\phi = 129.3°$ nominally. The deviation from these nominal phase advances is assumed to be 30%. The nominal and actual values of the twiss parameters for the PUs and the Kicker are shown in Table 1.

Table 1: Twiss Parameters

mode	$\beta_{\mathbf{k}}$	$\beta_{\mathbf{pu}}$	$\alpha_{\mathbf{k}}$	$\alpha_{\mathbf{pu}}$
Nominal	14.06	12.67	-1.36	1.24
Actual	9.84	16.47	-1.63	1.74

A bunch oscillation with initial status vector $\mathbf{x}(0) = [2\,cm,\ 0\,rad]^T$ and disturbances $\mathbf{n_p} \sim N(\mathbf{0},\ \mathbf{R_{nn}})$, where $\mathbf{R_{nn}} = 10^{-7} \cdot diag(0.73,\ 0.028)$, has been generated using the actual optics parameters. The measurements are disturbed by an additive white Gaussian noise with zero mean and standard deviation $\sigma_z = 1\ cm$.

Figure 1 shows the Mean Squared Error (MSE) of beam angle for three estimators. The blue curve corresponds to the MSE of our addressed robust approach by using the Extended Kalman Filter, where the black curve corresponds to the MSE of the vector summation approach addressed in [3] by using the nominal optics parameters as a comparision reference, and the red curve corresponds to the MSE of the approach of applying the linear Kalman Filter by using the nominal values of the optics parameters as well as an other comparision references.

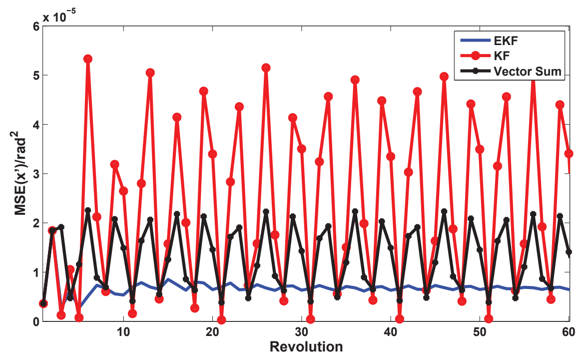

Figure 2: MSE of beam angle estimations.

The results show that the average value over time of the MSE for our addressed robust approach is about one third of the corresponding value for the vector summation approach, and one fifth of the corresponding value for the approach of applying the linear Kalman Filter. This leads to an important enhancement in the beam emittance growth, and therefore the collider luminosity using our approach.

Finally, it is very important to mention that using this new approach leads to enhancements with the price of more computational cost and implementation complexity. The impact of rounding errors on the performance must be investigated as well.

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