CREATION OF FELWI USING LARGE AMPLIFICATION REGIME

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Abstract

A threshold condition for amplification without inversion in a Free-Electron Laser Without Inversion (FELWI) is determined. This condition is found to be too severe for the effect to be observed in an earlier suggested scheme because a threshold intensity of the field to be amplified appears to be too high. This indicates that alternative schemes have to be found for making creation of FELWI realistic.

THRESHOLD FOR FELWI

According to the main idea of Ref. [1], a possibility of FELWI realization is strongly related to a deviation of electrons from their original direction of motion owing to interaction with the fields of undulator and co-propagating light wave. The deviation angle appears to be proportional to energy gained or lost by an electron during its passage trough the undulator. Owing to this, a subsequent regrouping of electrons over angles provides regrouping over energies. In principle, a proper installation of magnetic lenses and turning magnets after the first undulator in FELWI can be used in this case for making faster electrons running over a longer trajectory than the slower ones [2]. This is the *negative-dispersion* condition which is necessary for getting *amplification without inversion* [3].

It's clear that the described mechanism can work only if the interaction-induced deviation of electrons (with a characteristic angle α) is larger than the natural angular width α_{beam} of the electron beam,

$$\alpha > \alpha_{\text{beam}}. \tag{1}$$

As the energy gained/lost by electrons in the undulator and the deviation angle are proportional to the field strength amplitude of the light wave to be amplified, the condition (1) determines the threshold light intensity, only above which amplification without inversion can become possible. This threshold intensity is estimated below.

In the non-collinear FEL the electron slow-motion phase is defined as

$$\varphi = qz + \vec{k}\cdot\vec{r} - \omega t, \tag{2}$$

where $q = 2\pi/\lambda_0$ and λ_0 is the undulator period, $\vec{k}$ and ω are the wave vector and frequency of the wave to be amplified, $\left|\vec{k}\right| = \omega/c$, $\vec{r} = \vec{r}(t)$ is the electron position vector and $z = z(t)$ is its projection on the undulator axis. Let the initial electron velocity $\vec{v}_0$ be directed along the undulator axis $0z$. Let the undulator magnetic field $\vec{H}$ be directed along the x-axis. Let the light wave vector $\vec{k}$ be lying in the (xz) plane under an angle θ to the z-axis. Let the electric field strength $\vec{\varepsilon}$ of the wave to be amplified is directed along the y-axis, as well as its vector potential $\vec{A}_{\text{wave}}$ and the undulator vector potential $\vec{A}_{\text{und}}$, where

$$A_{\text{wave}} = \frac{c\varepsilon_0}{\omega}\cos\left(\vec{k}\cdot\vec{r} - \omega t\right), \quad A_{\text{und}} = \frac{H_0}{q}\cos(qz), \tag{3}$$

and ε_0 and H_0 are the amplitudes of the electric component of the light field and of the undulator magnetic field. The described geometry geometry corresponds to that considered in Ref. [1].

The slow motion phase (2) obeys the usual pendulum equation

$$\ddot{\varphi} = -a^2\sin\varphi, \tag{4}$$

where

$$a = \frac{ce\sqrt{\varepsilon_0 H_0}}{E_0}; \tag{5}$$

$E_0 \equiv \gamma mc^2$ is the initial electron energy and γ is the relativistic factor. If L is the undulator length, the ratio L/c is the time it takes for an electron to pass through the undulator. The product of this time by the parameter a of Eq. (5) is known [4] as the saturation parameter μ,

$$\mu = \frac{aL}{c} = \frac{eL\sqrt{\varepsilon_0 H_0}}{E_0}. \tag{6}$$

Amplification in FEL (with $H_0 = const$) is efficient one as long as $\mu \leq 1$. At $\mu > 1$ the FEL gain G falls. The condition $\mu \sim 1$ determines the saturation field $\varepsilon_{0\,\text{sat}}$ and intensity I_{sat}. For example, at $L = 3\,\text{m}$, $H_0 = 10^4\,\text{Oe}$, $\gamma = 10^2$ we have $\varepsilon_{0\,\text{sat}} \sim 1.2\times10^4\,\text{V/cm}$ and $I_{\text{sat}} \sim 2\times10^5\,\text{W/cm}^2$. In our further estimates of the FELWI threshold filed and intensity we'll have to keep in mind that it's hardly reasonable to consider fields stronger than the saturation field $\varepsilon_{0\,\text{sat}}$.

The pendulum equation (4) has the first integral of motion (kinetic + potential energy of a pendulum = *const*).

$$\frac{\dot{\varphi}^2(t)}{2} - a^2\cos[\varphi(t)] = const. \tag{7}$$

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Initial conditions to Eqs. (4) and/or (7) are given by

$$\varphi(0)=\varphi_0, \quad \dot\varphi(0)=\delta\equiv\frac{\omega-\omega_{\rm res}}{2\gamma^2}, \tag{8}$$

where φ_0 is an arbitrary initial phase, δ is the resonance detuning, and $\omega_{\rm res}$ is the resonance frequency for non-collinear FEL given by

$$\omega_{\rm res}=\frac{cq}{1-\frac{v_0}{c}\cos\theta}\approx\frac{2\gamma^2 cq}{1+\gamma^2\theta^2} \tag{9}$$

with $\theta=(\widehat{\vec{k},0z})$.

In the case of a not too long undulator and sufficiently small energy width of the electron beam a characteristic value of the detuning is evaluated as $|\delta|\sim 1/t\sim c/L$.

The rate of change of the electron energy is defined as the work produced by the light field per unit time, and as it's well known [4], this rate is connected directly with the second derivative of the slow-motion phase

$$\frac{dE}{dt}=\frac{E}{2cq}\,\ddot\varphi\approx\frac{E_0}{2cq}\,\ddot\varphi. \tag{10}$$

The last approximate expression is written down in the approximation of a small change of the electron energy, $|E-E_0|\ll E_0$. In this approximation Eq. (10) gives the following expression for the total gained or lost energy of a single electron after a passage trough the undulator

$$\Delta E=E\left(\frac{L}{c}\right)-E_0\approx\frac{E_0}{2cq}\left[\dot\varphi\left(\frac{L}{c}\right)-\delta\right]. \tag{11}$$

In the weak-field approximation ($\mu\ll 1$) one can use the iteration method with respect to the squared parameter a of Eq. (5) for solving Eq. (7). The zero-order solution is evident and very simple: $\dot\varphi^{(0)}\equiv\delta$. In the first order in a^2 one gets

$$\dot\varphi^{(1)}=\frac{a^2}{\delta}\left(\cos(\varphi_0+\delta\cdot t)-\cos(\varphi_0)\right)\sim\frac{a^2L}{c}=\frac{\mu^2 c}{L}. \tag{12}$$

By substituting this expression into Eq. (11) we find the first-order change of the electron energy

$$\Delta E^{(1)}=\frac{E_0}{2cq}\,\dot\varphi^{(1)}\sim\frac{E_0}{2cq}\,\frac{\mu^2 c}{L}=\mu^2E_0\frac{\lambda_0}{4\pi L}. \tag{13}$$

Of course, both $\dot\varphi^{(1)}$ and $\Delta E^{(1)}$ turn zero being averaged over an arbitrary initial phase φ_0. But here we are interested in maximal achievable rather than mean values of these quantities, and these maximal values are given just by estimates of Eqs. (12) and (13).

In accordance with the results of Refs. [1], Eq. (14), and [2], Eq. (13), a transverse velocity v_x and and energy ΔE acquired by an electron after a passage through the undulator are directly proportional to each other

$$v_x=c\,\theta\,\frac{\Delta E}{E_0}, \tag{14}$$

which gives in the first order the following estimate of the electron deviation angle α:

$$\alpha\approx\frac{v_x^{(1)}}{v_0}\approx\frac{v_x^{(1)}}{c}=\theta\,\frac{\Delta E^{(1)}}{E_0}\sim\theta\,\mu^2\,\frac{\lambda_0}{4\pi L}\sim\mu^2\,\frac{d\,\lambda_0}{4\pi L^2}, \tag{15}$$

where d is the electron beam diameter and we took $\theta\sim d/L$.

As said above, in the framework of a linear theory we can consider only such fields at which $\mu\leq 1$. Moreover, consideration of the case $\mu\gg 1$ has no sense at all because the corresponding fields are too strong and because saturation makes the gain too small. For these reasons let us take for estimates maximal value of the saturation parameter μ compatible with the weak-field approximation, $\mu\sim 1$. Let us take also $\lambda_0=3\,$cm, $d=0.3\,$cm, and $L=3\times 10^2\,$cm. Then, we get from Eq. (15) the following estimate of the electron deviation angle

$$\alpha\sim 10^{-6}. \tag{16}$$

CONCLUSIONS

At weaker fields and smaller values of the saturation parameter μ the deviation angle α is even smaller than that given by Eq. (16). But even at $\mu=1$ the angle α is very small. To make the estimate (16) compatible with the condition of Eq. (1) one has to provide the natural electron beam angular divergence smaller than 10^{-6}. Unfortunately, such weakly diverging electron beams hardly exist. For example, the microtron accelerator in Yerevan produces a beam with about 3 or 4 orders of magnitude larger angular divergence, and this can be a rather serious obstacle for attempts of creating FELWI. Hence, creation of FELWI requires invention of alternative schemes in which threshold restrictions would be much weaker than in the considered one.

We find that an FELWI cannot operate under a weak-amplification Thompson regime, for which the spatial amplification is small. Only a large-amplification regime, should be used to build an FELWI. It can be either the anomalous Thompson or the Raman regime of amplification, using an electron beam with overdense current density.

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