CHERENKOV RADIATION FROM A SMALL BUNCH MOVING IN A COLD MAGNETIZED PLASMA*

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Abstract

Investigation of the bunch radiation in plasma is important for the plasma wakefield acceleration (PWFA) technique and other applications in accelerator physics. We study the electromagnetic field of small relativistic bunch moving in a magnetized cold plasma along the magnetic field. The energy loss of the bunch was investigated earlier, however the structure of electromagnetic field was not analyzed. We perform analytical and numerical investigation of total field. Different equivalent representations for the field components are obtained. One of them allows separating quasistatic field and radiation one. Method of computation is developed as well. Some interesting physical effects are described. One of them is strong increase of some components of radiation field near the charge motion line (in the case of point charge). The case of a charged disc is considered as well. Prospects of use of obtained results for PWFA are discussed.

ANALYTICAL DETAILS

The energy loss of a point charge moving in a cold magnetized plasma was investigated half a century ago [1]. Curiously enough, since then no satisfactory analysis of the space-time structure of the electromagnetic field has been done. However, such an analysis is of interest in the context of further development of the plasma wakefield acceleration (PWFA) technique [2].

We suppose that a point charge q moves with constant velocity $\upsilon = \beta c$ (c is the light speed in vacuum) in a medium described by permeability $\mu = 1$ and a permittivity tensor [3]

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon_1 & -i\varepsilon_2 & 0 \\ i\varepsilon_2 & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_3 \end{pmatrix}, \tag{1}$$

where

$$\varepsilon_1 = 1 - \omega_p^2 / (\omega^2 - \omega_h^2), \quad \varepsilon_2 = \omega_p^2 \omega_h / [\omega(\omega^2 - \omega_h^2)], \quad \varepsilon_3 = 1 - \omega_p^2 / \omega^2, \tag{2}$$

$\omega_p^2 = 4\pi N e^2 / m$ is a plasma frequency (N is an electron density, e and m are charge and mass of electron respectively), and $\omega_h = |e| H_{ext} / (mc)$ is a gyrofrequency (H_{ext} is an external magnetic field). The charge moves along the main crystal axis which coincides with the z axis of Cartesian frame, therefore the charge density is

$$\rho = q\delta(x)\delta(y)\delta(z - \upsilon t). \tag{3}$$

Complex function theory methods being applied to the calculation of the field components in the ultra-relativistic case result in the decomposition of the total field into a sum of wave (W) and quasistatic (C) parts:

$$E_z = E_z^W + E_z^C, \tag{4}$$

$$E_z^W = \frac{q\beta \Phi(-\zeta)}{\omega_p^2 \omega_h} \int_{\omega_p}^{\omega_\Sigma} d\omega \frac{\omega\left(\omega^2 - \omega_h^2\right)}{\sqrt{\omega^2 - \omega_c^2}} J_0\left(\rho s_e\right) \times \left(f_1(\omega)\omega^2/c^2 + f_2(\omega) s_e^2 \right) \cos\left(\omega\zeta/\upsilon\right), \tag{5}$$

$$E_z^C = \frac{q\beta \operatorname{sgn}\zeta}{\omega_p^2 \omega_h} \operatorname{Re} \int_{\omega_*}^{\omega_* + i\infty} d\omega \frac{\omega\left(\omega^2 - \omega_h^2\right)}{\sqrt{\omega^2 - \omega_c^2}} \times J_0\left(\rho s_e\right) \left(f_1(\omega)\omega^2/c^2 + f_2(\omega) s_e^2 \right) \exp\left(i\omega\zeta/\upsilon\right), \tag{6}$$

$$f_1(\omega) = \varepsilon_1^2(\omega) - \varepsilon_2^2(\omega) - 2\varepsilon_1(\omega)/\beta^2 + \beta^{-4}, \tag{7}$$

$$f_2(\omega) = \beta^{-2} - \varepsilon_1(\omega), \tag{8}$$

$$s_e^2(\omega) = \upsilon^{-2}\left(\omega^2 - \omega_\Sigma^2\right)^{-1} \times \Big[\left(1-\beta^2\right)\left(-\omega^4 + \omega^2\omega_\Sigma^2 - \omega_p^2\omega_h^2/2\right) + \beta^2\omega_p^4 - \beta^2\omega^2\omega_p^2 + \beta\omega_p^2\omega_h\sqrt{\omega^2 - \omega_c^2} \Big] \tag{9}$$

$$\omega_\Sigma^2 = \omega_p^2 + \omega_h^2, \quad \omega_c^2 = \omega_p^2 - \omega_h^2\left(1-\beta^2\right)^2 / 4\beta^2. \tag{10}$$

$$\omega_* = \omega_h/2 + \sqrt{\omega_h^2/4 - \omega_p^2\beta^2/\left(1-\beta^2\right)}, \tag{11}$$

where J_0 is the Bessel function, $\zeta = z - \upsilon t$, $\rho = \sqrt{x^2 + y^2}$, and $\Phi(-\zeta)$ is Heaviside's step function. The quasistatic part is essential within relatively narrow region near the charge plane where

$$|\zeta| < \zeta_C \equiv c/(\gamma\omega_p), \quad \gamma = 1/\sqrt{1-\beta^2} \gg 1. \tag{12}$$

For $\zeta < 0$, $|\zeta| \gg \zeta_C$ and relatively small ρ the following approximate expressions can be obtained (in the cylindrical frame ρ, φ, z):

$$E_\rho \approx E_{\rho 0}\sin\left(\omega_\Sigma\zeta/\upsilon\right), \quad E_z \approx E_{z0}\cos\left(\omega_\Sigma\zeta/\upsilon\right), \quad H_z \approx H_{z0}\sin\left(\omega_\Sigma\zeta/\upsilon\right), \tag{13}$$

$$E_{\rho 0} = 2q\omega_p^2(\upsilon\omega_\Sigma\rho)^{-1}, \quad E_{z0} = 2q\omega_p^2\upsilon^{-2}\ln\left(\rho\omega_p/c\right), \quad H_{z0} = 2q\omega_p^2\omega_h(\upsilon c\omega_\Sigma)^{-1}\ln\left(\rho\omega_p/c\right). \tag{14}$$

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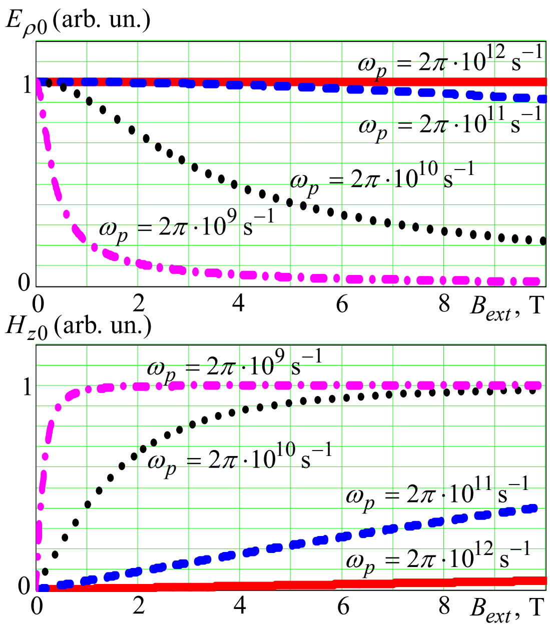

Figure 1. Magnitudes of orthogonal electric and longitudinal magnetic fields versus B_{ext}.

The rest of components (H_φ and $H_\rho = -\beta^{-1} E_\varphi$) vanishes $\sim \rho \ln\left(\rho\omega_p/c\right)$ as $\rho \to 0$.

Cherenkov radiation in plasma is a promising tool for acceleration of charged particles beams in accordance with the wakefield accelerating scheme [2]. Let us note how the external magnetic field affects the field of Cherenkov radiation in plasma. First, as seen from (14), the magnitude E_{z0} of the longitudinal (accelerating) field in the vicinity of charge motion line is independent on B_{ext}, i.e. magnitude of the accelerating field in magnetized plasma coincides with that in plasma without magnetic field. Second, the presence of B_{ext} results in the presence of H_z component which additionally focuses the bunch due to the Lorentz force. Magnitude of H_z can be increased significantly by B_{ext} (see Fig. 1). Third, as seen from Fig. 1, the magnitude $E_{\rho 0}$ of the orthogonal electric field can be reduced essentially by the external magnetic field.

NUMERICAL RESULTS

Along with analytical expressions the numerical algorithm based on the complex function theory methods was developed for the computation of the field components (details of this method as applied to the case of nongyrotropic plasma ($\varepsilon_2 = 0$) can be found in [4-6]).

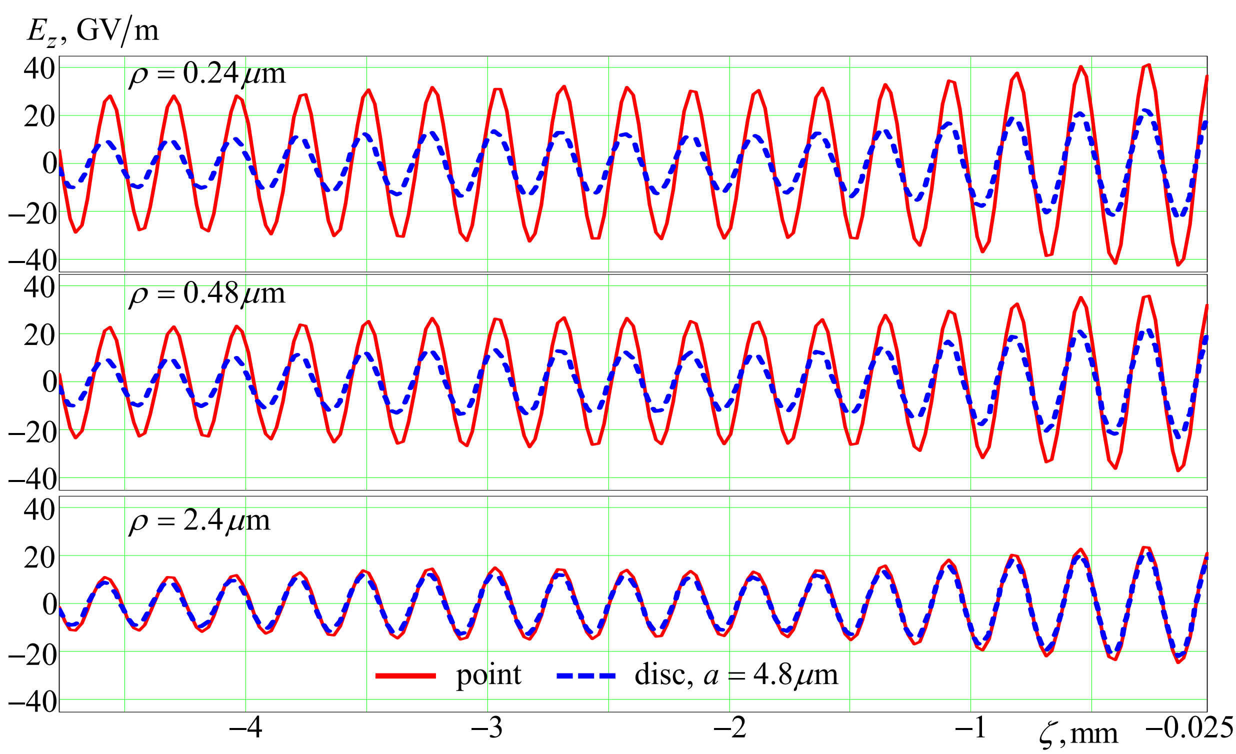

Figure 2. Longitudinal (accelerating) electric field E_z versus ζ for different offsets ρ from the motion line. Calculation parameters are: $\omega_p = 2\pi \cdot 10^{12}\mathrm{s}^{-1}$, $\omega_h = 0.5\omega_p$ ($B_{ext} = 18\,\mathrm{T}$), $\beta = 0.999$, $q = -1\mathrm{nC}$.

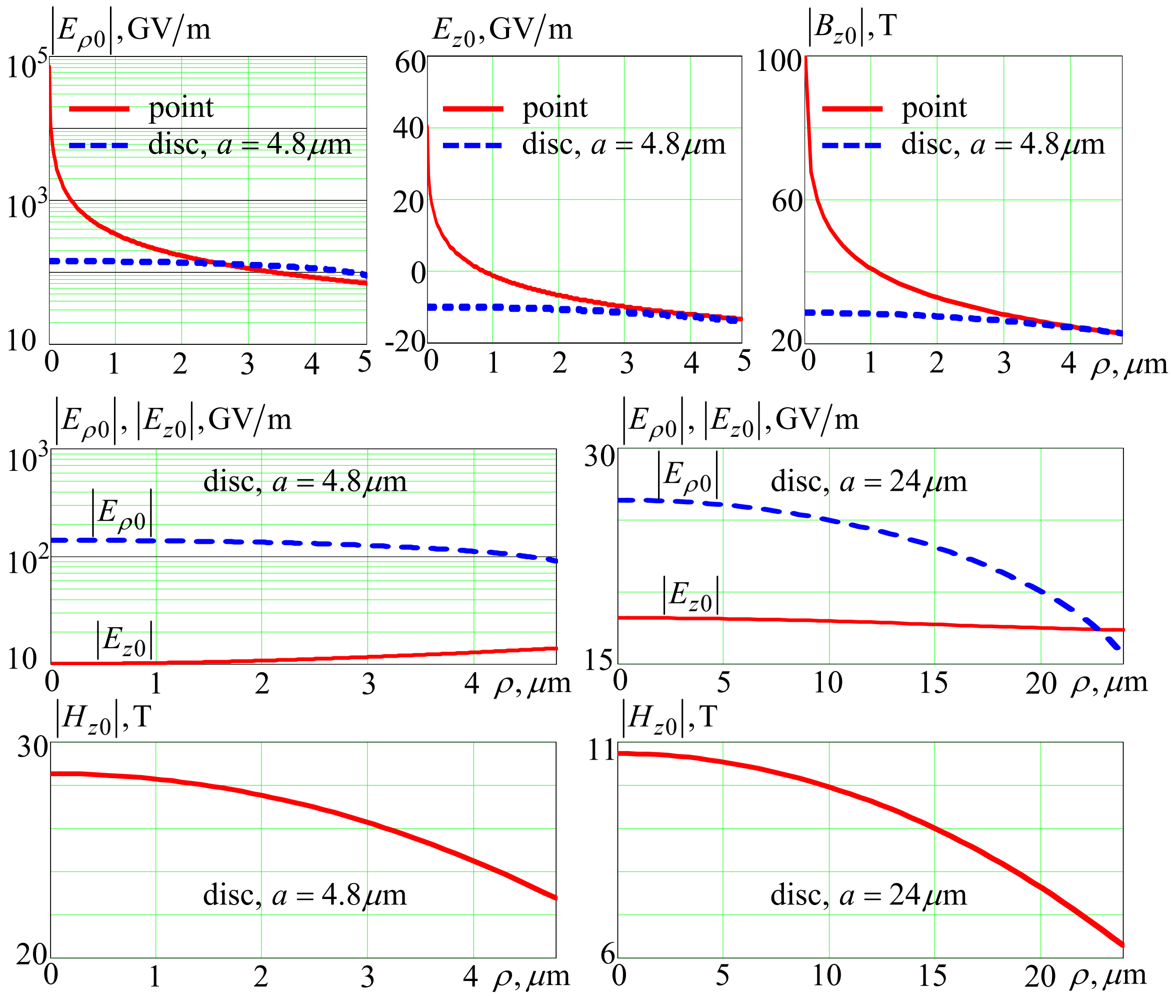

Figure 3. Magnitudes of the longitudinal and orthogonal electric field and longitudinal magnetic field versus ρ. Calculation parameters coincide with those of Fig. 2.

We have also calculated the field produced by uniformly charged infinitely thin disc with radius a:

$$\rho^{\mathrm{d}} = q\Phi\left(a - \sqrt{x^2 + y^2}\right)\delta(z - \upsilon t)/(\pi a^2) . \qquad (15)$$

Fig. 2 illustrates the dependence of the longitudinal (accelerating) electric field on ζ for different offsets from the motion line. For relatively large ρ the field produced by point charge practically coincides with that produced by charged disc. With decrease in ρ the difference becomes essential.

Fig. 3 illustrates the behaviour of the field component magnitudes versus ρ. As seen from Fig. 3, contrary to the field of point charge, field produced by disc remains finite at $\rho \to 0$. Further, for relatively small disc radius ($a = 4.8\,\mu\mathrm{m}$) E_ρ produced by disc is around an order larger compared with E_z, while for five times larger disc ($a = 24\,\mu\mathrm{m}$) these components become comparable. Magnetic field produced by small disc exceeds the external field B_{ext}, while the larger disc produces smaller field compared with the external one.

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