

RADIATION OF A BUNCH INTERSECTING A BOUNDARY BETWEEN VACUUM AND DIELECTRIC IN A CIRCULAR WAVEGUIDE*

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Abstract

Analysis of a field of a particle bunch in a waveguide loaded with a dielectric is important for the wakefield acceleration technique and for other problems in accelerator physics. We investigate the field of the bunch crossing a boundary between two dielectrics in a circular waveguide. We take into account the finite length of the bunch and analyze both the field structure and the energy loss. Special attention is paid to two cases: the bunch flies from vacuum into dielectric and from dielectric into vacuum. In the first case, investigation of formation of stationary wakefield is of interest (this is important for the wakefield acceleration technique). In the second case, quasi monochromatic wave is generated in the vacuum region. This effect can be used for elaboration of a quasi-monochromatic radiation generator of new type. In both cases we also study dynamics of the energy loss of the bunch.

INTRODUCTION

One of the problems being important for the wakefield acceleration technique consists in analysis of effect of the boundary on the wave field when a bunch flies into a dielectric structure. It should be noticed that a charge field structure in a regular waveguide loaded with a dielectric was analyzed frequently (see, for example, [1, 2]). However, the case of an irregular waveguide with two different media has not been examined sufficiently. The field in a semi-infinite waveguide with a metal end and the case of a dielectric cavity bounded by conducting planes was studied in papers [3-5]. But these problems differ essentially from the one considered here because matching of fields in different parts of the waveguide is of great importance. Analogous problem with a boundary between vacuum and cold plasma was investigated in [6]. But Cherenkov radiation (CR) is not generated in such situation; therefore it varies radically from the case under consideration.

Note that energetic characteristics of transition radiation (TR) at a single boundary in a waveguide and in the case of a dielectric plate were investigated in papers [7, 8]. However, the most attention was paid to study the energetic spectrums of generated modes. Dynamics of the energy losses was not analyzed. The field structure of the point charge was partially investigated in our works [9, 10]. Now we take into account the finite length of the bunch.

Let a bunch with a charge q moves in a metal circular waveguide of radius a along its axis (z -axis). The area

$z < 0$ is filled up with a homogeneous isotropic nondispersive dielectric with permittivity ε_1 , and the area $z > 0$ refers to a dielectric with permittivity ε_2 . The bunch moves uniformly with a velocity $\vec{v} = c\beta\vec{e}_z$ and intersects the boundary $z = 0$ at the moment $t = 0$. It is characterized by Gaussian distribution along z -axis and a negligible thickness. The charge density of such beam can be written in the form $\rho = q\delta(x)\delta(y)\exp(-\zeta^2/(2\sigma^2))$, where $\zeta = z - \beta ct$ and σ is a length of the beam.

The analytical solution of the problem is found traditionally as decomposition in an infinite series of normal modes [6, 9]. The finite bunch length σ leads to the fact that the amplitudes of waveguide modes are equal to amplitudes of modes of the point charge multiplied by $\exp[-\omega^2\sigma^2(2\beta^2c^2)^{-1}]$. This exponential factor results in decrease of significance of modes with large numbers.

We investigate the exact solution with analytical and computational methods. Analytical research is an asymptotic investigation with the steepest descent technique. Computations are based on original algorithm using some separation of the integration path and the singularities of the integrands. As distinct from our previous works [9, 10] we do not transform the initial integration contour (a real axis) but displace the singularities from the integration path by taking into account small absorption into the dielectric.

Here we give only some results for two cases: the bunch is flying from vacuum ($\varepsilon_1 = 1$) into dielectric ($\varepsilon_2 > 1$) and from dielectric ($\varepsilon_1 > 1$) into vacuum ($\varepsilon_2 = 1$).

THE CASE OF FLYING FROM VACUUM INTO DIELECTRIC

Next we present the behaviour of the first mode of the longitudinal component E_z of the total field in vacuum and dielectric for different lengths of the bunch. Figures 1, 2 illustrate the case of flying from vacuum $\varepsilon_1 = 1$ into dielectric with ε_2 . As we consider the ultra-relativistic case, the bunch velocity exceeds the Cherenkov threshold $\beta_{C2} = \varepsilon_2^{-1/2}$, Cherenkov radiation is generated in dielectric, and the whole field here is a combination of CR and TR. In the domain $0 < z < z_2 = ct/(\beta\varepsilon_2)$ the bunch wakefield is compensated by some part of TR which is equal to the wakefield taken with an opposite sign (Fig. 1 *a, b, c*). Note that the point z_2 is determined

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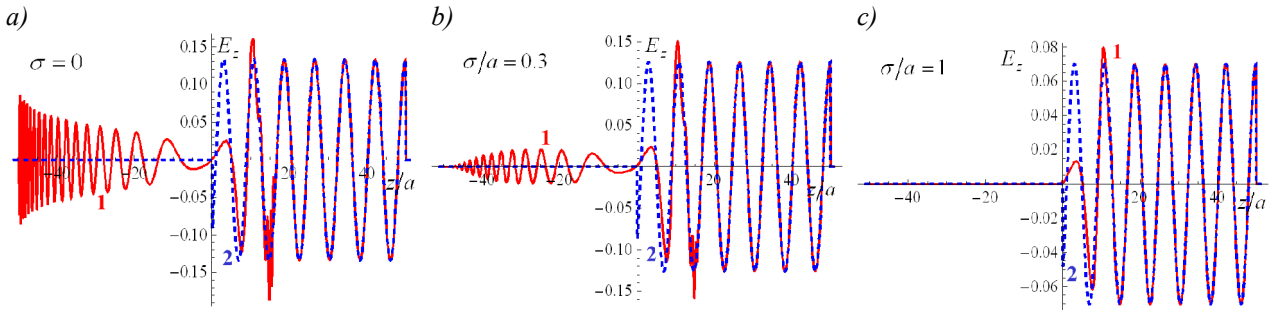


Figure 1: Dependence of longitudinal component E_z (MV/m) of the first mode of the total field (continuous red line 1) and the wakefield mode for an infinite regular waveguide (dashed blue line 2) on distance z/a for different bunch lengths σ ; $q = -1nC$, $\epsilon_1 = 1, \epsilon_2 = 10$, $a = 1cm$, $\gamma = 100$, $ct/a = 50$. Sighting point is on the waveguide axis.

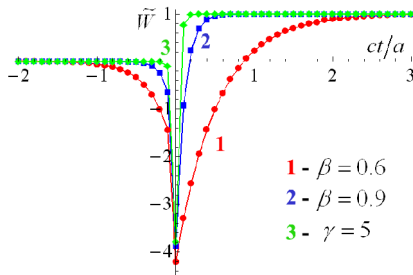


Figure 2: Dependence of the normalized total energy loss for the first mode on dimensionless time ct/a for different bunch velocities β (or $\gamma = (1 - \beta^2)^{-1/2}$); $\sigma = 0$, $\epsilon_1 = 1$, $\epsilon_2 = 10$, $\omega_0 = 2\pi \cdot 10GHz$, $a = 5mm$.

with the group velocity $V_{g2} = c/(\beta\epsilon_2)$ in a regular waveguide. “The compensation domain” is large for dielectrics with relatively small permittivity, and it is reduced with increase in ϵ_2 . As a result, there is the area $z_2 < z < c\beta t$ where the wave field practically coincides with the wakefield generated in an infinite regular waveguide. As well, there is the area $z < z_2$ where the boundary influence is principal.

Behaviour of longitudinal component E_z is demonstrated in Figure 1 for different bunch lengths. One

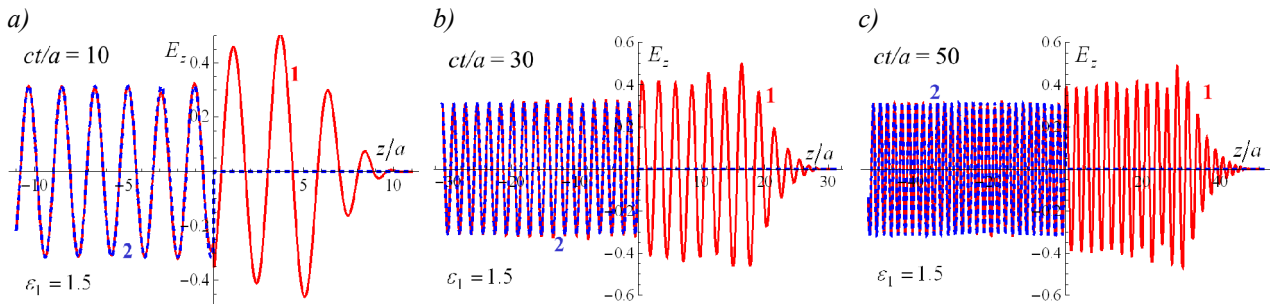


Figure 3: Dependence of longitudinal component E_z (MV/m) of the first mode of the total field (continuous red line 1) and the wakefield mode for an infinite regular waveguide (dashed blue line 2) on distance z/a for different dimensionless times ct/a ; $q = -1nC$, $\epsilon_1 = 1.5, \epsilon_2 = 1$, $a = 1cm$, $\sigma = 0.3cm$, $\gamma = 100$. Sighting point is on the waveguide axis.

can see decrease in the mode amplitude with increase in σ and also the effect of suppression of TR in the vacuum area (Fig. 1c).

The energy radiation loss of the bunch per unit length is defined as

$$W = -q E_z \Big|_{r \rightarrow 0}^{z \rightarrow \beta ct} \quad (1)$$

Dynamics of the total energy loss per unit length for the 1st mode is presented in Fig. 2. We consider the energy loss for the n^{th} mode in dimensionless unities:

$$\tilde{W} = W/W_C, \quad W_C = 2q^2 (a^2 \epsilon J_1^2(\chi_{0n}))^{-1}, \quad (2)$$

where χ_{0n} is the n^{th} zero of the Bessel function ($J_0(\chi_{0n}) = 0$), W_C is an energy loss for Cherenkov mode per the unity of the path [1].

One can see (Fig.2) that the energy loss is negative in some area near the boundary (where the bunch is attracted to the boundary). The dimension of this boundary area decreases with increase in β . Note that the total energy loss for the time interval $(-\infty, t)$ is determined by integral

$$\int_{-\infty}^t W(z) dz. \text{ It is negative up to some moment } t_0 > 0. \text{ It}$$

is getting positive only for $t > t_0$ when the Cherenkov loss is dominant.

THE CASE OF FLYING FROM DIELECTRIC INTO VACUUM

The case of flying from dielectric with ε_1 into vacuum ($\varepsilon_2 = 1$) is presented in Figures 3, 4. The bunch velocity exceeds the Cherenkov threshold $\beta_{C1} = 1/\sqrt{\varepsilon_1}$, so CR emerges in dielectric. Cherenkov radiation is reflected and refracted at the boundary and, as a result, so-called Cherenkov transition radiation (CTR) is generated [9, 11]. In vacuum, CTR exists under conditions

$$\beta_{C1} < \beta < \beta_{CT1} \equiv (\varepsilon_1 - 1)^{-1/2}$$

in the area

$$z < z_1 = ct\sqrt{1 - \beta^2(\varepsilon_1 - 1)}/\beta.$$

The dimension of this zone z_1 increases with group velocity of waveguide waves

$$V_{g1} = c\sqrt{1 - \beta^2(\varepsilon_1 - 1)}/\beta. \quad (3)$$

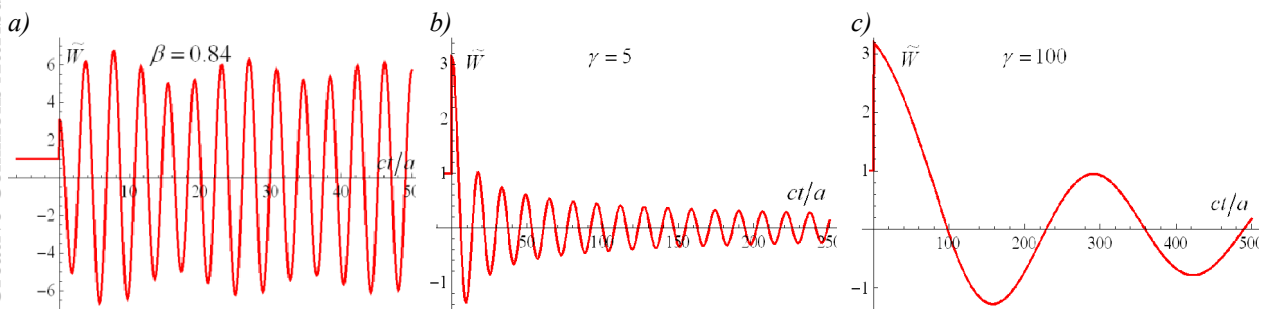


Figure 4: The same as in Fig.2 for $\varepsilon_1 = 1.5$, $\varepsilon_2 = 1$, $\beta_{C1} = 0.81$, $\beta_0 = 0.88$.

So, if $\varepsilon_1 < 2$ CTR emerges for ultra-relativistic bunches with $\beta \approx 1$ (Fig. 3 a,b,c).

If the group velocity (3) is more than the bunch velocity, that is

$$\beta < \beta_0 \equiv \sqrt{\left(\sqrt{(\varepsilon_1 - 1)^2 + 4 - \varepsilon_1 + 1}\right)/2},$$

the bunch interacts with CTR (Fig. 4 a). One can see that the energy loss oscillates with approximately constant amplitude in the vacuum area. The situation changes if $\beta > \beta_0$ (Fig. 4 b,c), when the charge leaves the CTR behind (in this case the charge interacts with TR only). In this case, the oscillation amplitude lessens as $1/\sqrt{t}$, and the oscillation period $T_0 = a\gamma(c\chi_{on})^{-1}$ is getting larger with increase in γ .

CONCLUSION

We have shown that, when the bunch flies from vacuum into dielectric, there is the area where the wave field practically coincides with the wakefield generated in an infinite regular waveguide. This area is large if the dielectric permittivity takes on a large value. The energy loss per unit length is negative in some area near the

border, and the bunch when approaching the boundary is attracted to it. The area where the boundary influence is principal decreases significantly with increase in the velocity of the bunch motion. These conclusions are important for the wakefield acceleration technique.

When the bunch flies from dielectric into vacuum with a certain velocity, a large quasi monochromatic radiation is generated in the vacuum region. This conclusion is of interest for development of a microwave generator of a new type. Study of the total energy loss shows that interaction between the bunch and radiation is especially intensive if the bunch velocity is less than the group velocity of CTR.

Finally note that recently the idea of self-amplified Cherenkov radiation in a waveguide filled with a periodic structure is discussed [12]. The present study can be useful for development of such technique.

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